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HAMBLIN SMITH'S
ARITHMETIC.

KIRKLAND & SCOTT.

W. J. Gage & Co.'s Mathematical Series.

A TREATISE
ON
ARITHMETIC.

BY

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ADAPTED TO CANADIAN SCHOOLS
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PREFACE.

The present edition of Hamblin Smith's Arithmetic is not simply a reprint of the English work. Several articles have been introduced with a view of making the book more useful and complete. Some of these will be found on pages 33–40, 45, 48, 64, 92–95, 115. From Simple Interest to the end, the work has been almost altogether re-written. In the English edition, the treatment of the subjects in this part of the book was too simple for our Canadian Schools. In this edition, the important subjects of Discount, Stocks and Shares, Exchange, &c., have been treated at greater length than in the ordinary text-books on Arithmetic. Some important articles of a practical business nature have been introduced; amongst these are Equation of Accounts and Partnership Settlements. While the scientific character of the work has been preserved, special care has been taken to adapt it to the wants of the business community.

Examination Papers have been added to each chapter. These have been carefully selected with a view of securing variety and avoiding sameness. They are designed to stimulate the student to think for himself, and to assist him in preparing for the different official examinations.

In order to render the work as complete as possible, an Appendix has been added in which the subjects of Interest, Discount, Equation of Payments, and Annuities have been treated algebraically, and in a manner which, it is hoped, will commend itself to the student. A method of finding the Cube Root by substitution in a simple and easily-remembered formula is also given.

Toronto, 1890.

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CONTENTS.

PURE ARITHMETIC.

Section. *Page.*

I. ON THE METHOD OF REPRESENTING NUMBERS BY FIGURES 1
II. ADDITION 9
III. SUBTRACTION 12
IV. MULTIPLICATION 15
V. DIVISION 22
VI. ON THE RESOLUTION OF NUMBERS INTO FACTORS 27
VII. INEXACT DIVISION 30
VIII. METHODS OF VERIFYING THE OPERATIONS AND SOME PRACTICAL METHODS OF SHORTENING LABOUR IN THE FUNDAMENTAL RULES 33
IX. HIGHEST COMMON FACTOR 43
X. LOWEST COMMON MULTIPLE 46
XI. FRACTIONS 51
XII. DECIMAL FRACTIONS 74
XIII. SQUARE ROOT 101
XIV. CUBE ROOT 108

COMMERCIAL ARITHMETIC.

XV. ON ENGLISH, CANADIAN, AND UNITED STATES CURRENCIES 114
XVI. ON MEASURES OF TIME, LENGTH, SURFACE, SOLIDITY, CAPACITY, AND WEIGHT 129
XVII. FRACTIONAL MEASURES 140
XVIII. DECIMAL MEASURES 142
XIX. PRACTICE 148

COMMERCIAL ARITHMETIC.—*Continued.*

Section.		*Page*
XX.	PROBLEMS	154
XXI.	SIMPLE INTEREST	166
XXII.	COMPOUND INTEREST	174
XXIII.	PRESENT WORTH AND DISCOUNT	178
XXIV.	EQUATION OF PAYMENTS	180
XXV.	AVERAGES AND PERCENTAGES	191
XXVI.	PROFIT AND LOSS	200
XXVII.	STOCKS AND SHARES	204
XXVIII.	DIVISION INTO PROPORTIONAL PARTS	218
XXIX.	ALLIGATION	226
XXX.	EXCHANGE	230
XXXI.	RATIO AND PROPORTION	241
XXXII.	THE METRIC SYSTEM	250
XXXIII.	MEASUREMENT OF AREA	253
	EXAMINATION PAPERS	263
	ANSWERS	294
	APPENDIX I.	326
	APPENDIX II.	346

ARITHMETIC.

I. On the Method of Representing Numbers by Figures.

1. ARITHMETIC is the science which teaches the use of numbers.

2. The number *one*, or *unity*, is taken as the foundation of all numbers, and all other numbers are derived from it by the process of *addition*.

Thus:

Two is the number that results from adding *one* to *one*;
Three is the number that results from adding *one* to *two*;
Four is the number that results from adding *one* to *three*;

and so on.

3. By means of the symbols or figures

1 2 3 4 5 6 7 8 9,

called the NINE SIGNIFICANT DIGITS, together with the symbol or figure 0, called ZERO, we can represent numbers of any magnitude.

4. First, each of the significant digits, standing by itself, represents a number *greater by one* than the number represented by the digit that immediately precedes it in the list of digits.

Thus 7 represents a number greater by *one* than the number represented by 6.

5. The symbol +, read PLUS, is used to denote the operation of ADDITION.

The symbol = stands for the words "is equal to" or "the result is."

Since

$2=1+1$, where unity is written *twice*.
$3=2+1=1+1+1$, where unity is written *three* times.
$4=3+1=1+1+1+1$, where unity is written *four* times;

and so on.

6. Numbers between nine and a hundred are represented by *two* figures, the one on the left-hand signifying how many *groups of ten units* are contained in the number represented, and the one on the right-hand signifying how many single units are contained in the number, in addition to the groups of ten units.

Thus, in the expression 69,

the figure 6 represents six groups of ten units,
the figure 9 represents nine single units.

These groups of ten units are for brevity called *Tens*, and the single units are for brevity called *Units*.

Numbers between ninty-nine and a thousand are represented by *three* figures.

In the expression 745,

the figure 7 represents seven groups of a hundred units,
the figure 4 represents four groups of ten units,
the figure 5 represents five single units.

In the expression 3475,

the figure 3 represents three groups of a *thousnad* units.

In the expression 23475,

the figure 2 represents two groups of *ten thousand* units.

In the expression 123475,

the figure 1 represents one group of a *hundred thousand* units.

In the expression 9123475,

the figure 9 represents nine groups of a *million* units; and so on.

7. To put the matter briefly: when we express a number in figures, and tell off the figures *from right to left*,

the *first* figure represents a number of *units*,
the *second* figure represents a number of *tens*,
the *third* figure represents a number of *hundreds*,

the *fourth* figure represents a number of *thousands*,
the *fifth* figure represents a number of *tens of thousands*,
the *sixth* figure represents a number of *hundreds of thousands*,

the *seventh* figure represents a number of *millions*,
the *eighth* figure represents a number of *tens of millions*,
the *ninth* figure represents a number of *hundreds of millions*,

the *tenth* figure represents a number of *billions*,
the *eleventh* figure represents a number of *tens of billions*,
the *twelfth* figure represents a number of *hundreds of billions*,

the *thirteenth* figure represents a number of *trillions*.

8. When the symbol 0 appears in an expression, it shows that the number, represented by the expression contains no single units, tens, hundreds, etc., according as the 0 is placed in the first, second, third place, the order of place being reckoned from right to left.

Thus:

20 represents the number which contains two groups of ten units and no single units;

300 represents the number which contains three groups of a hundred units, and no group of ten, and no single units;

4007 represents the number which contains four groups of a thousand units, and no group of a hundred, and no group of ten, and seven single units.

NUMERATION.

9. To write in words the meaning of a number expressed in figures, is called NUMERATION.

The remarks, which we have already made, ought to enable the learner to write in words all numbers expressed by ONE, TWO, or THREE figures.

Thus:

the number expressed by 8 is written EIGHT;
the number expressed by 27 is written TWENTY-SEVEN;
the number expressed by 304 is written THREE HUNDRED AND FOUR.

10. Next take the case of numbers expressed by FOUR, FIVE, or SIX figures, as 4237, 23509, 402675.

Draw a line, separating the *three figures on the right* of each expression from the rest of the expression, and over the figure or figures on the left of the line write the word *Thousand*, thus:

Thousand			Thousand			Thousand	
4	237	;	23	509	;	402	675.

Then the meaning of each expression can be written at once in words, thus:

Four thousand two hundred and thirty-seven;
Twenty-three thousand five hundred and nine;
Four hundred and two thousand six hundred and seventy-five.

11. Next take the case of numbers expressed by SEVEN, EIGHT, or NINE figures, as, for instance, the number expressed by 347295328.

Draw a line, separating the three figures on the right from the rest of the expression, and a second line, marking off the next three figures. Over these write the word *Thousand*, and over the figures on the left of this second line the word *Millions*, thus:

Millions	Thousand	
347	295	328.

Then we can write the meaning in words, thus:

Three hundred and forty-seven *millions*,
two hundred and ninety-five *thousand*,
three hundred and twenty-eight.

Again to express in words 20040030, write it thus:

Millions	Thousand	
20	040	030

and the number expressed in words is—

Twenty *millions*
forty *thousand*
and thirty.

12. If more than nine figures are in the given number, mark off the figures by *threes*, as before, and

over the *fourth* parcel write the word *billions*,
over the *fifth* parcel write the word *trillions*.

Thus to express in words 24003269407032, proceed thus:

Trillions	Billions	Millions	Thousand	
24	003	269	407	032

and the number expressed in words is—

Twenty-four *trillions*,
three *billions*,
two hundred and sixty-nine *millions*,
four hundred and seven *thousand*
and thirty-two.

Note. 1 followed by *three* zeros, 1000, represents a thousand.
1 followed by *six* zeros, 1000000, represents a million.
1 followed by *nine* zeros, 1000000000, represents a billion.

Examples. (i.)

Write in words the numbers expressed by the following figures:—

(1) 7, 13, 45, 59, 326, 4578.
(2) 90, 110, 207, 4300, 4036, 4306.
(3) 780, 609, 5360, 2020, 1101.
(4) 36497, 49532, 654321, 743269.
(5) 45000, 32600, 75230, 500000.
(6) 8572914, 3469218, 4629817.
(7) 9000000, 29000000, 715000000.
(8) 910307240, 307004205, 380503040.
(9) 243759268342, 307405006270.
(10) 417235682719435, 203056300072010.

NOTATION.

13. To represent by figures a number expressed in words is called NOTATION.

The method to be employed is this:

Prepare the divisions in which the figures representing *thousand*, *millions*, &c., are to be placed, thus:

Trillions	Billions	Millions	Thousand	

and place in each division, as well on the right and left of the outermost lines, the figures required.

Thus, to represent by figures forty-seven thousand three hundred and nine, we proceed thus:

Thousand	
47	309

and the number expressed in figures is 47309.

Again, to represent by figures four billions three hundred and two millions eighteen thousand and fifty-three, we proceed thus:

Billions	Millions	Thousand	
4	302	018	053

and the number expressed in figures is 4302018053.

Examples. (ii.)

Express in figures the following numbers:—

(1) Nine; twelve; seventeen; nineteen; thirteen; sixteen; eleven.

(2) Twenty-three; twenty-seven; thirty-five; thirty-eight; forty-four; forty; twenty-six; thirty-four.

(3) Sixty-seven; seventy-five; sixty-two; eighty-three; seventy-four; ninety-two; sixty-eight; ninety-five.

(4) Seventy-six; twenty-two; fifty; fifteen; twenty-eight; sixty-one; forty-nine; eighteen; ninety; seventy-three.

(5) One hundred and seven; one hundred and thirty; two hundred and forty-six; three hundred and seventy-two; six hundred and eight; seven hundred and forty; nine hundred and ninety.

(6) Eight hundred and thirty-six; seven hundred and forty-seven; four hundred and ten; nine hundred and thirteen; seven hundred and fifty; three hundred and eighty-four.

(7) Eight hundred and eighteen; eight hundred and eight; two hundred and six; four hundred and thirty; five hundred and twelve; seven hundred and eighty-seven.

(8) Seven thousand eight hundred and forty-five; nine thousand six hundred and thirty-seven; twelve thousand; eight thousand four hundred; six thousand and three; eighty-five thousand and forty.

(9) Five thousand four hundred and seventy; three thousand six hundred and fifty; eight thousand seven hundred and eighty; one thousand two hundred and forty-seven; four thousand eight hundred and eight.

(10) Six thousand and four; seven thousand and twenty-two; three thousand five hundred; nine thousand and forty-seven; two thousand and seventeen; nineteen thousand four hundred and two.

(11) Seventy thousand and seven; sixty thousand and sixty; fourteen thousand and fourteen; seventy thousand and seventeen; twelve thousand three hundred and three; sixteen thousand and five.

(12) Three hundred and fifty-six thousand seven hundred and twenty-eight; six hundred and forty thousand eight hundred and forty-two; nine hundred thousand; eight hundred thousand and forty.

(13) Seven millions; four millions five hundred and seventy-six thousand eight hundred and sixty-five; seventy-five millions eight hundred and six thousand nine hundred and forty.

(14) Three hundred and fifteen millions; five millions and forty thousand; eight millions and seven hundred; eighteen millions and twenty; seven hundred millions and two.

(15) Three hundred and fifteen billions six hundred and seventy-four millions eighteen thousand and three; thirty-five billions six hundred millions, five hundred and twenty.

(16) Seven billions; five trillions eight hundred billions six hundred thousand and forty-seven; eight trillions forty-three thousand and seven.

(17) Three hundred and five trillions five billions four millions six thousand and three; fifty-three trillions fifty-three millions fifty-three thousand and fifty-three.

(18) Nine trillions and nine ; ninety trillions and nine hundred ; nineteen trillions and nineteen thousand ; one trillion one million one thousand one hundred and one.

ROMAN NUMERALS.

14. In the Roman system of Notation, which is still used frequently in inscriptions, in references to chapters of books, and for other purposes, the symbols chiefly employed were I, V, X, L, C, D, M.

These symbols standing by themselves, represented respectively the numbers one, five, ten, fifty, a hundred, five hundred, and a thousand. Intermediate numbers were represented by means of an arrangement that the numbers represented by the symbols I and X when standing on the *right* of a higher symbol were to be *added* to the number represented by that symbol, and when standing on the *left* were to be *subtracted* from it.

Thus :

VI represented the number *six*,
IV represented the number *four*,
and LX represented the number *sixty*,
XL represented the number *forty*.

The following table will explain the method for numbers up to a thousand :

1	I.	11	XI.	21	XXI.	110	CX.
2	II.	12	XII.	30	XXX.	150	CL.
3	III.	13	XIII.	40	XL.	188	CLXXXVIII.
4	IV.	14	XIV.	44	XLIV.	200	CC.
5	V.	15	XV.	50	L.	300	CCC.
6	VI.	16	XVI.	60	LX.	400	CCCC.
7	VII.	17	XVII.	70	LXX.	500	D.
8	VIII.	18	XVIII.	80	LXXX.	600	DC.
9	IX.	19	XIX.	90	XC.	900	DCCCC.
10	X.	20	XX.	100	C.	1000	M.

Examples. (iii.)

Write in words:

(1) XXVII. (2) XLIX. (3) LXVIII.
(4) LXXIII. (5) XCII. (6) CXLIV.
(7) CLXIII. (8) CXCIX. (9) DCLXIV.
(10) MDCCCLXXII.

Write in Roman Numerals:

(1) 37. (2) 59. (3) 62. (4) 87. (5) 95.
(6) 139. (7) 145. (8) 179. (9) 846. (10) 1763.

II. Addition.

15. If we combine two or more groups of units, so as to make one group, the number of units in this single group is called the SUM of the numbers of units in the original groups.

To find the sum of 5 and 3, we reason thus:

$$\begin{aligned} \text{Since } 3&=1+1+1, && \text{(Art. 5)} \\ 5+3&=5+1+1+1 && \\ &=6+1+1 && \text{(Art. 4)} \\ &=7+1 && \\ &=8. && \end{aligned}$$

16. By practice we become able to express the result of adding a number less than ten to another number, without breaking up the number which we have to add, into units.

Thus we say

7 and 5 make 12,
15 and 8 make 23;

and so on.

Again, if we have three or four numbers, each less than ten, to add together, we perform the process mentally; thus, to add 4, 7, 9, and 6 together we say 4, 11, 20, 26.

17. We now proceed to explain the process of addition in the case of higher numbers.

Suppose we have to add together the four numbers 2475, 397, 486, and 3007.

We arrange them thus:

$$\begin{array}{r} 2475 \\ 397 \\ 486 \\ 3007 \\ \hline 6365 \end{array}$$

placing the figures that represent *units* in each number in the same vertical line, and those that represent *tens* in the same vertical line, and similarly for those that represent *hundreds* and *thousands*. We then draw a horizontal line under the last number, and under this line we place the number representing the sum of the given numbers, which is found in the following way:

Adding 7, 6, 7, and 5 units, the sum is twenty-five units, that is 2 tens and 5 units: we place the five under the line of units, and carry on the 2 tens for addition to the line of tens.

Adding 2, 0, 8, 9, and 7 tens, the sum is twenty-six tens, that is two hundreds and 6 tens: we place the 6 under the line of tens, and carry on the 2 hundreds for addition to the line of hundreds.

Adding 2, 0, 4, 3, and 4 hundreds, the sum is thirteen hundreds, that is one thousand and 3 hundreds: we place the 3 under the line of hundreds, and carry on the 1 thousand for addition to the line of thousands.

Adding 1, 3, and 2 thousands, the sum is six thousands, and we place 6 under the line of thousands.

Examples. (iv.)

Add together

(1) 4 and 7, 3 and 13, 5 and 15, 9 and 27.

(2) $\begin{array}{r} 62 \\ 36 \\ \hline \end{array}$ (3) $\begin{array}{r} 40 \\ 27 \\ \hline \end{array}$ (4) $\begin{array}{r} 36 \\ 24 \\ \hline \end{array}$

(5)	237	(6)	209	(7)	562
	349		140		70
	823		600		106

(8)	459	(9)	5462	(10)	24609
	6		723		3470
	237		8004		40052
	4269		9217		6207

(11)	429	(12)	364	(13)	253	(14)	143
	347		629		189		49
	425		488		567		257
	269		976		278		6
	538		853		384		428

(15)	6842	(16)	8750	(17)	8604	(18)	6843
	5679		4623		4007		4297
	8526		7988		5290		326
	5037		6543		3046		52
	2409		5729		7259		7008

(19) $64 + 43 + 7 + 85 + 9$.

(20) $247 + 356 + 28 + 423 + 97 + 12$.

(21) $425 + 3742 + 4236 + 39 + 847$.

(22) $7288 + 976 + 45 + 623 + 4000$.

(23) $8 + 97623 + 3407 + 5260 + 86$.

(24) $41537 + 9215 + 48 + 6077 + 23 + 2413$.

(25) $275413 + 3126 + 725 + 5007$.

(26) $74259 + 346274 + 30000 + 1000001 + 207$.

(27) $4692 + 72430 + 80000729 + 40 + 600000000$.

(28)	46243	(29)	748325	(30)	5629
	35297		54297		426589
	825649		532684		37259
	246728		20047		506
	815		4207		670492
	43376		617043		37987
	645980		3025		6493

(31)	(32)	(33)
256497	654297	625493
648098	248643	75862
720430	380469	5436
630689	472586	87294
407246	582987	4859
864928	639458	862
254384	498468	13

(34)	(35)	(36)
7462594	4697498	6572043
8625837	527	2869257
4398025	4307046	436
6702403	27209	698206
5124917	152372	45297
6219806	4058	3526084
4390143	7265204	57002
7409425	4372943	852968

(37) Seven hundred and forty ; forty thousand and fifteen ; six hundred and forty-seven ; fifty-three thousand three hundred and three ; seventeen thousand five hundred and forty-six.

(38) Five hundred and eight ; six thousand and nine ; fifty-five thousand and fourteen ; eight hundred and nineteen ; seven hundred thousand and six ; two thousand and twelve.

(39) Six hundred and forty-five thousand eight hundred and forty-five ; seventy thousand and forty-seven ; sixty thousand and forty ; seven hundred and fifty thousand ; three hundred thousand and fifteen.

(40) Two hundred and one millions ninety-six thousand three hundred and forty-two ; fifty-four thousand three hundred and four ; eighteen millions six thousand and three ; five hundred thousand and forty ; eight millions and eight.

III. Subtraction.

18. If from a number we take away a smaller number, the process is called *Subtraction*.

Strictly we ought to take away each of the units, of which the smaller number is composed, separately from the larger number : thus, to subtract 3 from 5, we reason thus :

$$3 = 1+1+1,$$

if we take away one of these units from 5, we have 4 left; if we take away the second unit from 4, we have 3 left; if we take away the third unit from 3, we have 2 left.

The Symbol –, read *minus*, is used to denote the operation of Subtraction. Thus the operation of subtracting 3 from 5, and its connection with the result, may be briefly expressed thus:

$$5 - 3 = 2.$$

19. By practice we become able to subtract a number, less than ten, from another number, without breaking up the smaller number into units; thus we say,

$$7 - 4 = 3,$$
$$18 - 5 = 13,$$
$$49 - 8 = 41;$$

and so on.

20. Before we proceed to explain the process of Subtraction in the case of higher numbers, we must notice the principle on which a certain step in the process is founded.

If we are comparing two numbers, with a view to discover the number by which one exceeds the other, we may add ten single units to the greater, if we also add one group of ten units to the less; and we may add ten groups of ten units to the greater, if we also add one group of a hundred units to the less; and so on.

Suppose, for example, we want to find the number by which 56 exceeds 29, we might reason thus:

56 = five tens together with six units.
29 = two tens together with nine units.

To the former add *ten single units*, and to the latter add *one group of ten units*.

Then the resulting numbers will be,

in the first case, five tens together with sixteen units,
in the second case, three tens together with nine units.

Hence the excess of the former over the latter will be the number, made up of two tens together with seven units, and will therefore be represented by 27.

Let us now take an example, to show the *practical* way of performing the operation of subtraction, accompanied by a complete explanation of the process.

Suppose we have to take 589 from 926;

From	926
Take	589
Remainder	337

We arrange the numbers, placing the figures that represent units in each in the same vertical line, and doing the same with those that represent tens and hundreds.

We then reason thus: we cannot take 9 units from 6 units; we therefore add *ten units* to the 6 units, making *sixteen* units, and we take 9 units from the sixteen units, and set down the result, which is 7 units, under the line of units.

Having increased the upper number by ten units, we add, by way of compensation, 1 *ten* to the lower number, changing 8 tens into 9 tens. We proceed thus: we cannot take 9 tens from 2 tens; we therefore add *ten tens* to the 2 tens, making *twelve* tens, and from these we take 9 tens, and set down the result, which is 3 tens, under the line of tens.

Having increased the upper number by *ten tens*, we add, by way of compensation, 1 *hundred* to the lower number, changing 5 hundreds into 6 hundreds.

We then take 6 hundreds from 9 hundreds, and set down the result, which is 3 hundreds, under the line of hundreds.

Examples. (v.)

Find the difference between the following pairs of numbers:

(1) 13 and 6. (2) 15 and 7. (3) 23 and 4.
(4) 3 and 32.

(5) 57	(6) 96	(7) 74	(8) 87	(9) 92
23	42	39	58	47

(10)	313 247	(11)	704 195	(12)	630 548	(13)	7426 3618
(14)	6239 4127	(15)	4729 501	(16)	6258 36	(17)	65472 4001
(18)	357 249	(19)	4625 1846	(20)	72649 43821	(21)	20004 17243

(22) 437 – 56 (23) 529 – 483 (24) 827 – 795
(25) 3000 – 958 (26) 7040 – 583 (27) 6259 – 479
(28) 58623 – 7428 (29) 64295 – 53296 (30) 70000 – 68904
(31) 52764 and 34297. (32) 42456 and 102479.
(33) 624300 and 14000702. (34) 99999 and 100000.
(35) A million and a thousand.
(36) A hundred millions and a hundred thousand.
(37) Ten billions and a thousand and one.
(38) What number must be taken from 26 to leave 18 ?
(39) What number must be taken from 427 to leave 401 ?

(40) What number must be taken from three thousand and fifteen to leave two thousand four hundred and five ?

(41) By how many does a thousand exceed four hundred and seven ?

(42) The greater of two numbers is 427, and the sum of the numbers is 586, what is the smaller of the two numbers ?

(43) What number must be added to 7428 to make 8047 ?

IV. Multiplication.

21. Multiplication is the process by which we find the sum of two, three, four, or more numbers, which are *equal*.

Thus, if we have to find the sum of three numbers each equal to 7, we call the process *the* MULTIPLICATION *of* 7 *by* 3.

This sum is called the PRODUCT of the multiplication of 7 by 3.

The number 3 is called the MULTIPLIER.

The number 7 is called the MULTIPLICAND.

The following table must be committed to memory.

The Multiplication Table.

Twice	Three times	Four times	Five times	Six times	Seven times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2.. 4	2.. 6	2.. 8	2..10	2..12	2..14
3.. 6	3.. 9	3..12	3..15	3..18	3..21
4.. 8	4..12	4..16	4..20	4..24	4..28
5..10	5..15	5..20	5..25	5..30	5..35
6..12	6..18	6..24	6..30	6..36	6..42
7..14	7..21	7..28	7..35	7..42	7..49
8..16	8..24	8..32	8..40	8..48	8..56
9..18	9..27	9..36	9..45	9..54	9..63
10..20	10..30	10..40	10..50	10..60	10..70
11..22	11..33	11..44	11..55	11..66	11..77
12..24	12..36	12..48	12..60	12..72	12..84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 .. 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 ..108
10 .. 80	10 .. 90	10 ..100	10 ..110	10 ..120
11 .. 88	11 .. 99	11 ..110	11 ..121	11 ..132
12 .. 96	12 ..108	12 ..120	12 ..132	12 ..144

22. Let it be carefully observed that Multiplication is a short form of Addition. When we say that 3 times 4 is twelve, we assert that, if 4 and 4 and 4 be *added* together, the result is 12.

Each of the numbers 3 and 4 is called a FACTOR of the product 12.

Again, if we had to find the value of 4 times 67, we might proceed thus:

$$\begin{array}{r}67\\67\\67\\67\\\hline 268\end{array}$$

Now since the figures in each vertical line are the same, we may save ourselves the trouble of addition, by learning, from the Multiplication Table, the numbers that result from adding the same number four times. Then we may write the operation in a shorter form, thus:

$$\begin{array}{r}67\\4\\\hline 268\end{array}$$

The process will stand thus:

Four times 7 is twenty-eight; we set down 8 in the place of units, and carry on two for addition to the line of tens. Four times 6 tens is 24 tens, and adding 2 tens the result is twenty-six tens, that is two hundreds and six tens; we set down 6 in the place of tens, and 2 in the place of hundreds, and the final result is 268.

Here 67 is called the Multiplicand,
4 is called the Multiplier,
268 is called the Product.

23. The symbol ×, placed between two numbers, expresses that the second is multiplied by the first, and the whole operation in the example just given is briefly expressed thus:

$$4 \times 67 = 268.$$

24. Next observe that the multiplier and multiplicand may change places, without altering the value of the product.

Thus $3 \times 4 = 4 \times 3$, or 3 times 4 = 4 times 3.

$$\begin{aligned}\text{For 3 times } 4 &= 4+4+4.\\ &\left.\begin{array}{l}=1+1+1+1\\+\,1+1+1+1\\+\,1+1+1+1\end{array}\right\}\text{I.}\\ \text{And 4 times } 3 &= 3+3+3+3\\ &\left.\begin{array}{l}=1+1+1\\+\,1+1+1\\+\,1+1+1\\+\,1+1+1\end{array}\right\}\text{II.}\end{aligned}$$

Now the results obtained from I. and II. must be the same, for the horizontal columns of the one are identical with the vertical columns of the other.

25. If we multiply a number by 10, the product is obtained by annexing 0 to the number, that is,

$$10 \times 364 = 3640.$$

If we multiply a number by 100, the product is obtained by annexing 00 to the number, that is,

$$100 \times 364 = 36400.$$

So by annexing 000 to a number we multiply it by 1000, and so on.

If we have to multiply a number by 20, we may first multiply it by 2 and then annex 0 to the result, and the final result will be the product required.

Again, if we have to multiply a number by 200, we may first multiply it by 2 and then annex 00 to the result.

The method of expressing the result of multiplications of this kind in practice is as follows :

We multiply 4276 by 700 and 14239 by 6000 thus :

$$\begin{array}{r} 4276 \\ 700 \\ \hline 2993200 \end{array} \qquad\qquad \begin{array}{r} 14239 \\ 6000 \\ \hline 85434000 \end{array}$$

Examples. (vi.)

Find the Product in the following cases of Multiplication :

(1) 3 times 15. (2) 4 times 76. (3) 5 times 98.
(4) 10 times 87. (5) 6 times 114. (6) 7 times 123.
(7) 11 times 843. (8) 12 times 947.

(9) Multiply 25 by 2, 3, 7, 9.
(10) Multiply 127 by 5, 8, 10, 11, 20, 70.
(11) Multiply 2467 by 4, 6, 11, 12, 500, 7000.
(12) Multiply 37429 by 9, 11, 12, 50000, 80000000.

26. Ex. (1). *To multiply* 347 *by* 23.

The form of the operation is:

$$\begin{array}{r} 347\\ 23\\ \hline 1041\\ 694\\ \hline 7981\\ \hline \end{array}$$

The explanation it this:

The multiplier is made up of two parts, 3 and 20; we therefore multiply 347 first by three, and then by 20, and add the two results.

Now $3 \times 347 = 1041$.
and $20 \times 347 = 6940$.

In setting down this second result *we omit the zero*, because it will have no effect on the addition which has to be performed.

Examples. (vii.)

Multiply

(1) 23 by 15.
(2) 37 by 29.
(3) 45 by 36.
(4) 70 by 26.
(5) 125 by 24.
(6) 327 by 42.
(7) 205 by 43.
(8) 307 by 98.
(9) 2684 by 35.
(10) 57296 by 27.
(11) 84293 by 88.
(12) 7629302 by 76.

Ex. (2). *To multiply* 34007 *by* 213.

$$\begin{array}{r} 34007\\ 213\\ \hline 102021\\ 34007\\ 68014\\ \hline 7243491\\ \hline \end{array}$$

Here when we multiply 34007 by 200, the result is 6801400, and *we omit the two zeros* at the end, being careful to put the 4 in the place of hundreds.

Observe that, in all cases, the first figure on the right of each partial product will be in the same vertical line with the figure by which we are multiplying: thus, in the example just given, the 4 in the third product is in the same vertical line with the 2 by which we multiplied.

Ex. (3). *To multiply* 30047 *by* 21009.

$$\begin{array}{r}
30047\\
21009\\
\hline
270423\\
30047\\
60094\\
\hline
631257423\\
\hline
\end{array}$$

Here the first figure on the right of the second product stands in the place of thousands, because we are then multiplying 30047 by 1000.

Ex. (4). *To multiply* 70407 *by* 3700.

$$\begin{array}{r}
70407\\
3700\\
\hline
49284900\\
211221\\
\hline
260505900\\
\hline
\end{array}$$

Examples. (viii.)

Multiply

(1) 326 by 532.
(2) 704 by 176.
(3) 809 by 506.
(4) 917 by 406.
(5) 5376 by 423.
(6) 7846 by 340.
(7) 7859 by 5006
(8) 85639 by 598.
(9) 79802 by 4007.
(10) 30207 by 5060.
(11) 642867 by 90807.
(12) 8637405 by 40300.
(13) 970052 by 40072.
(14) 980740 by 3406.
(15) 9864302 by 300071.
(16) 870120506 by 700403.
(17) 32804070 by 409300.
(18) 742349 by 947.
(19) 578628 by 6205.
(20) 428734 by 8057.
(21) 984236 by 5009.
(22) 872469 by 70043.
(23) 385704 by 36479.
(24) 423796 by 57243.
(25) 620072 by 400205.
(26) 270403 by 502049.
(27) 427964 by 582978.

Ex. (5). *To find the* continued product *of* 14, 8, *and* 70.

Here we first multiply 14 by 8, and then multiply the product by 70, thus:

$$\begin{array}{r}14\\8\\\hline 112\\70\\\hline 7840\end{array}$$

that is, $14 \times 8 \times 70 = 7840$.

Examples. (ix.)

Find the continued product of

(1) 18, 19, and 20. (2) 436, 73, 12, and 5.
(3) 3476, 2300, 70010, and 2003.

27. When a number is multiplied *by itself* once, twice, three times, . . . the resulting products are called the second, third, fourth, . . . POWERS of the number. The process is called *Involution*, and the Power to which the number is raised is expressed by the number of times the number has been employed as a factor in the operation.

The term *square* is usually employed instead of *second* power.

The term *cube* is usually employed instead of *third* power.

Thus, 144 is the square of 12, because $12 \times 12 = 144$.
64 is the cube of 4, because $4 \times 4 \times 4 = 64$.
81 is the fourth power of 3, because $3 \times 3 \times 3 \times 3 = 81$.

Examples. (x.)

Find the squares of

(1) 15. (2) 24. (3) 40. (4) 57.
(5) 69. (6) 72. (7) 87. (8) 100.
(9) 114. (10) 237. (11) 625. (12) 897.
(13) 789.

And the cubes of

(14) 11. (15) 13. (16) 25. (17) 47
(18) 68. (19) 193. (20) 100. (21) 257.
(22) 356. (23) 539. (24) 704. (25) 987.

V. Division.

28. DIVISION is the process by which, when *a product* is given, and we know *one* of the factors, the *other* factor is determined.

The product is, with reference to this process, called the DIVIDEND.

The given factor is called the DIVISOR.

The factor, which has to be found, is called the QUOTIENT.

29. The operation of division is denoted by the sign $\div$.

Thus $12 \div 3$ signifies that 12 is to be divided by 3.

The same operation is denoted by writing the Dividend over the Divisor, with a line drawn between them, thus, $\frac{12}{3}$

In this Chapter we shall treat only of cases in which the Dividend contains the Divisor an *exact* number of times.

30. For small numbers, the Multiplication Table affords the means of solving questions in Division.

For instance, since $12 = 4 \times 3$,
$12 \div 4 = 3$, and $12 \div 3 = 4$;
and since $96 = 12 \times 8$,
$96 \div 12 = 8$, and $96 \div 8 = 12$.

31. When we divide one number by another, we find how many times the latter is contained in the former, and therefore any process by which we can discover how many times one number is contained in another will furnish a rule for division. Such a process is explained by the examples, which we shall now give.

Ex. (1). *Divide* 408 *by* 17.

Since $17 \times 20 = 340$,
and $17 \times 30 = 510$,

it is plain that 17 is contained in 408 more than *twenty* times, and less than *thirty* times.

If then we take away 340 from 408, and find how many times 17 is contained in the number that remains, we shall find how many times, more than *twenty*, the Divisor is contained in the Dividend 408.

Now 408 – 340 = 68, and this number contains 17 just *four* times.

Hence 17 is contained in 408 twenty times, and also four times, that is, the Quotient resulting from the division of 408 by 17 is 24.

This process is represented more briefly thus :

$$\begin{array}{r@{\,}l}
17\,)\,408 & (\,20+4 \\
340 & \\
\hline
68 & \\
68 & \\
\hline
\end{array}$$

Hence 408 ÷ 17 = 24.

And yet more briefly, availing ourselves of the notation by which the *local* value of digits is represented, and we are enabled to omit zeros,

$$\begin{array}{r@{\,}l}
17\,)\,408 & (\,24 \\
34 & \\
\hline
68 & \\
68 & \\
\hline
\end{array}$$

Ex. (2). Suppose we have to divide 89012 by 10 :

$$\begin{array}{lll}
\text{Divisor.} & \text{Dividend.} & \text{Quotient.} \\
17) & 89012 & (5236 \\
 & 85 & \\
 & \overline{40} & \\
 & 34 & \\
 & \overline{61} & \\
 & 51 & \\
 & \overline{102} & \\
 & 102 & \\
 & \overline{} &
\end{array}$$

We first find how often 17 is contained in 89, and as it is contained five times, we set down 5 as the first figure in the quotient, then multiply 17 by 5, and subtract the result 85 from the 89: to the remainder 4 we annex the next figure in the dividend; then as 17 is contained in 40 twice, we set down 2 as the second figure in the quotient, then multiply 17 by 2, and subtract the result 34 from the 40; and proceed by similar steps to the end of the operation.

Ex. (3). Divide 920575 by 23.

```
23) 920575 (40025
    92
    ---
      057
       46
      ---
       115
       115
       ---
```

Here, when we bring down 0, the *third* figure of the dividend, 23 is not contained in it; we therefore set down 0 as the second figure of the quotient, and when we bring down 5, the *fourth* figure of the dividend, 23 is not contained in 5; we therefore set down another 0 as the third figure of the quotient. When we then bring down 7, the next figure of the dividend, 23 is contained in 57 twice, and the operation proceeds easily.

Examples. (xi.)

Divide

(1) 18 by 6.
(2) 27 by 9.
(3) 84 by 7.
(4) 132 by 12.
(5) 182 by 13.
(6) 238 by 17.
(7) 456 by 19.
(8) 3708 by 36.
(9) 3996 by 37.
(10) 6499 by 493.
(11) 431376 by 817.
(12) 976272 by 940.
(13) 19249470 by 342.
(14) 86366784 by 358.
(15) 224009433 by 489.
(16) 4690325214 by 618.
(17) 2880376 by 1369.
(18) 10781526 by 6142.
(19) 98955005667 by 4123.
(20) 4076361 by 2019.
(21) 13312053 by 237.
(22) 505350366 by 89.
(23) 360919856 by 83.
(24) 4600304 by 907.
(25) 218860161 by 689.
(26) 337103025 by 861
(27) 39916424548 by 1001.
(28) 152847420 by 5060.
(29) 26540538445 by 7649.

(30) 1165584398000 by 17072.
(31) 35088008823434 by 74291.
(32) 369187022085112 by 65432.
(33) 837741356152459 by 98989.
(34) 58376823669 by 642867.
(35) 2959990965442 by 9864302.
(36) 261449109180 by 8723694.

32. If any two of the three numbers that form the Divisor, Dividend, and Quotient be given, we can find the third.

For Dividend ÷ Divisor = Quotient.
Dividend ÷ Quotient = Divisor.
Divisor × Quotient = Dividend.

Examples. (xii.)

(1) The Dividend is 1171692, the Divisor 342. Find the Quotient.

(2) The Dividend is 149201, the Quotient 23. Find the Divisor.

(3) The Divisor is 987, the Quotient 64852. Find the Dividend.

SHORT DIVISION.

33. When the Divisor is not greater than 12, the process of division may be greatly abridged.

Suppose we have to divide 92368 by 8.

The operation is set down in the following form:

$$\begin{array}{r|l} 8 & 92368 \\ \hline & 11546 \quad \text{Quotient.} \end{array}$$

The following is the process:

Since 8 is contained *once* in 9, with 1 as remainder, we set down 1 under the 9, and mentally prefix the remainder 1 to the 2, reading the result as 12; then since 8 is contained *once* in 12, with 4 as remainder, we set down one under the 2, and prefix 4 to the 3, reading the result as 43; then since 8 is contained *five times* in 43, with 3 as remainder, we set down 5 under the three, and prefix 3 to the 6, reading the result as 36; then since 8 is contained *four times* in 36, with 4 as remainder, we set down 4 under the 6, and prefix 4 to the 8, reading the result as 48; then since 8 is contained *six times* in 48, with no remainder, we set down 6 under the 8, and our operation is completed.

Next, suppose we have to divide 11042304 by 12.

The operation is set down thus:

12|11042304

920192 Quotient.

The following is the process:

We must take three figures before we obtain a number which contains 12; then we say 12 is contained *nine times* in 110, with 2 to carry on; then 12 is contained *twice* in 24, and there is nothing to carry on; then 12 is *not contained at all* in 2, we therefore set down 0 under the 2, and carry on 2; then 12 is contained in 23 *once*, with 11 to carry on; then 12 is contained in 110 *nine times*, with 2 to carry on; lastly, 12 is contained in 24 *twice* exactly.

Examples. (xiii.)

Divide

(1) 7652 by 2.
(2) 725961 by 3.
(3) 8650232 by 4.
(4) 8749320 by 5.
(5) 7463424 by 6.
(6) 3504221 by 7.
(7) 713406960 by 9.
(8) 4362017 by 11.
(9) 7912464 by 12.
(10) 4000623070905 by 9.
(11) 7642300721 by 11.
(12) 36089882405604 by 12.

Divide each of the following numbers by 2, 3, and 4 separately:

(13) 4263924. (14) 620437548. (15) 27540918264.

Divide each of the following numbers by 5, 8, and 9 separately:

(16) 46528920. (17) 981754200. (18) 234567000.

Divide each of the following numbers by 7, 11, and 12 separately:

(19) 7971348. (20) 29574468. (21) 6736387812.

VI. On the Resolution of Numbers into Factors.

34. We shall discuss in this section an operation, which is the opposite of that which we call Multiplication. In Multiplication we determine the product of two given factors; in the operation, of which we have now to treat, the product is given, and the factors have to be found.

35. For small numbers the factors may be determined by inspection:

thus, the factors of 21 are 3 and 7.
the factors of 55 are 5 and 11.

36. When we have found two factors that make up a product, one or both of these factors may be themselves reducible to simpler factors:

thus 9 and 6 are factors of 54:

and the factors of 9 being 3 and 3,
and the factors of 6 being 2 and 3,

the number 54 can be split up into *four* factors, 2, 3, 3, 3.

37. *Prime* numbers are those which have no exact divisor but themselves and unity:

thus, 2, 3, 5, 7, 11, 13, 17, 19 are Prime Numbers.

Composite numbers are those which can be resolved into factors, each of which is greater than 1:

thus 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 are Composite Numbers.

38. Every composite number can be resolved into factors which are prime numbers, thus:

$$4=2\times2;\ 6=2\times3;\ 8=2\times2\times2;\ 9=3\times3.$$

Hence, in resolving a large number into factors, we divide it by any small prime number by which we know it is exactly divisible, and then divide the quotient by any small prime number by which it is exactly divisible, and proceed in this way till the quotient is 1; then the divisors are the factors required.

Thus, to find the factors of 2520:

$$\begin{array}{r|r} 2 & 2520 \\ \hline 2 & 1260 \\ \hline 2 & 630 \\ \hline 3 & 315 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \\ \hline\hline \end{array}$$

Hence $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$.

In practical arithmetic we seldom require to find *all* the factors of a composite number, but very frequently we want to know whether a number is exactly divisible by *a particular number*.

The student will find it of use to remember the following properties of numbers.

A number is exactly divisible

by 2 when its last figure is 0 or an *even* digit, as 426;
3 when the sum of its digits is divisible by 3, as 579;
4 when its last *two* figures are divisible by 4, as 2364;
8 when its last *three* figures are divisible by 8, as 25256,
5 when its last figure is 0 or 5, as 30 and 135;
9 when the sum of its digits is divisible by 9, as 275265;
10 when its last figure is 0;
11 when the difference between the sum of the digits in the *odd* places (reckoning from the right) and the sum of the digits in the *even* places is either 0 or divisible by 11. Thus 24794 and 829191 are divisible by 11.

Examples. (xv.)

Find whether the following numbers be exactly divisible by 2, 3, 4, 5, 8, 9, 10, or 11.

(1) 117.
(2) 288.
(3) 495.
(4) 1050.
(5) 23472.
(6) 42345.
(7) 27464.
(8) 32495.
(9) 84732.
(10) 6480.
(11) 619182718.

Note.—We have inserted these remarks at this point, because, in attempting to resolve a large number into factors, it is well to know whether the attempt to divide it by 2 or 3, or 5, &c., will be successful.

The student may now, following the instructions given in Art. 38, work another set of Examples.

Examples. (xvi.)

Resolve into *prime* factors:

(1) 18.
(2) 24.
(3) 27.
(4) 32.
(5) 36.
(6) 39.
(7) 42.
(8) 51.
(9) 54.
(10) 57.
(11) 72.
(12) 85.
(13) 91.
(14) 99.
(15) 100.
(16) 105.
(17) 108.
(18) 112.
(19) 132.
(20) 176.
(21) 288.
(22) 432.
(23) 525.
(24) 625.
(25) 729.
(26) 999.
(27) 1296.
(28) 1760.
(29) 5760.

39. The process of Multiplication may often be made shorter when the Multiplier is a composite number, by resolving it into *two or more factors.*

Thus, if we have to multiply 2579825 by 56, we may resolve 56 into the factors 8 and 7, and proceed thus;

$$\begin{array}{r} 2579825 \\ 8 \\ \hline 20638600 \\ 7 \\ \hline 144470200 \\ \hline \end{array}$$

The advantage of this method will be more apparent when we come to multiplication of sums of money, weights, and measures.

Examples. (xvii.)

Multiply, after resolving the multiplier into factors not greater than 12,

(1) 347 by 14.
(2) 423 by 22.
(3) 5462 by 27.
(4) 8497 by 36.
(5) 8573 by 49.
(6) 28472 by 56.
(7) 49273 by 63.
(8) 90728 by 132.
(9) 90725 by 360.
(10) 40207 by 108.
(11) 36729 by 1320.
(12) 704075 by 14400.

40. So also we may often simplify the process of Division, when the Divisor, though greater than 12, can be made up by factors each not greater than 12. For we can divide the Dividend first by one of these factors, and then divide the Quotient by a second factor, and so on.

Suppose we have to divide 47268540 by 45.

Here 45 can be made up of the factors 9 and 5:

$$\begin{array}{r|r} 9 & 47268540 \\ \hline 5 & 5252060 \\ \hline & 1050412 \end{array}$$

Examples. (xviii.)

Apply the process just explained in the division of

(1) 34608 by 14.
(2) 6791040 by 15.
(3) 752364576 by 18.
(4) 1143995886 by 27.
(5) 285216822 by 33.
(6) 2095501072 by 49.
(7) 4157028792 by 56.
(8) 1200130008 by 84.
(9) 22039992 by 108.
(10) 57667632 by 132.
(11) 472634500 by 125.
(12) 565184160 by 720.
(13) 537062400 by 14400.

VII. Inexact Division.

41. Hitherto we have chosen examples, in which the Divisor is contained an *exact* number of times in the Dividend.

Now suppose we have to divide 23 by 7.

Since 3 × 7 = 21, it follows that we can divide 23 units into 3 parcels, each containing 7 units, and when we have done this, 2 units out of the 23 remain over.

In such a case we call 3 the Quotient, and 2 the Remainder.

Again, if we have to divide 72469 by 53, we proceed thus,

```
53) 72469 (1367
    53
    --
    194
    159
    ----
     356
     318
     ----
      389
      371
      ----
       18
       --
```

Hence the Quotient is 1367, and the Remainder 18.

Note.—If we multiply the Quotient by the Divisor, and add the Remainder to the product, the sum must be equal to the Dividend.

Examples. (xix.)

Divide

(1) 3492 by 37. (2) 486296 by 41.
(3) 872968 by 47. (4) 57092 by 65.
(5) 7492736 by 71. (6) 82749325 by 98.
(7) 87467 by 103. (8) 978462 by 409.
(9) 8276253 by 723. (10) 974004562 by 10[illegible].
(11) 48237654 by 4821. (12) 68725642903 by [illegible]371.

42. When we employ, in cases of *inexact* division, the method of Short Division, after breaking up the Divisor into component factors, as in Art. 40, the Remainder will be found by a process now to be explained.

Ex. (1). Divide 43276 by 21.

```
   ⎧3| 43276
21 ⎨ |-------
   ⎩7| 14425 and 1 unit over,
     |-------
        2060 and 5 parcels of 3 units, or 15 units over ;
```

whence the Quotient is 2060, and the Remainder is 15 + 1, or 16.

Ex. (2). Divide 572948 by 125.

$$125\begin{cases}5\,\underline{|\,572948} \\ 5\,\underline{|\,114589} \text{ and 3 units over,} \\ 5\,\underline{|\;\;22917} \text{ and 4 parcels of 5 units, or 20 units over,}\end{cases}$$

4583 and 2 parcels of 25 units, or 50 units over ;

whence the Quotient is 4583, and the Remainder is $50+20+3$, or 73.

Examples. (xx.)

Divide, employing Short Division,

(1) 4153 by 15.
(2) 587595 by 16.
(3) 42813 by 18.
(4) 423672 by 21.
(5) 724972 by 25.
(6) 569024971 by 27.
(7) 2825780 by 33.
(8) 8642396 by 35.
(9) 356599 by 48.
(10) 8274913 by 64.
(11) 230047914 by 77.
(12) 419421 by 99.
(13) 44487 by 105.
(14) 95379 by 189.
(15) 1194477 by 210.

43. In dividing a number by 10, we have merely to mark off the *last* figure, the other figures giving the Quotient, and the figure marked off the Remainder:

thus $2460197 \div 10 = 246019$ with remainder 7.

Again, to divide 42395675 by 20, we might proceed thus:

$$\begin{array}{r|l} 10 & \underline{42395675} \\ 2 & \underline{\;\;4239567} \text{ and 5 units over,} \\ & \;\;2119783 \text{ and 1 parcel of 10 units over,} \end{array}$$

whence the Quotient is 2119783, and Remainder $10+5$, or 15.

But the operation is written more briefly thus:

$$\begin{array}{r|l} 2,0 & \underline{4239567,5} \\ & 2119783 \quad \text{and 15 remainder.} \end{array}$$

Again,

in dividing by 100, we mark off the *last two* figures,
in dividing by 1000, we mark off the *last three* figures,

from Divisor and Dividend, and find the Quotient and Remainder by a similar process.

44. If any *three* of the four numbers that form the Divisor, Dividend, Quotient, and Remainder be given, we can find the *fourth*.

1. Let Divisor, Dividend, and Quotient be given. Multiply the Divisor by the Quotient, subtract the result from the Dividend, and you have the Remainder.

2. Let Divisor, Quotient, and Remainder be given. Multiply the Divisor by the Quotient, add the Remainder to the result, and you have the Dividend.

3. Let Divisor, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Divisor, and you have the Quotient.

4. Let Quotient, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Quotient, and you have the Divisor.

Examples. (xxi.)

(1) The Divisor is 25, the Dividend 4276, the Quotient 171. Find the Remainder.

(2) The Divisor is 342, the Quotient 1381, the Remainder 67. Find the Dividend.

(3) The Divisor is 596, the Dividend 372149, the Remainder 245. Find the Quotient.

(4) The Quotient is 2910, the Dividend 8765237, the Remainder 317. Find the Divisor.

VIII. Methods of Verifying the Operations and some Practical Methods of Shortening Labor in the Fundamental Rules.

45. ADDITION. The usual verification is to add both upwards and downwards, and see if the sums agree. This is generally sufficient. Another method is to draw a horizontal line across the middle of the sum and add it in two separate parts, then find the sum of the two answers, which must agree with the work it is to verify. If it be a very long sum, it may be divided into three parts by two horizontal lines, and the three separate sums found, &c.

46. Subtraction. The correctness of the result in Subtraction may be tested by adding the remainder or difference to the Subtrahend, when the result ought to be the same as the top line or Minuend.

47. Multiplication. The proof of Multiplication by *casting out the nines* depends on the following property of numbers :—

Any number divided by nine will leave the same remainder as the sum of its digits divided by nine.

This will be evident from the following example:

$$\frac{6783}{9}=\frac{6000}{9}+\frac{700}{9}+\frac{80}{9}+\frac{3}{9}$$

$$=(666+\tfrac{6}{9})+(77+\tfrac{7}{9})+(8+\tfrac{8}{9}+\tfrac{3}{9})$$

$$=666+77+8+\frac{6}{9}+\frac{7}{9}+\frac{8}{9}+\frac{3}{9}$$

$$=751+\frac{6+7+8+3}{9}$$

Hence it is clearly seen that the remainder, arising from the division of 6783 by 9, is the same as that arising from the division of the sum of the digits by 9.

This test may be given in the form of the following rule:

Divide the sum of the digits in the Multiplicand by 9, and set down the remainder. Divide the sum of the digits in the Multiplier by 9, and set down the remainder. Multiply the two remainders together, divide the result by 9, and set down the remainder. If the process be correct, this remainder will be the same as the remainder obtained by taking the sum of the digits in the Product and dividing it by 9.

For example, if we multiply 76371 by 854 the product is 65220834.

Sum of digits in Multiplicand = 24,
and 24 ÷ 9 gives remainder 6.
Sum of digits in Multiplier = 17,
and 17 ÷ 9 gives remainder 8.
First remainder × second remainder = 48,
and 48 ÷ 9 gives remainder 3.
Sum of digits in the Product = 30,
and 30 ÷ 9 gives remainder 3.

This so-called proof is defective as a proof in the following, as it fails to detect errors in the product—

1. If the order of figures in the product be misplaced, as 37 for 73.

2. If errors be made which counterbalance each other, as 35 written for 62, the sum of digits in each case being the same.

3. If 9 be written for 0, or 0 for 9, or either be omitted or inserted too often.

48. Division.—To prove Division, multiply the divisor by the quotient, and add the remainder, if there is one, to the product. If the result is equal to the dividend, we have a verification of the first operation. Division may also be proved by casting out the nines, but the proof is less direct than in Multiplication. For instance, if we divided 417 by 29, the quotient is 14 with remainder 11. The most convenient form in which to apply the proof of nines is to write this in the form of 29 × 14 + 11 = 417. The remainder gives 2 × 5 + 2, or 12. This remainder and the dividend, 417, divided by 9, give a remainder 3, which therefore proves the work.

49. Arithmetical Complement.—The arithmetical complement of a number is defined to be the difference between any given number and the unit of the next superior order; thus 6 is the arithmetical complement of 4, 47 of 53, 8468 of 1532, and so on, being the differences respectively of 4, 53, 1532, and 10, 100, 10000, the next superior units of these numbers. Conversely, also, 4, 53, 1532 are the arithmetical complements of 6, 47, 8468 respectively.

The arithmetical complement of a number may be found by the following rule:—

Begin at the left hand and subtract every figure from 9 until the last; subtract that from 10.

The arithmetical complement may be used to find the difference between two numbers, thus: if 239 be subtracted from 576 the remainder is 337. But if 761, the arithmetical complement of 239, the less number, be added to 576, the greater, the sum will be 1337, one unit (1000 in this case) of the next superior order greater than the difference of the two numbers. By removing this unit, the number will be left equal to the difference of 239 and 576; so that the difference of the two numbers can be found by addition. The arithmetical complement may be written thus $\bar{1}761$, with the subtractive unit on the left, which when added to 576, the sum will be 337, the additive and subtractive units being together equal to zero.

This method is employed with great advantage to find the aggregate of several numbers when some of them are additive and some subtractive. Thus, if we have—

$$3795 - 1532 - 2019 + 8759 - 5104,$$

we arrange them as follows:—

	3795
A. C. of 1532 is	$\bar{1}8468$
" 2019 "	$\bar{1}7981$
	8759
" 5104 "	$\bar{1}4896$
	3899

the aggregate required.

50. Contractions in Multiplication.—The multiplication by any number from 12 to 19 inclusive, may be effected as follows:

Multiply by the figure of the Multiplier in the units' place, and to the number to be carried add the figure of the Multiplicand just multiplied.

Ex. 1. Multiply 2384 by 19.

$$\begin{array}{r} 2384 \\ 19 \\ \hline 45296 \end{array}$$

4 × 9 = 36 ; set down 6 and carry 3.

8 × 9 + 3 carried + 4, the units' figure of the multiplicand = 79 ; set down 9 and carry 7.

3 × 9 + 7 carried + 8, the tens' figure of the multiplicand = 42 ; set down 2 and carry 4.

2 × 9 + 4 carried + 3, the hundreds' figure of the multiplicand = 25 ; set down 5 and carry 2.

2 carried + 2, the thousands' figure of the multiplicand = 4 ; set down 4.

The *back figure system*, as it is sometimes called, may be extended to numbers between 20 and 30, and between 30 and 40, by adding to the number to be carried the *double* or the *treble* of the figure of the multiplicand just multiplied.

Ex. 2. Multiply 34578 by 999.

Here 34578000 = 1000 times 34578.
and 34578 = 1 " "

34543422 = 999 times 34578.

Ex. 3. Find the product of 34578 by 699.

Here 699 = 700 − 1
and 24204600 = 700 times 34578.
34578 = 1 " "

24170022 = 699 times 34578.

Hence, any number can be multiplied by 99, 999, 9999, &c., by annexing 2, 3, 4, &c., ciphers to the multiplicand, and subtracting the multiplicand from this product. And in a similar way any number can be multiplied by another composed of a repetition of the figure with any other figure in the highest place.

Ex. 4. Multiply 9643287 by 378427.

9643287
(378)(42)(7)

7 times the multiplicand = 67503009

42 times the multiplicand = 6 times 7 times multiplicand = 6 times 67503009 = 405018054

378 times the multiplicand = 9 times 42 times the multiplicand = 9 times 405018054 = 3645162486

3649280169549

To Square any Number ending in 5.

Square the 5 and write down the result; then increase the number to the left of 5 by 1, and multiply this sum by the number to which the 1 was added. Set this product to the left of the 25 and the number thus formed will be the result required.

Ex. Find the square of 75.

5 squared = 25.

Add 1 to 7 and multiply by 7 and place the 56 to the left of the 25. 5625 is the result required.

51. Abbreviations in Division.—Since 4 × 25 is 100, and 8 × 125 is 1000, the division by 25 will be effected by multiplying the dividend by 4, and cutting off the last two figures from the product. The division by 125 will be effected by multiplying the dividend by 8, and cutting off the last three figures from the product. In each case the figures cut off, when divided respectively by 4 or by 8, will be the remainder, and those left will be the quotient.

Any number can be divided by 9, 99, 999, &c., by successively dividing the given number by 10, 100, 1000, &c., respectively, and taking the sum of the successive remainders for the true remainder; except when the sum of the latter exceeds the next higher unit; in that case both the quotient and remainder must be increased by unity.

Ex. Divide 65874 by 99.

$$\begin{array}{r} 100\ )\ 658,74 \\ 6,58 \\ 6 \\ \hline 665,39 \end{array}$$

Here the sum of the partial remainder is 138, and both the quotient and remainder must be increased by unity. The reason of this we leave as an exercise for the student.

There is a method of dividing one number by another, termed the Italian method, which materially shortens the process. In this method all the partial subtrahends are omitted, and only the partial remainders retained in the working.

Ex. Divide 108419716121 by 5783.

$$\begin{array}{l} 5783\ )\ 108419716121\ (\ 18748005 \\ \hline \quad\ \ 50589 \\ \hline \quad\ \ \ \ 43257 \\ \hline \quad\ \ \ \ \ \ 27761 \\ \hline \quad\ \ \ \ \ \ \ \ 46296 \\ \hline \quad\ \ \ \ \ \ \ \ \ \ \ \ 32121 \\ \hline \quad\ \ \ \ \ \ \ \ \ \ \ \ \ \ 3206 \text{ final remainder.} \end{array}$$

The first step is simply subtraction, giving 5058 for remainder. The work of the next step is as follows: 8 times 3 is 24; 4 from 9 gives 5 (which put down) and carry 2. 8 times 8 and 2 gives 66; 6 from 8 gives 2 (which put down) and carry 6. 8 times 7 and 6 gives 62; 2 from 5 gives 3 (put down) and carry 6. 8 times 5 and 6 gives 46; 46 from 50 gives 4 (put down).

It sometimes happens that *one* has also to be carried from the subtraction. For instance in this case—

$$\begin{array}{l} 5783\ )\ 50581\ (\ 8 \\ \hline \qquad\quad\ 4317 \end{array}$$

We say: 8 times 3 is 24; 4 from 11 gives 7 (put down) and carry 3 (instead of 2). Then 8 times 8 and 3 gives 67; 7 from 8 gives 1 (put down) and carry 6, &c.

Examination Papers.

I.

(1) Express in words, 4237496; and in figures, six hundred and fifty-three thousand eight hundred and twelve.

(2) Find the sum of 24753, 86729, 4237, and 80462.

(3) Find the difference between 86293 and 78464.

(4) Multiply 8627 by 493, and 50042 by 307.

(5) Divide 8423793 by 9, and 2659582 by 358.

II.

(1) Write in figures, twenty-five millions two hundred and fifty-seven thousand six hundred and thirty; and in words, 402050407.

(2) From seventeen millions and seventeen take eight thousand and eight.

(3) Multiply 6549 by 4037, and 27004 by 3700.

(4) Divide 32456789 by 96, first by Long Division and then by Short Division, and show that the results agree.

(5) Find the sum of one million and six, fifteen thousand and eleven, one hundred thousand and ten, and sixty thousand four hundred; and divide the result by 9.

III.

(1) Write in words, 10010201401; and in figures, one million twenty-three thousand and one. Add together the two numbers, and from the sum subtract their difference.

(2) Multiply 740296 by 2089, and 426004 by 3704.

(3) Divide 78297426 by 35, employing Short Division.

(4) From one hundred and twenty-six millions four hundred and six thousand and three take ninety-five millions and four.

(5) Divide the product of 723 and 347 by 48.

IV.

(1) Express in figures the number represented by MDCCCLXXXVIII.

(2) Divide 987654321 by 132, using Short Division.

(3) Reduce to prime factors 56, 78, and 114.

(4) Multiply the sum of 86297 and 40025 by the difference between 789 and 694.

(5) By how many does one million exceed one hundred and one?

V.

(1) Divide three hundred and fifty-three billions eight millions nine hundred and seventy-two thousand six hundred and two by 5406.

(2) Multiply 8976589 by 9876.

(3) Resolve into elementary factors (*i.e.*, prime numbers) 40, 90, and 126.

(4) Express in Roman Notation 24, 47, and 178.

(5) How many bricks may be taken away in 24 carts, each taking 500 bricks?

VI.

(1) Explain the method for the multiplication of two numbers, each consisting of several figures, and multiply 30071 by 20590, explaining the reason for each step of the process.

(2) Multiply 76894754 by 112756 in three lines of partial products.

(3) By what number must the product of the sum and difference of 8376 and 5684 be increased so that the result may be exactly divisible by 7859?

(4) A drover bought 527 sheep at $2 per head; twice as many calves at thrice as much per head, 19 cows at $29 per head, and thrice as many horses as cows at four times as much apiece. How much did the whole drove cost him?

(5) One-half the sum of two numbers is 4331, and one-half their difference is 3353. Find the numbers.

VII.

(1) Eight head of cattle at $23 each, and 7 horses at $89 each, were given for 3 acres of land. What was the land worth per acre?

(2) If 18 men can reap a field in 76 days, how long will it take 19 men to reap the same field?

(3) A man bought an equal number of sheep and cows for $6300. Each sheep cost $3.50, and each cow $21.50. How many of each did he buy?

(4) It was found that after 789 had been subtracted 375 times from a certain number that the remainder was 562. Find the number.

(5) The ages of three brothers are 19, 17, and 15 years, and their father wills them his property worth $35,700 according to their ages. What does each get?

VIII.

(1) There is a number which, when divided by 4, and the quotient diminished by 35^2 and the result multiplied by 10, and the product decreased by the difference between the arithmetical complements of 7846 and 3479 gives 883. Find the number.

(2) If 5 lbs. of tea are worth 15 lbs. of coffee, and 4 lbs. of coffee are worth 8 lbs. of sugar, how many pounds of sugar are worth 75 lbs. of tea?

(3) Find the number from which if 13675 be taken the remainder will be 45209 less 27645.

(4) A horse is worth 8 times as much as a saddle, and both together are worth $261. Find the value of the horse.

(5) A dealer in cattle gave $6400 for a certain number, and sold a part of them for $3600 at $18 each, and by so doing lost $2 per head. For how much a head must he sell the remainder to gain $800 on the whole?

IX.

(1) Any number may be multiplied by 5, 25, 125, &c., by annexing 1, 2, 3, &c., ciphers respectively to the number, and then dividing it by 2, 4, 8, &c. Explain the reason of this rule.

(2) Of what number is 99995 both divisor and quotient?

(3) A person bequeathed his property to his 3 sons. To the youngest he gave $1789; to the second 5 times as much as to the youngest; and to the eldest 3 times as much as to the second; find the value of the property.

(4) In walking a certain distance John takes 17694 steps; how many steps will James take in walking half the distance, John taking 3 steps for every four of James's?

(5) A merchant failed and his goods were worth $7770. Out of this he can pay his creditors 37 cents on the dollar. One of his creditors got $1998 as his share. Find the merchant's indebtedness, and what he owed the one creditor.

X.

(1) In the multiplication of numbers, how do you prove the correctness of the operation by casting out the nines? Explain and give reasons for the rule, and show the errors to which it is liable.

(2) Multiply together 172814412 and 987654321 in three lines of partial products.

(3) Simplify $1-2+4-8+16-32+64-128+256-512+1024-2048+4096-8192+16384-32768+65536-131072+262144-524288+1048576-2097152+4194304$.

(4) Divide 7864643457 by 9999.

(5) The quotient is equal to 6 times the divisor, and the divisor to 6 times the remainder, and the three together amount to 516; find the dividend.

IX. On the Method of Finding the Highest Common Factor of two or more numbers.

52. A number is said to be a *Factor* of another number when the latter is exactly divisible by the former. Thus 3 is a factor of 12.

A number is said to be a *Common Factor* of two or more numbers when each of the latter is exactly divisible by the former. Thus 3 is a Common Factor of 9, 12, and 15.

The *Highest Common Factor* of two or more numbers is the highest number which will exactly divide each of them.

Thus 6 is the Highest Common Factor of 6, 12, and 18, and 9 is the Highest Common Factor of 27, 36, and 108.

The words Highest Common Factor we shall write briefly H. C. F.

For small numbers the H. C. F. may be found by inspection, and by way of practice the student may work the following examples, applying the tests of divisibility given in Art. 38.

Examples. (xxii.)

Find the H. C. F. of

(1) 8 and 14.
(2) 12 and 30.
(3) 40 and 60.
(4) 36 and 90.
(5) 48 and 144.
(6) 7, 14, 21.
(7) 15, 27, 105.
(8) 32, 48, 128.
(9) 16, 64, 256, 1024.
(10) 24, 51, 105, 729.

53. In large numbers, the factors cannot often be determined by inspection, and if we have to find the H. C. F. of *two* such numbers, we have recourse to the following rule:

Divide the greater of the two numbers by the less, and the divisor by the remainder, repeating the process until no remainder is left: the last divisor is the H. C. F. *required.*

Thus, to find the H. C. F. of 689 and 1573, we proceed thus:

```
689 ) 1573 ( 2
      1378
      ----
       195 ) 689 ( 3
             585
             ---
             104 ) 195 ( 1
                   104
                   ---
                    91 ) 104 ( 1
                          91
                         ---
                          13 ) 91 ( 7
                               91
                               --
```

Hence 13 is the H. C. F. of 689 and 1573.

The reason of the above process depends upon the following proposition:

A common factor of any two numbers is also a factor of their sum, of their difference, and of any multiples of either of them.

Thus, 7 is a common factor of 28 and 91;
7 is also a factor of their *sum*, 28+91, or 119;
7 is also a factor of their *difference*, 91 – 28, or 63.

Also, 7 is a factor of 5 times 91, and of any other *multiple* of 91.

And 7 is a factor of 8 times 28, and of any other *multiple* of 28.

Any number which is a factor of 689 and 1573 is a factor also of their difference, 195,
and is therefore a factor of any multiple of 195—*e.g.*, 585,
and therefore of 585 and 689,
and therefore of their difference, 104,
and therefore of 104 and 195,
and therefore of their difference, 91,
and therefore of 91 and 104,
and therefore of their difference, 13,
and therefore of 91 and 13,
and therefore since 13 is a *factor* of itself and 91, it is a factor of the given numbers 689 and 1573.

Also, 13 is the Highest Common Factor of the given numbers, for it has been shown that any number which is a *factor* of 689 and 1573 is also a factor of 13, and since 13 is the highest factor of itself, it is the *Highest Common Factor* of 689 and 1573.

In the preceding proof it may be observed that the *quotients* are of no importance to the result. We are simply finding the difference between a certain number used as a dividend and a *multiple* of another number used as a divisor. This multiple, therefore, need not always be less than the dividend, and it will be sufficient to find the difference between the dividend and the *nearest* multiple of the divisor. Attention to this will sometimes shorten labor. Thus in the preceding example,

```
195 ) 689 ( 4
      780
     ———
       91 ) 195 ( 2
            182
           ———
             13 ) 91 ( 7
                  91
                  —
```

Examples. (xxiii.)

Find the H. C. F. of

(1) 384 and 1296.
(2) 2272 and 3552.
(3) 7455 and 47223
(4) 12321 and 54345.
(5) 6906 and 10359.
(6) 1908 and 2736.
(7) 49608 and 169416.
(8) 126025 and 40115.
(9) 1581227 and 16758766.
(10) 35175 and 236845.

54. If the H. C. F. of *three* numbers be required, we first find the H. C. F. of two of the numbers. Then the H. C. F. of this result and the third number will be the H. C. F. required.

For example, if we require the H. C. F. of 351, 459, and 1017, we first find the H. C. F. of 351 and 459 to be 27; and then we find the H. C. F. of 27 and 1017 to be 9, which is therefore the H. C. F. required.

Examples. (xxiv.)

Find the H. C. F. of

(1) 16, 20, 28. (2) 14, 42, 56, 138.
(3) 365, 511, 803. (4) 232, 290, 493.
(5) 492, 1476, 1763. (6) 148, 444, 592, 703.

X. Lowest Common Multiple.

55. A number is called the *Multiple* of another number when the former is exactly divisible by the latter. Thus 12 is a multiple of 3.

A number is said to be a *Common Multiple* of two or more numbers when the former is exactly divisible by each of the latter. Thus 12 is a Common Multiple of 2, 3, and 4.

The Lowest Common Multiple of two or more numbers is the lowest number which is exactly divisible by each of them.

Thus 12 is the Lowest Common Multiple of 4, 6, and 12; and 60 is the Lowest Common Multiple of 15, 20, and 30.

The words Lowest Common Multiple we shall write briefly L. C. M.

56. To find the L. C. M. of two numbers we have the following Rule:

Divide one of the numbers by the H. C. F. *and multiply the quotient by the other number. The result is the* L. C. M.

For example, to find the L. C. M. of 24 and 36.

The H. C. F. of 24 and 36 is 12.

$$\text{Now } 24 \div 12 = 2.$$
$$\therefore \text{L. C. M. of 24 and } 36 = 36 \times 2 = 72.$$

NOTE.—The symbol ∴ stands for the word *therefore*.

Since 12 is the H. C. F. of 24 and 36, then $24 = 12 \times 2$ and $36 = 12 \times 3$, also $24 \times 36 = 12 \times 2 \times 12 \times 3$, and obviously the L. C. M. of the two numbers will consist of the product of all the prime factors in the two numbers; or the least common multiple of 24 and $36 = 12 \times 3 \times 2$ or $\frac{24 \times 36}{12} = 72$.

And there is no integral number less than 72 which is a multiple of 24 and 36.

For 72 contains 24 3 times, 36 2 times, and 3 and 2 being prime to each other:

Wherefore the L. C. M. of 24 and $36 = \frac{24 \times 36}{12}$, or the least common multiple of two numbers is equal to their product divided by their highest common factor.

The following form is, perhaps, more convenient in practice:

$$\text{L. C. M. of 24 and } 36 = \frac{24 \times 36}{12} = 24 \times \frac{36}{12}.$$

The L. C. M. of two numbers is equal to the product of either of the numbers multiplied by the quotient arising from dividing the other by their highest common factor.

Examples. (xxv.)

Find the L. C. M. of

(1) 27 and 54.
(2) 88 and 108.
(3) 633 and 844.
(4) 195 and 735.
(5) 1000 and 2125.
(6) 3432 and 3575.
(7) 936 and 2925.
(8) 2304 and 4032.
(9) 2443 and 4537.

57. To find the L. C. M. of three or more numbers, we might find the L. C. M. of any two, and then find the L. C. M. of the resulting number and of a third of the original numbers, and so on, the final result being the L. C. M. required.

Thus, to find the L. C. M. of 12, 20, 36, and 54, we might proceed thus:

The L. C. M. of 12 and 20 is 60,
of 60 and 36 is 180,
of 180 and 54 is 540;
∴ the L. C. M. of 12, 20, 36, and 54 is 540.

But *in practice* it is generally more convenient to proceed by the following Rule :

Set down the given numbers side by side ; divide by any number, commencing with 2, 3, 5 . . . *which will exactly divide two at least of the numbers ; set down the quotients and the numbers that are not exactly divisible by the divisor, side by side ; and proceed in this way till you get a line of numbers which are prime to one another. Then the continued product of all the divisors and the numbers in this line will be the* L. C. M. *required.*

Thus, to find the L. C. M. of 12, 20, 30, 54.

$$\begin{array}{r|r} 2 & 12,\ 20,\ 30,\ 54 \\ \cline{2-2} 2 & 6,\ 10,\ 15,\ 27 \\ \cline{2-2} 3 & 3,\ \ 5,\ 15,\ 27 \\ \cline{2-2} 5 & 1,\ \ 5,\ \ 5,\ \ 9 \\ \cline{2-2} \multicolumn{1}{r}{} & 1,\ \ 1,\ \ 1,\ \ 9 \end{array}$$

$\therefore$ L. C. M. $= 2 \times 2 \times 3 \times 5 \times 9 = 540$.

The following is somewhat shorter :

Set down the numbers in a line, then strike out any that are contained in any of the others. Divide those not struck out by any number that will exactly divide one of them ; under any that it exactly divides, place the quotient ; under any which contain some factor common to it, set down the quotient, after striking out this factor ; and bring down all the other numbers.

Proceed in this way with the new line ; and so on, until all the numbers left in any line have no common measure, but unity. Then the continued product of the numbers in this line and all the divisors is the L. C. M. *of the given numbers.*

Thus, taking the numbers in Section 57

$$\begin{array}{r|r} 12 & 12,\ 20,\ 30,\ 54 \\ \cline{2-2} \multicolumn{1}{r}{} & 5,\ \ 5,\ \ 9 \end{array}$$

$\therefore$ L. C. M. $= 9 \times 5 \times 12 = 540$.

To find the L. C. M. of 4, 8, 10, 12, 16, 20, 24, 25, 30

Again,

$$\begin{array}{r|rrrrrrrrr} 12 & \cancel{4}, & \cancel{8}, & \cancel{10}, & \cancel{12}, & \cancel{16}, & 20, & 24, & 25, & 30 \\ \hline & & & & & & 4, & \cancel{5}, & \cancel{2}, 25, & \cancel{5} \end{array}$$

$$\therefore \text{L. C. M.} = 25 \times 4 \times 12 = 1200.$$

Examples. (xxvi.)

Find the L. C. M. of

(1) 6, 9, 24, 40.

(2) 8, 12, 22, 55.

(3) 12, 18, 96, 144.

(4) 16, 30, 48, 56, 72.

(5) 84, 156, 63, 99.

(6) 27, 33, 54, 69, 132.

(7) 17, 51, 119, 210.

(8) 15, 26, 39, 65, 180.

(9) 44, 126, 198, 280, 330.

(10) 50, 338, 675, 702, 975.

Examination Papers.

I.

(1) Find the least number which, divided by 13, 15, and 17, gives remainder 12 in each case.

(2) If A, B, and C walk 103950 inches together, how often will they step at the same moment, A taking 33 inches at a step, B 27, and C 30?

(3) How many rails will enclose a field 23023 feet long by 17765 feet wide, the fence being straight, and 6 rails high, the rails of equal length, and the longest that can be used?

(4) Two cog-wheels containing 210 and 330 cogs respectively are working together. After how many revolutions of the larger wheel will two cogs which once touch, touch again?

(5) Three numbers between 30 and 140 have 12 for their H. C. F., and 2772 for their L. C. M. Find the numbers.

II.

(1) Explain how to find (1) the H. C. F. and (2) the L. C. M. of a series of numbers by resolving them into their prime factors.

(2) A farmer has 600 bushels of wheat; what are the three smallest-sized bags, and the three largest bins, holding an exact number of bushels, that will each measure the same without a remainder?

(3) What is the smallest sum of money with which I can buy sheep at $5 each, cows at $22 each, or horses at $75 each?

(4) Three horses are running round a race-course of 5280 yards; the first horse runs 440 yards a minute, the second

352 yards, and the third 264 yards; find the time between their once coming all together, and their coming all together again.

(5) Find the least number which divided by 675, 1050, and 4368, will leave the same remainder, 32.

III.

(1) Explain how you would find all the divisors which a number has. Find those of 1800.

(2) The L. C. M. of 2, 3, 4, 5, 6, 8, 9, and another number prime to them is 10440. What is this number?

(3) How do you determine whether a number is prime or composite?

Which of the following numbers are prime and which composite:—3391, 2699, 14787, and 1477?

(4) Three men, A, B, and C, start together from the same place to walk round an island 60 miles in circumference; they walk in the same direction, A at the rate of 5 miles per hour, B at 4, and C at 3. In what time will all be together for the first time after starting, and how many miles will each have gone?

(5) Find the greatest weight, in grains, that will measure both pounds Avoirdupois and pounds Troy, there being 5760 grains in one pound Troy, and 144 lbs. Avoirdupois contain as many grains as 175 lbs. Troy.

IV.

(1) Define Factor, Measure, Multiple, and explain when a number is Prime, and when Composite. In what digits must prime numbers end?

(2) The product of two numbers is 1270374, and half of one of them is 3129. What is the other?

(3) The fore-wheel of a carriage was 11 feet in circumference, and the hind one 13 feet. There being 5280 feet in a mile, how many miles had a carriage gone when the same spots which were on the ground at the time of starting had been on the ground 360 times at the same instant?

(4) A can dig 36 post-holes in a day; B can dig 32, and C 30 in the same time. What is the smallest number which will furnish exact days' labor, either for each working alone, or for all working together?

(5) How many firkins of butter, each containing 56 lbs., at 23 cents a pound, must be given for 14 bbls. of sugar, each containing 276 lbs., at 8 cents per pound.

V.

(1) Explain the use of zero in decimal notation.

(2) Find the greatest number which will divide 10974 and 15336, leaving as remainders respectively 54 and 36

(3) The digits in the units' and millions' places of a number are 2 and 7 respectively ; what will be the digits in the same places when 999999 is taken from the number?

(4) An avenue 3 miles long is planted with 5 rows of trees. The trees are placed in the different rows at the distances of 6 8, 9, 10, and 12 feet respectively. If the rows start from the same straight line, (1) how often will 5 trees be in a line, there being 5280 feet in a mile? and (2) how many trees will there be in the avenue?

(5) A number is composed of the following factors : 2^4, 3^2, 5^6, 11, and 17 ; find the number.

XI.—On Fractions.

58. Numbers are the measures of quantities.

A *Quantity* is anything which may be regarded as being made up of parts, like the whole.

Thus a sum of money is a quantity, because we may regard it as made up of parts like the whole.

To measure any quantity we fix upon some known quantity of the same kind for our standard or UNIT, and the NUMBER, which expresses how many times this Unit is contained in the quantity, is called the MEASURE of the quantity.

To put this in a more *practical* shape, we give the following illustration : We measure large sums of money by the *Unit* which we call a *Dollar,* and when we say that a man's income is *two thousand a year,* we mean that he receives yearly a sum of money which contains the unity two thousand times, and we call the number Two Thousand the *measure* of his income.

59. Now we can conceive that a unit of measurement may be divided into a number of parts of *equal* magnitude.

For instance, if we take a dollar as the *Unit* by which we measure sums of money, we suppose this Unit to be divided into one hundred equal parts, and we call each of these parts *one-hundredth* of a Dollar; *two* such parts will be two-hundredths, *three* will be three-hundredths of a Dollar. Such parts are called Fractions of a Dollar, or other Unit, and we give the following definition:

DEF.—A FRACTION is an expression representing one or more of the equal parts of a Unit.

The number of equal parts into which the Unit is divided is called the DENOMINATOR of the Fraction, and the number expressing how many of these parts are taken to form the Fraction is called the NUMERATOR of the Fraction.

These operations are denoted by the following symbols: we represent a fraction by writing the numerator above the denominator, and separating them by a horizontal line.

Thus $\frac{3}{4}$ represents the Fraction of which the Numerator is 3 and the Denominator 4.

Such Symbols are called Fraction-Symbols, or, for brevity, Fractions.

60. The symbol $\frac{1}{2}$ is read one-half.
The symbol $\frac{1}{3}$ is read one-third.
The symbol $\frac{3}{4}$ is read three-fourths.
The symbol $\frac{6}{7}$ is read six-sevenths,

and so on.

61. The Numerator and Denominator of a Fraction are called the TERMS of the Fraction.

A PROPER Fraction is one in which the Numerator is less than the Denominator, as $\frac{3}{5}$.

An IMPROPER Fraction is one whose Numerator is not less than its Denominator, $\frac{7}{4}$.

In our explanation of the fundamental operations performed with fractions we shall make use, as far as is possible, of *proper* fractions only.

62. To show that $\frac{2}{3}=\frac{8}{12}$.

Suppose a UNIT to be divided into 3 equal parts.

Then $\frac{2}{3}$ will represent 2 of these parts . . . . . . (1).

Next, let each of the 3 parts be subdivided into 4 equal parts.

Thus the UNIT has been divided into 12 equal parts, and $\frac{8}{12}$ will represent 8 of these subdivisions . . . . . . (2).

Now, one of the parts in (1) is equal to 4 of the subdivisions in (2).

$\therefore$ 2 parts are equal to 8 subdivisions,

and $\therefore \frac{2}{3}=\frac{8}{12}$.

We draw from this proof two inferences:

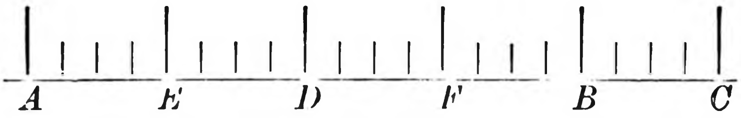
I. If the numerator and denominator of a fraction be *multiplied* by the same number, the value of the fraction is not altered.

Thus $\frac{3}{7}=\frac{12}{28}$ and $\frac{4}{15}=\frac{40}{150}$.

II. If the numerator and denominator of a fraction be *divided* by the same number, the value of the fraction is not altered.

Thus $\frac{14}{20}=\frac{7}{10}$ and $\frac{90}{100}=\frac{9}{10}$.

63. To make the important theorem established in Article 62 more clear, we shall give a *practical* proof that $\frac{4}{5}=\frac{16}{20}$, by taking a straight line as the unit of length.



Let the line AC be divided into 5 equal parts.

Then, if B be the point of division nearest to C,

AB is $\frac{4}{5}$ of AC . . . . (1).

Next, let each of the parts be subdivided into 4 equal parts.

Then AC contains 20 of these subdivisions,

and AB contains 16 of these subdivisions;

$\therefore AB$ is $\frac{16}{20}$ of AC . . . . (2).

Comparing (1) and (2) we conclude that

$$\frac{4}{5}=\frac{16}{20}$$

64. A fraction is in its *lowest terms* when the numerator and denominator have no common factor except unity:

Thus $\frac{2}{3}$, $\frac{5}{7}$, $\frac{17}{19}$, represent fractions in their lowest terms.

To reduce a fraction to its lowest terms we have the following Rule:

Divide the Numerator and Denominator by their H. C. F.

Thus, if we have to reduce $\frac{18}{81}$ to its lowest terms, we know that 9 is the H. C. F. of 18 and 81, and dividing the numerator and denominator by 9, we have the resulting fraction $\frac{2}{9}$.

Again, to reduce $\frac{25}{500}$ to its lowest terms, we find 25 to be the H. C. F. of 25 and 500, and therefore $\frac{1}{20}$ will be the reduced fraction.

When we see, by inspection or by an application of the tests of divisibility given in Art. 38, that a factor is common to both Numerator and Denominator, we may divide both by this factor and reduce the fraction to *lower* terms, without going through the process of finding the H. C. F.

Thus, to reduce the fraction $\frac{270}{9360}$ we see that both terms are divisible by 10, and $\therefore \frac{270}{9360}=\frac{27}{936}$.

Now 27 and 936 are both divisible by 9 (Art. 38),

$$\text{and } \therefore \frac{27}{936}=\frac{3}{104}.$$

Examples. (xxvii.)

Reduce to their lowest terms the following fractions:

(1) $\frac{24}{80}$. (2) $\frac{72}{280}$. (3) $\frac{42}{210}$ (4) $\frac{192}{576}$.
(5) $\frac{5184}{6912}$. (6) $\frac{1680}{1920}$. (7) $\frac{6409}{7395}$. (8) $\frac{319}{5687}$.
(9) $\frac{9405}{15615}$. (10) $\frac{3178}{5221}$.

65. Two fractions may be replaced by two equivalent fractions with a Common Denominator by the following rule:

Find the L. C. M. *of the denominators of the given fractions.*

Divide the L. C. M. *by the denominator of each fraction.*

Multiply the first Numerator by the first Quotient.

Multiply the second Numerator by the second Quotient.

The two Products will be the Numerators of the equivalent fractions whose common denominator is the L. C. M. *of the original denominators.*

The same rule holds for three, four, or more fractions.

Ex. (1). Reduce to equivalent fractions with the lowest common denominator, $\frac{3}{8}$ and $\frac{4}{7}$.

Denominators 8, 7,
L. C. M. 56.
Quotients 7, 8.
New numerators 21, 32.
Equivalent fractions $\frac{21}{56}$, $\frac{32}{56}$.

Ex. (2). Reduce to equivalent fractions with the lowest common denominator, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{13}{72}$.

Denominators 3, 9, 72.
L. C. M. 72.
Quotients 24, 8, 1.
New numerators 48, 32, 13.
Equivalent fractions $\frac{48}{72}$, $\frac{32}{72}$, $\frac{13}{72}$.

Examples. (xxviii.)

Reduce to equivalent fractions with the lowest common denominator,

(1) $\frac{3}{4}$, $\frac{5}{7}$. (2) $\frac{4}{9}$, $\frac{5}{18}$, $\frac{7}{27}$. (3) $\frac{3}{5}$, $\frac{4}{7}$, $\frac{6}{11}$.
(4) $\frac{5}{12}$, $\frac{13}{20}$, $\frac{17}{80}$, $\frac{19}{120}$. (5) $\frac{4}{7}$, $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.
(6) $\frac{1}{3}$, $\frac{3}{5}$, $\frac{1}{6}$, $\frac{5}{18}$. (7) $\frac{3}{10}$, $\frac{5}{27}$, $\frac{7}{90}$, $\frac{11}{360}$

66. To compare the values of two or more fratcions, we convert them into equivalent fractions with a common denominator: then the comparison of the values of the original fractions can be made by comparing the numerators of the new fractions.

For example, to compare the value of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$.

The equivalent fractions are $\frac{56}{84}$, $\frac{63}{84}$, $\frac{60}{84}$.

The descending order of value of the numerators is 63, 60, 56;

∴ the descending order of value of the given fractions is

$$\frac{3}{4},\ \frac{5}{7},\ \frac{2}{7}.$$

67. We may also compare fractions by reducing them to fractions with a common *Numerator*, and assigning the greatest value to that one of the resulting fractions which has the *least* denominator.

Thus, to compare the values of

$$\frac{3}{5},\ \frac{27}{31},\ \text{and}\ \frac{81}{95}.$$

The equivalent fractions are

$$\frac{81}{135},\ \frac{81}{93},\ \text{and}\ \frac{81}{95},$$

∴ the descending order of the given fractions is

$$\frac{27}{31},\ \frac{81}{95},\ \frac{3}{5}.$$

Examples. (xxix.)

Compare the values of

(1) $\frac{3}{7}$, $\frac{4}{5}$, $\frac{9}{13}$.

(2) $\frac{5}{6}$, $\frac{7}{9}$, $\frac{12}{17}$.

(3) $\frac{9}{11}$, $\frac{13}{15}$, $\frac{17}{21}$.

(4) $\frac{3}{20}$, $\frac{7}{40}$, $\frac{11}{70}$.

(5) $\frac{7}{33}$, $\frac{9}{43}$, $\frac{11}{53}$.

(6) $\frac{2}{17}$, $\frac{5}{34}$, $\frac{7}{51}$.

ADDITION OF FRACTIONS.

68. The rule for adding two or more fractions together is this:

Reduce the Fractions to equivalent fractions having the Lowest Common Denominator.

Then add *the numerators of the equivalent fractions, and place the result as the Numerator of a fraction whose Denominator is the common denominator of the equivalent fractions.*

The fraction will be equal to the sum of the original fractions.

For example, to find the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

$$\frac{1}{3}=\frac{4}{12}\ \text{and}\ \frac{1}{4}=\frac{3}{12}.$$

$$\therefore\ \frac{1}{3}\times\frac{1}{4}=\frac{4}{12}\times\frac{3}{12}=\frac{7}{12}.$$

Examples. (xxx.)

Find the sum of the following fractions:

(1) $\frac{1}{7}$ and $\frac{2}{5}$. (2) $\frac{2}{3}$ and $\frac{3}{14}$.

(3) $\frac{2}{7}$, $\frac{3}{11}$ and $\frac{2}{33}$. (4) $\frac{4}{9}$, $\frac{5}{14}$ and $\frac{5}{42}$

(5) $\frac{5}{13}$, $\frac{2}{39}$, $\frac{25}{78}$ and $\frac{7}{156}$.

(6) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$.

(7) $\frac{5}{11}$, $\frac{2}{27}$, $\frac{7}{55}$ and $\frac{4}{135}$.

(8) $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{1}{11}$.

(9) $\frac{2}{3}$, $\frac{3}{40}$, $\frac{7}{240}$ and $\frac{11}{2880}$.

SUBTRACTION OF FRACTIONS.

69. The rule for subtracting a fraction from a greater fraction is this:

Reduce the fractions to equivalent fractions having the Lowest Common Denominator. Then subtract *the numerator of the smaller of the equivalent fractions from the numerator of the greater, and place the result as the numerator of a fraction, whose denominator is the common denominator of the equivalent fractions. This fraction will be equal to the difference of the original fractions.*

For example, to find the difference between $\frac{2}{3}$ and $\frac{5}{7}$.

$$\frac{2}{3}=\frac{14}{21} \text{ and } \frac{5}{7}=\frac{15}{21},$$

$$\therefore \frac{5}{7}-\frac{2}{3}=\frac{15}{21}-\frac{14}{21}=\frac{1}{21}.$$

Examples. (xxxi.)

Find the difference of the following fractions:

(1) $\frac{4}{5}$ and $\frac{5}{6}$. (2) $\frac{3}{7}$ and $\frac{15}{19}$.

(3) $\frac{11}{12}$ and $\frac{12}{13}$. (4) $\frac{13}{51}$ and $\frac{235}{357}$.

(5) $\frac{17}{63}$ and $\frac{29}{108}$. (6) $\frac{9}{38}$ and $\frac{43}{209}$.

(7) $\frac{146}{273}$ and $\frac{268}{637}$. (8) $\frac{199}{200}$ and $\frac{359}{360}$.

(9) $\frac{347}{1242}$ and $\frac{835}{1998}$.

MULTIPLICATION OF FRACTIONS.

70. A fraction is multiplied by a whole number by multiplying the numerator by that number and leaving the denominator unchanged.

Thus $\frac{2}{7}$ multiplied by 3 becomes $\frac{6}{7}$,
for each of the symbols $\frac{2}{7}$ and $\frac{6}{7}$ implies that a unit has been divided into 7 equal parts, and three times as many of those parts are taken to form the fraction represented by the latter as are taken to form the fraction represented by the former.

71. *To prove that* of $\frac{4}{5}=\frac{8}{15}$.

$$\tfrac{2}{3} \text{ of } \tfrac{4}{5}=\tfrac{2}{3} \text{ of } \tfrac{12}{15}. \qquad \text{Art. 62}$$

Now, suppose a unit to be divided into 15 equal parts,

then $\frac{2}{3}$ of $\frac{4}{5}=\frac{2}{3}$ of 12 of such parts,
$=\frac{8}{12}$ of 12 of such parts, Art. 62.
$=8$ of such parts ; Art. 62.
but $\frac{8}{15}=8$ of such parts ; Art. 62.
$\therefore \frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

Hence we derive the Rule for what is called MULTIPLICATION OF FRACTIONS.

We extend the meaning of the sign ×, and define $\frac{2}{3}\times\frac{4}{5}$ (which, according to our definition in Art. 21, would have no meaning) to mean $\frac{2}{3}$ of $\frac{4}{5}$, and we conclude that $\frac{2}{3}\times\frac{4}{5}=\frac{2\times4}{3\times5}$, which in words gives us this rule :

"*Take the product of the numerators to form the numerator of the resulting fraction, and the product of the denominators to form the denominator.*"

The same rule holds good for the multiplication of three or more fractions.

Before effecting the multiplication, common factors should be removed from the numerator and denominator. It will be well for the learner to be familiar with the principles laid down in Art. 38.

For example, to find the value of $\frac{14}{25}$ of $\frac{35}{51}$ of $\frac{17}{49}$ we proceed thus :

$$\tfrac{14}{25} \text{ of } \tfrac{35}{51} \text{ of } \tfrac{17}{49} = \frac{14\times35\times17}{25\times51\times49}$$
$$= \frac{2\times7\times5\times7\times17}{5\times5\times3\times17\times7\times7}$$

and removing common factors from numerator and denominator,

$$= \frac{2}{5\times3}$$
$$= \frac{2}{15}.$$

Examples. (xxxii.)

Reduce to their simplest form :

(1) $\frac{3}{7}$ of $\frac{5}{9}$.
(2) $\frac{3}{4}\times\frac{5}{7}\times\frac{9}{11}$.
(3) $\frac{42}{65}\times\frac{39}{56}\times\frac{24}{27}$.
(4) $\frac{84}{85}\times\frac{102}{117}\times\frac{91}{132}$.
(5) $\frac{17}{82}\times\frac{27}{38}\times\frac{123}{153}$.
(6) $\frac{100}{161}\times\frac{35}{72}\times\frac{33}{40}$.
(7) $\frac{9}{10}$ of $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{29}$.
(8) $\frac{292}{465}$ of $\frac{573}{657}$ of $\frac{248}{1719}$.
(9) $\frac{1205}{2763}$ of $\frac{712}{2169}$ of $\frac{1535}{1068}$.

DIVISION OF FRACTIONS.

72. A fraction is divided by a whole number by multiplying the denominator by that number, and leaving the numerator unchanged.

Thus $\frac{2}{7}$ divided by 3 becomes $\frac{2}{21}$,

for $\frac{2}{7}$ implies that a unit has been divided into 7 equal parts. $\frac{2}{21}$ implies that a unit has been divided into 21 equal parts, and hence each part in the former is three times as great as each part in the latter, and since the same number of parts is taken in both cases, the latter fraction is one-third of the former.

73. *To show that* $\frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}$.

The quotient resulting from the division of $\frac{2}{3}$ by $\frac{4}{5}$ is such a number that, when it is multiplied by the divisor $\frac{4}{5}$, the product must be equal to the dividend $\frac{2}{3}$, that is,

$$\begin{aligned}
\tfrac{4}{5}\text{ of the Quotient} &= \tfrac{2}{3},\\
\therefore\ \tfrac{5}{4}\text{ of }\tfrac{4}{5}\text{ of the Quotient} &= \tfrac{5}{4}\text{ of }\tfrac{2}{3},\\
\therefore\ \tfrac{20}{20}\text{ of the Quotient} &= \tfrac{5}{4}\text{ of }\tfrac{2}{3},\\
\therefore\ \text{the Quotient} &= \tfrac{5}{4}\text{ of }\tfrac{2}{3},\\
\text{that is, }\tfrac{2}{3}\div\tfrac{4}{5} &= \tfrac{5}{4}\text{ of }\tfrac{2}{3},\\
\text{or, }\tfrac{2}{3}\div\tfrac{4}{5} &= \tfrac{2}{3}\times\tfrac{5}{4},
\end{aligned}$$

Hence we obtain the following rule for what is called DIVISION OF FRACTIONS.

Invert the divisor, and proceed as in Multiplication.

Thus, $\frac{12}{49}\div\frac{16}{35}=\frac{12}{49}\times\frac{35}{16}=\frac{15}{28}$.

Examples. (xxxiii.)

Divide

(1) $\frac{12}{49}$ by $\frac{3}{7}$

(2) $\frac{25}{39}$ by $\frac{10}{13}$

(3) $\frac{85}{256}$ by $\frac{51}{64}$

(4) $\frac{16}{261}$ by $\frac{4}{27}$

(5) $\frac{38}{275}$ by $\frac{133}{385}$

(6) $\frac{91}{369}$ by $\frac{78}{287}$

(7) $\frac{49}{550}$ by $\frac{343}{1450}$

(8) $\frac{695}{1544}$ by $\frac{1251}{2316}$

(9) $\frac{1535}{2421}$ by $\frac{921}{1076}$

74. Having now established the elementary rules for operations performed with fractions, we proceed to notice some other points belonging to this branch of Arithmetic.

75. A whole number, or integer, can be written as a fraction, by putting 1 beneath the number as a denominator: thus 5 may be written as a fraction, thus $\frac{5}{1}$.

Also, since $\frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4}$, and so on, it is clear that we can represent a whole number by a fraction whose denominator is any whole number we please to select.

76. A *Mixed Number* is a number made up of an integer and a fraction, as $4\frac{2}{7}$. This may be read thus, *four and two-sevenths*, and must be regarded as the *sum* of 4 and $\frac{2}{7}$.

A mixed number can be brought into the form of an improper fraction, by multiplying the integer by the denominator of the fraction, adding to the product the numerator of the fraction, and making the sum the numerator of a fraction of which the denominator is the denominator of the original fraction.

Thus, $4\frac{2}{7} = \frac{30}{7}$,

for $4\frac{2}{7} = 4 + \frac{2}{7} = \frac{28}{7} + \frac{2}{7} = \frac{30}{7}$.

Conversely, an improper fraction can be reduced to a mixed number, by dividing the numerator by the denominator, setting down the quotient as the integral part, and making the remainder the numerator of the fractional part of the mixed number, the denominator being the denominator of the original fraction.

Thus, $\frac{25}{7} = 3\frac{4}{7}$.

for $\frac{25}{7} = \frac{21+4}{7} = \frac{21}{7} + \frac{4}{7} = 3 + \frac{4}{7} = 3\frac{4}{7}$.

Examples. (xxxiv.)

Convert into improper fractions:

(1) $7\frac{4}{9}$. (2) $23\frac{15}{47}$. (3) $216\frac{13}{29}$. (4) $173\frac{19}{1000}$.

and into mixed numbers:

(5) $\frac{427}{10}$. (6) $\frac{3477}{1000}$. (7) $\frac{4293}{137}$. (8) $\frac{65943}{71}$.

77. The rules for the Addition, Subtraction, Multiplication, and Division of Fractions are applicable to *Improper* Fractions.

Thus $\frac{7}{4} + \frac{13}{10} = \frac{35}{20} + \frac{26}{20} = \frac{61}{20} = 3\frac{1}{20}$

$$\frac{9}{5} - \frac{13}{12} = \frac{108}{60} - \frac{65}{60} = \frac{43}{60}$$

$$\frac{13}{9} \times \frac{27}{26} = \frac{13\times27}{9\times26} = \frac{13\times9\times3}{9\times13\times2} = \frac{3}{2} = 1\frac{1}{2}.$$

$$\frac{117}{110} \div \frac{91}{33} = \frac{117}{110} \times \frac{33}{91} = \frac{9\times13\times11\times3}{11\times10\times7\times13} = \frac{9\times3}{10\times7} = \frac{27}{70}.$$

78. In the application of the rules to *Mixed Numbers*, we *may* in all cases change the Mixed Numbers into Improper Fractions, and proceed as in the foregoing Examples. In Division we *must* proceed thus:

For example,

$$4\frac{5}{9} \div 12\frac{3}{10} = \frac{41}{9} \div \frac{123}{10} = \frac{41}{9} \times \frac{10}{123} = \frac{10}{27}$$

$$16 \div 12\frac{4}{5} = \frac{16}{1} \div \frac{64}{5} = \frac{16}{1} \times \frac{5}{64} = \frac{5}{4} = 1\frac{1}{4}.$$

In Multiplication it is usually the best course thus.

$$7\frac{2}{3} \times 5\frac{4}{7} = \frac{23}{3} \times \frac{39}{7} = \frac{23\times13}{7} = \frac{299}{7} = 42\frac{5}{7}.$$

In Addition it is often advantageous to proceed thus:

$$\begin{aligned}4\frac{2}{3}+3\frac{4}{7} &= 4+\frac{2}{3}+3+\frac{4}{7}\\ &= 4+3+\frac{2}{3}+\frac{4}{7}\\ &= 7+\frac{14}{21}+\frac{12}{21}\\ &= 7+\frac{26}{21}\\ &= 7+1\frac{5}{21}\\ &= 8\frac{5}{21}.\end{aligned}$$

and, similarly, when three or more numbers are to be added, we may separate the fractions from the integers, and make a distinct operation for each class.

In Subtraction we can employ the same method, but a little care is necessary. Suppose we have to take

$$3\tfrac{4}{7} \text{ from } 4\tfrac{2}{3},$$

Reducing the *fractional* parts of the numbers to equivalent fractions with a common denominator, we have

$$3\tfrac{12}{21} \text{ and } 4\tfrac{14}{21}.$$

We can now take the integral part of the first number from the integral part of the second, and the fractional part of the first from the fractional part of the second, and we have

$$4\tfrac{14}{21}-3\tfrac{12}{21}=1\tfrac{2}{21}.$$

But suppose we have to take $3\frac{5}{7}$ from $10\frac{2}{5}$,

Since $\frac{5}{7}=\frac{25}{35}$ and $\frac{2}{5}=\frac{14}{35}$

$\frac{5}{7}$ is *greater* than $\frac{2}{5}$,

and we cannot take away the fractional part of $3\frac{25}{35}$ from the fractional part of $10\frac{14}{35}$. We escape from the difficulty by the device of adding *unity* to each expression, to $3\frac{25}{35}$ in the form of 1, and to $10\frac{14}{35}$ in the form of $\frac{35}{35}$.

$$\text{Thus } 10\tfrac{14}{35}-3\tfrac{25}{35}=10\tfrac{49}{35}-4\tfrac{25}{35}=6\tfrac{24}{35}.$$

Take another illustration of a *practical* nature.

From $5\frac{1}{4}d.$ take away $3\frac{3}{4}d.$

We add four farthings, *i.e.*, $\frac{4}{4}$ of a penny, to the former sum, and 1 penny to the latter, and reason thus:

$$5\tfrac{1}{4}d.-3\tfrac{3}{4}d.=5\tfrac{5}{4}d.-4\tfrac{3}{4}d.=1\tfrac{2}{4}d.=1\tfrac{1}{2}d.$$

Examples. (xxxv.)

Simplify the following fractions:

(1) $4\frac{2}{7}\div 3\frac{7}{9}$. (2) $8\frac{3}{5}\div 6\frac{1}{7}$. (3) $104\frac{2}{9}\div 53\frac{7}{13}$.

(4) $6\frac{2}{3}\times 9\frac{5}{8}$. (5) $14\times 3\frac{2}{21}$. (6) $9\frac{4}{15}\times 19\frac{2}{7}$.

(7) $2\frac{1}{3}+3\frac{1}{4}$. (8) $5\frac{2}{7}+6\frac{3}{11}+1\frac{4}{15}$.

(9) $16\frac{2}{5}+4\frac{4}{9}+17\frac{13}{45}$. (10) $4\frac{3}{7}-2\frac{1}{5}$.

(11) $14\frac{4}{9}-5\frac{7}{8}$. (12) $6\frac{7}{13}-5\frac{9}{14}$.

The following examples should be carefully noticed:

I. From 17 take $4\frac{5}{21}$.

$$\begin{aligned}17-4\tfrac{5}{21}&=16+1-4\tfrac{5}{21}=16-4+1-\tfrac{5}{21}.\\&=12+\tfrac{16}{21}=12\tfrac{16}{21}.\end{aligned}$$

II. From 317 take $\frac{5}{49}$.

$$317-\tfrac{5}{49}=316+1-\tfrac{5}{49}=316+\tfrac{44}{49}=316\tfrac{44}{49}.$$

III. Multiply $\frac{999}{1000}$ by 397.

$$\text{Since } \tfrac{999}{1000}=1-\tfrac{1}{1000}$$

$$397\times\tfrac{999}{1000}=397-\tfrac{397}{1000}=396+1-\tfrac{397}{1000}$$

$$=396+\tfrac{603}{1000}=396\tfrac{603}{1000}.$$

79. A COMPOUND FRACTION is defined to be the fraction of a fraction.

Thus $\frac{2}{3}$ of $\frac{5}{7}$ and $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7}$ are compound fractions.

They are reduced to simple fractions by the process of Multiplication.

Thus $\frac{3}{4}$ of $2\frac{1}{4}$ of $5\frac{2}{7}=\frac{3}{4}\times\frac{9}{4}\times\frac{37}{7}=\frac{3\times9\times37}{4\times4\times7}=\frac{999}{112}=8\frac{103}{112}$.

80. A COMPLEX FRACTION is one of which the Numerator or Denominator is itself a fraction or a mixed number.

Thus $\dfrac{\frac{3}{4}}{7}$ $\dfrac{2}{\frac{5}{9}}$ and $\dfrac{4\frac{2}{7}}{5\frac{3}{8}}$ are complex fractions.

They are reduced to simple fractions by the process of Division.

$$\text{Thus } \frac{\frac{3}{4}}{7}=\tfrac{3}{4}\div7=\tfrac{3}{4}\div\tfrac{7}{1}=\tfrac{3}{4}\times\tfrac{1}{7}=\tfrac{3}{28}$$

$$\text{and } \frac{2}{\frac{5}{9}}=2\div\tfrac{5}{9}=\tfrac{2}{1}\times\tfrac{9}{5}=\tfrac{18}{5}=3\tfrac{3}{5}.$$

Examples. (xxxvi.)

Simplify the following fractions :

(1) $\frac{3}{4}$ of $5\frac{1}{9}$ of $7\frac{1}{5}$.

(2) $4\frac{3}{7}$ of $11\frac{1}{5}$ of 15.

(3) $\frac{7}{8}$ of $2\frac{5}{9}$ of $3\frac{4}{7}$ of 90.

(4) $\dfrac{\frac{2}{5}}{\frac{7}{8}}$.

(5) $\dfrac{6\frac{2}{9}}{4\frac{2}{3}}$.

(6) $\dfrac{14}{3\frac{1}{2}}$.

(7) $\dfrac{30\frac{1}{4}}{11}$.

(8) $\dfrac{16\frac{2}{3}}{\frac{25}{39}}$.

THE HIGHEST COMMON FACTOR AND THE LEAST COMMON MULTIPLE OF FRACTIONS.

81. The H. C. F. or L. C. M. of fractions can be readily found by considering that the denominator is simply the name of so many units represented by the numerator No difficulty is ever experienced in finding the H. C. F. or L. C. M. of \$12 and \$16, or of 12 apples and 16 apples. In fractions the name is written *under* the number representing the collection of units of that name.

Thus to find the H. C. F. of $\frac{12}{36}$ and $\frac{16}{36}$, proceed as in whole numbers; find the H. C. F. of 12 and 16, which is 4, and call it by its name, which in this case is thirty-sixths. Hence the H. C. F. is $\frac{4}{36}$.

Similarly to find the L. C. M. of $\frac{12}{36}$ and $\frac{16}{36}$, find the L. C. M. of 12 and 16, which is 48, and call it by its proper name. Hence the L. C. M. is $\frac{48}{36}$. Hence to find the H. C. F. of fractions we have the following rule:

Change them to others having the same name or denominator, and find the H. C. F. *of their numerators. This placed over the common denominator will be the* H. C. F. *of the fractions.*

To find the L. C. M. of fractions: *Change them to others having a common denominator, and find the* L. C. M. *of the numerators. This placed over the common denominator will be the* L. C. M. *of the fractions.*

The following is somewhat shorter: *Find the* L. C. M. *of the numerators, and under this place the* H. C. F. *of the denominators of the fractions. The resulting fraction will be the* L. C. M. required.

Examples. (xxxvii.)

Find the H. C. F. of the following fractions:

(1) $\frac{1}{7}$ and $\frac{2}{5}$. (2) $\frac{17}{63}$ and $\frac{29}{108}$.

(3) $\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{8}$, and $5\frac{2}{3}$.

(4) $\frac{2}{3}$, $\frac{2}{7}$, $\frac{12}{13}$, $4\frac{1}{5}$, and $5\frac{1}{4}$.

Find the L. C. M. of the following fractions:

(5) $\frac{2}{3}$ and $\frac{5}{6}$. (6) $2\frac{1}{2}$ and $7\frac{1}{8}$.

(7) $4\frac{1}{3}$, $5\frac{2}{9}$, and $3\frac{5}{12}$.

(8) $\frac{1}{2}$ of $2\frac{3}{4}$ of $\dfrac{7}{3\frac{1}{2}}$ and $\frac{3}{7}$ of $\dfrac{8}{1\frac{1}{3}}$ of $2\frac{1}{5}$.

ON THE USE OF BRACKETS.

82. When an expression is inclosed in a bracket ( ), it is intended to show that the whole of the expression is affected by some symbol which precedes or follows the bracket.

Thus $24\times(3\frac{1}{2}+7\frac{1}{4})$ means, that 24 times the sum of the numbers $3\frac{1}{2}$ and $7\frac{1}{4}$ is to be taken, which we may effect by combining $3\frac{1}{2}$ and $7\frac{1}{4}$ by addition, and multiplying the result by 24.

Again, $2\frac{5}{7}\div(4\frac{3}{4}-2\frac{1}{2})$ signifies that $2\frac{5}{7}$ is to be divided by the difference between $4\frac{3}{4}$ and $2\frac{1}{2}$; and therefore the result will be

$$2\tfrac{5}{7}\div 2\tfrac{1}{4},\text{ or }\tfrac{19}{7}\div\tfrac{9}{4},\text{ or }\tfrac{19}{7}\times\tfrac{4}{9},\text{ or }\tfrac{76}{63}.$$

And, generally, we may say, that when numbers are included in a bracket, the expression within the bracket must be brought into the simplest form before combining it with expressions not in the bracket.

83. The methods of denoting a bracket are various; thus, the marks [ ] and { } are often employed. Brackets are made to enclose one another, as in the expression,

$$[3\div 2+3\div\{4+5\div(2+\tfrac{1}{3})\}].$$

In removing such brackets it is best to commence with the *innermost*, and to remove the brackets one by one, thus,

$$\begin{aligned}&3\div[2+3\div\{4+5\div(2+\tfrac{1}{3})\}]\\&=3\div[2+3\div\{4+5\div\tfrac{7}{3}\}]\\&=3\div[2+3\div\{4+\tfrac{15}{7}\}]\\&=3\div[2+3\div\tfrac{43}{7}]\\&=3\div[2+\tfrac{21}{43}]\\&=3\div\tfrac{107}{43}=\tfrac{129}{107}.\end{aligned}$$

We have worked out this example at length because it will teach the learner how to simplify with neatness a peculiar class of fractions called *Continued* Fractions, which appear in a form like the following:

$$\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{2-\frac{9}{16}}}}$$

This fraction, by the aid of brackets, may be represented thus,

$$1\div[4+1\div\{1-1\div(2-\tfrac{9}{10})\}],$$

and then we can simplify it by the gradual removal of the brackets, the final result being $\frac{7}{51}$.

84. There is another method of simplifying Complex and Continued Fractions, which we may explain by the following examples:

Ex. (1). To simplify $\dfrac{5}{2+\frac{3}{7}}$.

Multiply all the terms of the fraction by 7, and it becomes

$$\frac{35}{14+3} \text{ or } \frac{35}{17}.$$

Ex. (2). To simplify $\dfrac{\frac{2}{3}}{5+\frac{3}{10}}$.

Multiply the terms by 30, and we get

$$\frac{20}{150+9} \text{ or } \frac{20}{159}.$$

Ex. (3). To simplify $\dfrac{\frac{2}{3}-\frac{3}{7}}{\frac{5}{6}-\frac{5}{14}}$.

Multiply all the terms by 42, and we get

$$\frac{28-18}{35-15} \text{ or } \frac{10}{20} \text{ or } \frac{1}{2}.$$

Ex. (4). To simplify $\dfrac{3}{3+\dfrac{4}{9+\frac{2}{7}}}$

$$\frac{3}{3+\dfrac{4}{9+\frac{2}{7}}}=\frac{3}{3+\frac{28}{65}}=\frac{195}{195+28}=\frac{195}{223}.$$

Ex. (5). To simplify $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}$

$$\frac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{2}}}}=\frac{1}{1+\dfrac{1}{1+\frac{2}{3}}}=\frac{1}{1+\frac{3}{5}}=\frac{5}{8}$$

Examples. (xxxviii.)

Simplify the following fractions:

(1) $\dfrac{6}{5+\frac{3}{4}}$ (2) $\dfrac{7}{19-\frac{3}{11}}$ (3) $\dfrac{\frac{4}{5}}{7-\frac{2}{3}}$

(4) $\dfrac{6\frac{7}{8}}{11-\frac{5}{12}}$ (5) $\dfrac{\frac{4}{5}-\frac{3}{20}}{\frac{7}{10}-\frac{9}{40}}$ (6) $\dfrac{\frac{11}{16}-\frac{5}{24}}{\frac{7}{12}+\frac{3}{4}}$

(7) $\dfrac{2}{5+\dfrac{6}{9+\frac{3}{4}}}$ (8) $\dfrac{3}{2+\dfrac{1}{3+\frac{1}{3}}}$

(9) $\dfrac{5}{2-\dfrac{1}{4-\frac{2}{5}}}$ (10) $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{5}}}}$

85. If two brackets stand side by side, *with no sign between them*, as $(\frac{2}{3}+\frac{3}{4})\ (\frac{5}{6}-\frac{2}{7})$, it is implied that the contents of one bracket are to be *multiplied* by the contents of the other.

The following cases will illustrate the generally received usage in Arithmetic respecting these signs:

(1) *The operations indicated by "of," ×, and ÷ should be performed before adding or subtracting.*

Ex. (1).
$$\begin{aligned} & \tfrac{3}{4} + \tfrac{2}{3} \text{ of } \tfrac{9}{11} - \tfrac{1}{4} \div \tfrac{1}{3} + \tfrac{2}{3} \times \tfrac{6}{11} \\ &= \tfrac{3}{4} + (\tfrac{2}{3} \text{ of } \tfrac{9}{11}) - (\tfrac{1}{4} \div \tfrac{1}{3}) + (\tfrac{2}{3} \times \tfrac{6}{11}) \\ &= \tfrac{3}{4} + \tfrac{6}{11} - \tfrac{3}{4} + \tfrac{4}{11} \\ &= \tfrac{10}{11}. \end{aligned}$$

(2) *The operations indicated by × and ÷ should be performed in the order in which they occur.*

Ex. (2).
$$\begin{aligned} & \tfrac{2}{3} \times \tfrac{6}{11} \div \tfrac{3}{4} \\ &= \tfrac{2}{3} \times \tfrac{6}{11} \times \tfrac{4}{3} \\ &= \tfrac{16}{33}. \end{aligned}$$

Ex. (3).
$$\begin{aligned} & \tfrac{2}{3} \div \tfrac{6}{11} \times \tfrac{3}{4} \\ &= \tfrac{2}{3} \times \tfrac{11}{6} \times \tfrac{3}{4} \\ &= \tfrac{11}{12}. \end{aligned}$$

Ex. (4).
$$\begin{aligned} & \tfrac{3}{4} \times \tfrac{5}{6} \div \tfrac{2}{3} \times \tfrac{1}{4} \\ &= \tfrac{3}{4} \times \tfrac{5}{6} \times \tfrac{3}{2} \times \tfrac{1}{4} \\ &= \tfrac{15}{64}. \end{aligned}$$

(3) *The operation indicated by "of" should be performed before that indicated by* ÷; this is the only case in which custom makes a distinction between × and "of."

Ex. (5).

$$\begin{aligned} &\tfrac{3}{8} \text{ of } 2\tfrac{5}{7} \div 1\tfrac{6}{7} \text{ of } \tfrac{3}{4} \\ &= (\tfrac{3}{8} \times \tfrac{19}{7}) \div (\tfrac{13}{7} \times \tfrac{3}{4}) \\ &= \tfrac{3}{8} \times \tfrac{19}{7} \times \tfrac{7}{13} \times \tfrac{4}{3} \\ &= \tfrac{19}{26}. \end{aligned}$$

Examples. (xxxix.)

Simplify the following expressions:

(1) $3\tfrac{2}{5} \div (2\tfrac{1}{3} + 1\tfrac{5}{7})$.

(2) $(4\tfrac{3}{11} + 2\tfrac{1}{5}) \div 35\tfrac{3}{5}$.

(3) $\dfrac{2}{1 + \dfrac{5}{7 + \frac{2}{3}}}$.

(4) $\dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{2}}}$.

(5) $5 - \dfrac{3}{4 - \dfrac{5}{7 + \frac{2}{5}}}$.

(6) $\tfrac{3}{7} + \tfrac{5}{9}$ of $\tfrac{3}{10} - \tfrac{2}{21}$.

(7) $\tfrac{2}{3}$ of $\tfrac{5}{9} + \tfrac{3}{7} \div \tfrac{4}{5}$.

(8) $(\tfrac{11}{13} \div \tfrac{2}{7})$ of $7\tfrac{7}{12} - 1\tfrac{3}{5}$.

(9) $(\tfrac{4}{9} - \tfrac{3}{11})(2\tfrac{3}{4} + 3\tfrac{2}{3})$.

(10) $(\tfrac{3}{13} - \tfrac{2}{39}) \div (\tfrac{5}{78} + \tfrac{7}{156})$

(11) $\dfrac{(2 + \frac{1}{5}) \div (3 + \frac{1}{7})}{(\frac{1}{2} - \frac{1}{3}) \times (4 - 3\frac{3}{7})}$.

(12) $\dfrac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$.

86. We shall conclude this Chapter with a set of Miscellaneous Examples on Fractions.

Examples. (xl.)

(1) Add together

$$\tfrac{17}{33},\ \tfrac{5}{12},\ \tfrac{8}{44},\ \tfrac{3}{28},\ \tfrac{15}{56}.$$

(2) Add

$$\tfrac{2}{5} \text{ of } \tfrac{3}{7} \text{ to } \tfrac{3}{7} \text{ of } 2\tfrac{1}{3},$$

and multiply the result by

$$(\tfrac{2}{3} \text{ of } \tfrac{5}{6}) \div (\tfrac{3}{4} + \tfrac{4}{5}).$$

(3) Subtract

$$\tfrac{2}{3} \text{ of } \tfrac{5}{6} \text{ from } 1\tfrac{1}{2} \text{ of } \tfrac{4}{9},$$

and divide the result by

$$(\tfrac{2}{3} - \tfrac{4}{7}) \times (\tfrac{1}{5} - \tfrac{5}{8}).$$

(4) Simplify the fractions $\frac{221}{629}$, $\frac{72816}{8528}$ and find their product.

(5) Divide the product of $3\frac{2}{5}$ and $3\frac{3}{7}$ by the product of of $1\frac{5}{7}$ and $1\frac{13}{21}$.

(6) Multiply together the fractions of $4\frac{1}{3}$ $2\frac{3}{4}$, and add the result to $4\frac{3}{4}+3\frac{1}{3}$.

(7) Multiply the difference between $\frac{21}{72}$ and $\frac{201}{701}$ by the sum of $4\frac{7}{15}$ and $1\frac{3}{8}$; and multiply the result by the difference between $10\frac{2}{5}$ and $5\frac{2}{3}$.

(8) Simplify

$$\left(\tfrac{1}{3}+\tfrac{4}{7}\right)\frac{20\frac{1}{4}}{3\frac{6}{7}+2\frac{1}{4}}.$$

(9) Simplify

$$\left(3\tfrac{4}{5}+5\tfrac{1}{9}-\tfrac{1}{45}\right)\left(4\tfrac{1}{5}-3\tfrac{1}{4}\right)$$

divided by

$$1\tfrac{5}{11}+2\tfrac{1}{8}-\left(2\tfrac{9}{16}-\tfrac{1}{8}-\tfrac{1}{22}\right).$$

(10) Simplify

$$\left(1\tfrac{1}{3}+2\tfrac{2}{7}\right)\left(\frac{5\frac{1}{16}}{4\frac{6}{7}+1\frac{1}{4}}\right).$$

(11) Simplify

$$\left(7\tfrac{1}{9}+1\tfrac{4}{5}-\tfrac{1}{45}\right)\left(2\tfrac{1}{4}-\tfrac{4}{5}\right)$$

divided by

$$\left(4\tfrac{1}{8}-\tfrac{6}{11}\right)-\left(2\tfrac{7}{8}-\tfrac{7}{16}-\tfrac{1}{22}\right).$$

(12) Simplify

$$\frac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{7}} \text{ and } \left(\tfrac{5}{7}\text{ of }1\tfrac{6}{13}\right)\div\frac{2\frac{5}{7}}{3\frac{1}{4}}.$$

(13) Simplify

$$\cfrac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\frac{5}{13}}}} \text{ and } \cfrac{1}{4+\cfrac{1}{1-\cfrac{1}{2-\frac{9}{16}}}}.$$

(14) Simplify

$$\frac{10\frac{2}{5}-1\frac{5}{7}}{7\frac{1}{8}+3\frac{3}{40}} \text{ and } \left(\tfrac{3}{7}\text{ of }2\tfrac{1}{17}\right)\div\frac{1\frac{2}{3}}{2\frac{3}{7}}.$$

(15) Simplify

$$\frac{8\frac{7}{8}-7\frac{6}{7}+5\frac{5}{6}-4\frac{4}{5}}{9\frac{9}{10}-8\frac{13}{15}+7\frac{7}{8}-6\frac{6}{7}} \text{ and } 1\tfrac{21}{43}\times\tfrac{37797}{75008}.$$

(16) Simplify

$$\frac{5-\dfrac{1}{5-\frac{1}{5}}}{3-\dfrac{1}{3-\frac{1}{3}}}\times\tfrac{9}{23}\text{ of }7\text{ and }\frac{6+\dfrac{1}{6-\frac{1}{6}}}{4-\dfrac{1}{4-\frac{1}{4}}}\times 10\tfrac{8}{9}.$$

(17) Simplify

$$\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}}\times\tfrac{2}{11}\text{ of }365.$$

(18) Simplify

$$\frac{\frac{1}{21}\times5\frac{17}{23}\times6\frac{3}{11}+6\frac{19}{51}\times1\frac{23}{49}\div2\frac{5}{17}+1\frac{10}{49}}{9\frac{16}{57}\times1\frac{22}{23}\div5\frac{17}{38}+3\frac{11}{78}\times6\frac{17}{21}\div7\frac{21}{32}}\times12\tfrac{4}{9}.$$

(19) Simplify

$$\frac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times3\frac{33}{34}\div3\frac{37}{96}}{8\frac{17}{19}\times5\frac{14}{39}\div4\frac{15}{32}-7\frac{19}{20}\times5\frac{11}{65}\div14\frac{21}{25}}\times4\tfrac{8}{23}.$$

(20) Simplify

$$\frac{19}{7\times\dfrac{2}{3-1\frac{2}{3}}}\times\tfrac{7735}{67184}\div(1\tfrac{3}{16}-\tfrac{47}{48}).$$

(21) Simplify

$$\frac{1}{2+\dfrac{3}{4+\frac{5}{6}}}\times\tfrac{4862}{41147}\div(1\tfrac{1}{2}-\tfrac{23}{38}).$$

(22) Simplify

$$\frac{\dfrac{7}{4-\frac{5}{6}}-\dfrac{5}{6-\frac{3}{8}}}{\dfrac{4}{7-\frac{4}{7}}+\dfrac{2}{4-\frac{2}{5}}}\times\frac{\dfrac{1}{\frac{1}{2}-\frac{27}{59}}-13}{19-\dfrac{1}{\frac{1}{2}-\frac{6}{31}}}.$$

(23) Simplify

$$\frac{\dfrac{2}{3-\frac{4}{5}}+\dfrac{3}{4-\frac{5}{6}}}{\dfrac{3}{2-\frac{1}{4}}-\dfrac{1}{3-\frac{4}{5}}}\times\frac{\dfrac{1}{\frac{2}{3}-\frac{2}{9}}-\dfrac{1}{1\frac{1}{2}-\frac{19}{22}}}{\dfrac{1}{1\frac{1}{7}-\frac{1}{8}}-\dfrac{2}{6\frac{2}{3}-2\frac{5}{11}}}.$$

Examination Papers.

I.

(1) Explain how to reduce a mixed number to an improper fraction, and show the reason for each step.

(2) Bought $18\frac{3}{4}$ yards of silk at \$$2\frac{2}{5}$ a yard, and $27\frac{1}{2}$ lbs. of cheese at \$$\frac{3}{20}$ per lb.; how much money did I spend?

(3) How many times does the sum of $12\frac{4}{5}$ and $8\frac{7}{8}$ contain their difference?

(4) B, who owns $\frac{5}{11}$ of a ship, sells $\frac{4}{7}$ of his share for \$3600; what is the ship worth?

(5) There are two numbers whose sum is $4\frac{1}{5}$ and whose difference is $2\frac{4}{7}$; find the numbers.

II.

(1) What is meant by expressing one number as the fraction of another? Explain how to express $3\frac{1}{2}$ as the fraction of $9\frac{1}{5}$.

(2) How may the relative magnitude of two or more fractions be compared? Arrange the fractions $\frac{7}{15}$, $\frac{8}{35}$, $\frac{23}{44}$, $\frac{29}{56}$ in the order of magnitude.

(3) Add together $\frac{12}{91}$, $\frac{6}{77}$, and $\frac{9}{143}$, and find what is the least fraction with denominator 1000 which must be added in order that the sum may be greater than unity.

(4) Show that the value of $\frac{2+5}{3+7}$ lies between $\frac{2}{3}$ and $\frac{5}{7}$.

(5) A ship and her cargo are valued at \$60,000, and $\frac{3}{8}$ of the value of the ship is equal to $\frac{1}{4}$ of the value of the cargo; find the value of each.

III.

(1) Define Numerator and Denominator, and explain why they are appropriately applied to the terms of a fraction.

(2) If $\frac{4}{7}$ of $\frac{3}{5}$ of $2\frac{1}{2}$ bbls. of flour is worth \$$7\frac{1}{3}$, what is the value of $2\frac{2}{11}$ bbls.?

(3) If any number of fractions be equal, then any of them is equal to the fraction whose numerator is equal to the sum of all the numerators, and whose denominator is equal to the sum of all the denominators. Exemplify this in the case of six equal fractions.

(4) Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, and subtract the sum from 2; multiply the difference by $\frac{3}{5}$ of $\frac{27}{40}$ of 88, and find what fraction the product is of 999.

(5) A's age is $\frac{5}{12}$ of B's, and B's is $\frac{4}{7}$ of C's, and C 12 years ago was 72; what are their respective ages?

IV.

(1) Before adding fractions together, why is it necessary to change them to others having the same denominator?

(2) What number must be taken from $17\frac{1}{5}$ so that it may contain $3\frac{2}{7}$ an exact number of times?

(3) There is a number which divided by $8\frac{4}{17}$, and the quotient increased by $2\frac{3}{4}$ and the sum multiplied by $\frac{2\frac{1}{2}}{3}$, and the result diminished by $\frac{1}{2}$ of $\frac{3}{7}$ of $14\frac{4}{9}$ gives $2\frac{2}{3}$. Find the number.

(4) A bought a horse and carriage for $225, and paid for the harness $\frac{2\frac{1}{2}}{12}$ of what he paid for the horse. The carriage cost $\frac{7}{8}$ of the value of the horse. What was the price of each?

(5) Divide $8888 among A, B, and C, so that A may receive $88 less than 3 times B's share and C $176 more than one-half of A and B's shares.

V.

(1) Explain each step in the process of reducing a complex fraction to a simple one.

(2) Simplify $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 1$ divided by $3\frac{1}{3} \times 3\frac{1}{3} - 1$.

(3) What is the smallest sum of money with which A can purchase sheep at $$4\frac{1}{3}$ each, calves at $$5\frac{1}{6}$ each, or pigs at $$2\frac{1}{2}$ each; and how many of each can be bought with this sum?

(4) John spent $80 less than $\frac{2}{3}$ of his money at one time, and at another $40 more than $\frac{3}{7}$ of the remainder, and now has $40 left. How much had he at first?

(5) One-fourth of $\frac{2\frac{1}{3}}{7}$ of the length of a pole is in the mud; two-thirds of the remainder is in the water, and there are $5\frac{1}{2}$ feet in the air; what is the length of the pole?

VI.

(1) Show that $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5}$.

(2) Find three fractions whose numerators shall be 3, 5, and 7 respectively, and their sum equal to unity.

(3) From the sum of $3\frac{3}{4}$ and $4\frac{5}{6}$ subtract $6\frac{6}{7}$, multiply the difference by $2\frac{2}{5}$, and divide the product by $4\frac{1}{7}$.

(4) A sold a watch for $\frac{1}{5}$ more than it cost him to B, who sold it to C for $36, which was $\frac{1}{4}$ less than it cost him. What did the watch cost A?

(5) There are three rooms $21\frac{2}{3}$, $18\frac{3}{4}$, and $17\frac{7}{9}$ feet long respectively. Find the longest plain ruler with which the three rooms can be measured.

VII.

(1) Give a definition of multiplication that will apply to fractions.

(2) A person dies worth $40000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{3}{5}$ of her legacy to the son and the rest to the daughter; but the son adds his fortune to his sister's, and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this? and what fraction will her gain be of the whole?

(3) One half of a population can read; $\frac{21}{50}$ of the remainder can read and write; $\frac{4}{25}$ of the remainder can read, write, and cipher, while the rest, 243600, can neither read, write, nor cipher; what is the population?

(4) Three men, A, B, C, run round a circle in 5, 6, and $7\frac{1}{2}$ minutes respectively. If they start from the same point at the same time and run in the same direction, how long will they run before they are all together again? and how often will each have gone round it?

(5) A owned $\frac{5}{9}$ of a ship, and sold $\frac{3}{4}$ of his share to B, who sold $\frac{4}{7}$ of what he bought to C, who sold $\frac{9}{11}$ of what he bought to D; what part of the whole ship did each now own?

VIII.

(1) What are the advantages in arithmetical operations of employing fractions expressed by the smallest number possible? State how fractions expressed by large numbers may be reduced to equivalent fractions expressed by smaller numbers. Is this always possible?

(2) Is $\frac{4}{21}$ more nearly equal to $\frac{1}{6}$ or to $3\frac{1}{2} - 2\frac{20}{21} + \frac{1\frac{1}{4}}{3\frac{1}{4}}$ of $2\frac{1}{3} - 1\frac{1}{7}$, and by how much?

(3) Of the sovereigns who have reigned in England since the Norman conquest, there are $\frac{1}{6}$th of one name, $\frac{2}{9}$ths of another, $\frac{1}{12}$ of another, $\frac{1}{9}$ of each of two others, and $\frac{1}{18}$ of each of three others, and there are 5 besides; find how many sovereigns have reigned in England since the conquest.

(4) Three horses start from the same point, and at the same time, upon a race course 300 rods in circuit; the first horse passing over $\frac{1}{2}$ the circuit, the second $\frac{2}{5}$, the third $\frac{1}{3}$, in a minute. In how many minutes will they all be together again, and how far will each have travelled?

(5) Divide the difference of $13\frac{1}{3} \div \{(2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{1}{7}\}$ and $13\frac{1}{3} \div (2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{1}{7}$ by $13\frac{1}{3} \div 2\frac{6}{7} - 2\frac{8}{11} \times 1\frac{1}{7}$.

XII.—Decimal Fractions.

87. The multiples of 10 are 10, 20, 30, 40, 50, and so on. (Art. 39.)

The Powers of 10 are 10, 100, 1000, 10000, and so on, and these are called the first, second, third, fourth...... Powers of 10. (Art. 27.)

88. A Fraction, which has for its denominator one of the Powers of 10, is called a Decimal Fraction, or for shortness' sake, a Decimal. All other fractions are, by way of distinction, called Vulgar Fractions.

89. To save the trouble of writing the denominators of decimal fractions, a method of notation is used, by which we can express the value of the denominator in every case.

This method will be best explained by the following examples:

·3 stands for $\frac{3}{10}$, and is read thus, *three-tenths.*

·25 stands for $\frac{25}{100}$, and is read thus, *twenty-five hundredths.*

·347 stands for $\frac{347}{1000}$, and is read thus, *three hundred and forty-seven thousandths*

The figures which follow the Point · are those which form the *Numerator* of the fraction in each case.

The *number* of the figures which follow the Point corresponds to the number denoting the particular Power of 10, which forms the *Denominator* of the fraction in each case.

Now, as the first power of 10 is 1 followed by one zero, and the second power of 10 is 1 followed by two zeros, and the third power of 10 is 1 followed by three zeros, and so on, we can in every case write the denominator by affixing to 1 a number of zeros equal to the number of figures that follow the Point.

Thus, ·426789 stands for $\frac{426789}{1000000}$,

six zeros being affixed to the 1, because the number of figures that follow the Point is in this case *six*.

Again, ·07 stands for $\frac{7}{100}$,

·005 stands for $\frac{5}{1000}$,

·00025 stands for $\frac{25}{100000}$,

the zeros which come between the Point and the figures 7, 5, and 25, not being set down in the numerators of the fraction, as having no effect on the value of the numerators, seeing that 07 and 7 stand for the same number, and that 005 and 5 stand for the same number.

But these zeros affect the value of the denominators, as for instance,

$$\cdot 7 = \tfrac{7}{10}, \text{ while } \cdot 07 = \tfrac{7}{100}, \text{ and } \cdot 007 = \tfrac{7}{1000}.$$

90. Zeros *affixed* to a decimal have no effect on its value: that is,

·7, ·70, ·700 are all equal:

$$\text{for } \cdot 7 = \tfrac{7}{10},$$
$$\cdot 70 = \tfrac{70}{100} = \tfrac{7}{10},$$
$$\cdot 700 = \tfrac{700}{1000} = \tfrac{70}{100} = \tfrac{7}{10}.$$

91. The method of representing Decimal Fractions is merely an extension of the method by which Integers are represented, as will be seen from the following considerations.

As the *local* value of each digit increases tenfold as we advance from right to left, so does the local value of each decrease in the same proportion as we advance from left to right.

If, then, we affix a line of digits to the right of the units' place, each one of these having from its position a value, one-tenth part of the value which it would have if it were one place farther to the left, we shall have on the right hand of the units' place a series of fractions of which the denominators are successively 10, 100, 1000, ... while the numerators may be any numbers between 9 and zero.

Thus 246·4789

$$= 2 \times 100 + 4 \times 10 + 6 + \tfrac{4}{10} + \tfrac{7}{100} + \tfrac{8}{1000} + \tfrac{9}{10000}.$$

92. A number made up of an integer and a decimal, as 4·5, may be expressed in a fractional form by writing as the Numerator all the figures in the number, and as the Denominator 1 followed by as many zeros as there are figures *after the point.*

$$\text{Thus, } 4\cdot5 = \frac{45}{10},$$

$$\text{for } 4\cdot5 = 4 + \frac{5}{10} - \frac{40}{10} + \frac{5}{10} = \frac{45}{10}.$$

$$\text{Again, } 14\cdot075 = \frac{14075}{1000},$$

$$\text{for } 14\cdot075 = 14 + \frac{75}{1000} = \frac{14000}{1000} + \frac{75}{1000} = \frac{14075}{1000}.$$

Examples. (xli.)

Express, by means of fraction-symbols in their lowest terms,

(1) ·5. (2) ·25. (3) ·75. (4) ·375.
(5) ·00243. (6) ·0000725. (7) 14·8. (8) 104·235.
(9) 50·0004. (10) 100·001.

Express in the abbreviated form

(11) $\frac{9}{10}$. (12) $\frac{37}{100}$. (13) $\frac{4579}{10000}$.
(14) $\frac{3}{1000}$. (15) $\frac{17295}{100}$. (16) $\frac{59}{100000000}$.
(17) $\frac{25679}{1000000}$. (18) $\frac{325793}{100000}$. (19) $\frac{19}{10000}$.

93. We call

·5, 3·7, 15·9 decimal expressions of the *first* order,
·25, 4·39, 143·73 decimal expressions of the *second* order,
·043, 5·006, 27·009 decimal expressions of the *third* order,

the number of the order depending on the number of figures that *follow the point*.

The number denoting the order we call the INDEX of the order: thus 1 is the index of the *first* order, 2 of the *second* order, and so on.

94. From what is stated in Art. 90 we learn that a decimal of any order may be made into an equivalent decimal of a *higher* order by affixing one, two, three zeros, according as the index of the higher exceeds the index of the lower by 1, 2, 3.

Thus ·43 may be made into an equivalent decimal of the *fifth* order by affixing *three* zeros, thus, ·43000, and ·047 may be made into an equivalent decimal of the *seventh* order, by affixing *four* zeros, thus, ·0470000.

ADDITION OF DECIMAL FRACTIONS.

95. To add ·27 to ·45 we might proceed thus:

$$\cdot 27 = \tfrac{27}{100}$$
$$\cdot 45 = \tfrac{45}{100};$$
$$\therefore \cdot 27 + \cdot 45 = \tfrac{27}{100} + \tfrac{45}{100} = \tfrac{72}{100} = \cdot 72.$$

But we obtain the same result if we set down the decimals one under another, point under point, add the figures as if they stood for whole numbers, and place the point in the result under the other points, thus:

```
·27
·45
———
·72
———
```

96. If the decimals to be added be not of the same order, as for instance ·37 and ·049, we reason thus:

·049 is a decimal of the third order,

·37 is a decimal of the second order, but it can be made into an equivalent decimal of the third order by affixing a cipher, thus, ·370.

Then we proceed to add the decimals thus:

```
·370
·049
————
·419
————
```

Now, suppose we have to add more than two decimal expressions, as ·0074, ·72, ·05, and ·123456.

Of these four expressions the last is of the *sixth* order, and we may make the other three into equivalent decimals of the sixth order, and set them down thus:

```
·007400
·720000
·050000
·123456
———————
·900856
———————
```

When the learner is thoroughly acquainted with the principle on which this process of addition depends, he may omit the affixed zeros, since they have no effect on the result, and may write the sum just worked out in the following way:

·0074
·72
·05
·123456

·900856

If the numbers to be added be made up of integers combined with decimals, we keep the points in a vertical line, and proceed as in addition of integers.

Thus to add 4·27, 15·004, ·9007, and 23, we proceed thus:

4·2700	or thus,	4·27
15·0040		15·004
·9007		·9007
23·0000		23·
43·1747		43·1747

Examples. (xlii.)

Find the sum of

(1) ·275 and ·425. (2) ·007 and ·2394.
(3) ·001 and ·0002.
(4) 13·279, 3·00046, 742·000372.
(5) ·000493, 3·24, 15, 42·6, 324·42037.
(6) 49·327, ·458, 3317·05, 341·875, 32·4962.
(7) 700·372, 894·0009, ·347, ·00082, 5370·006.
(8) 560·379, ·45687, 350·0036, 7·074, 52·257.

SUBTRACTION OF DECIMAL FRACTIONS.

97. If we have to find the difference between ·47 and ·35, where both decimals are of the same order, and ·47 is the larger of the two, we proceed thus:

From	·47
Take	·35
Result	·12

performing an operation like that of Subtraction of Integers, and keeping the points in a vertical line.

That this method gives the correct result is evident, for

$$\cdot 47 - \cdot 35 = \tfrac{47}{100} - \tfrac{35}{100} = \tfrac{12}{100} = \cdot 12.$$

98. If we have to find the difference between ·888 and ·9, we may make the latter into a decimal of the third order, thus, ·900, and since this is larger than ·888, we proceed thus:

From ·900
Take ·888

Result ·012

If we have to find the difference between ·998 and 1, we observe that 1, being an integer, must be greater than ·998, which is a Proper Fraction, *i.e.*, $\frac{998}{1000}$, and we proceed thus:

From 1·000
Take ·998

Result ·002

Examples. (xliii.)

Find the difference between

(1) 56·429 and 5·218.
(2) 9·005 and 7·462.
(3) 53·316 and 5·0867.
(4) ·799 and ·8.
(5) 6·047 and 5·9863.
(6) 850·007 and 270·8796.
(7) ·0000086 and ·00001.
(8) ·00537 and ·000985.
(9) 10 and ·0002.
(10) ·09999 and ·101.

MULTIPLICATION OF DECIMALS.

99. In finding the product of ·12 and ·11, we might proceed thus:

$$\cdot 12 \times \cdot 11 = \tfrac{12}{100} \times \tfrac{11}{100} = \frac{12 \times 11}{100 \times 100} = \tfrac{132}{10000} = \cdot 0132,$$

the result being a decimal of the *fourth* order.

Again, if we have to find the product of 4·32 and ·00012:

$$4\cdot 32 \times \cdot 00012 = \tfrac{432}{100} \times \tfrac{12}{100000} = \tfrac{5184}{10000000} = \cdot 0005184,$$

the result being a decimal of the *seventh* order.

And, generally, the product of any two decimal expressions is a decimal expression of an order whose index is the sum of the indices of the orders of the two expressions.

Hence, we deduce the following rule for Multiplication of Decimals:

Multiply as in the case of integers, and mark off in the product a number of decimal places equal to the sum of the number of decimal places in the two factors.

For example, to multiply 2·4327 by 4·23.

```
   2·4327
     4·23
 --------
    72981
   48654
  97308
 --------
 10·290321
```

Again, to multiply 43·672 by ·00000047.

```
    43·672
 ·00000047
 ---------
    305704
   174688
 ---------
   2052584
```

We have now to mark off eleven decimal places from this product, and as the product contains only seven figures, we must prefix four zeros, and put the point on the left of these, thus, ·00002052584, and this will be the required product.

One more case must be considered.

Suppose we have to multiply ·235 by ·48.

```
  ·235
   ·48
 -----
  1880
  940
 -----
 ·11280
```

This decimal of the *fifth* order is equivalent to a decimal of the *fourth* order, ·1128 (Art. 90), and this is the simplest form of the result.

Examples. (xliv.)

Multiply

(1) 7·5 by 4·7. (2) 3·62 by 5·23.
(3) ·427 by ·235. (4) ·562 by ·00074.
(5) 3·00704 by 4·0205. (6) ·0009 by 1000.
(7) 623·4075 by 24·0259. (8) ·00746 by ·006235.
(9) 1432·6749 by ·00004030705.
(10) 50704·042 by ·004007090061.

Find the value of the following:

(11) ·407 × 4·03 × ·006.
(12) 1·01 × 1000 × ·001.
(13) ·52 × ·007 × 4·3 × ·02.

Find the continued product of

(14) ·07, 4·6, ·009, and 52·47.
(15) 42·6, ·795, 4·03, and ·00074.
(16) What is the cube of 2·74?
(17) Raise 3·5 to the fourth power.

DIVISION OF DECIMALS.

100. If we have to divide ·27 by 3, we might proceed thus:

$$\cdot 27 \div 3 = \tfrac{27}{100} \div 3 = \tfrac{9}{100} = \cdot 09.$$

Again, if we have to divide ·00625 by 25, we might proceed thus:

$$\cdot 00625 \div 25 = \tfrac{625}{100000} \div 25 = \tfrac{25}{100000} = \cdot 00025.$$

In both cases *the Quotient is a decimal of the same order as the dividend.*

Hence we derive the following Rule:

If the Divisor be an integer, perform the operation of Division as if the Dividend were also an integer, and mark off in the Quotient as many decimal places as there are decimal places in the Dividend.

For example, suppose we have to divide ·0086751 by 243.

$$\begin{array}{r@{\;}l}
243\ )\ \cdot0086751\ (\ 357 & \\
729\ \ \ \ \ \ \ & \\
\hline
1385\ \ \ \ \ \ \ & \\
1215\ \ \ \ \ \ \ & \\
\hline
1701\ \ \ \ \ \ \ & \\
1701\ \ \ \ \ \ \ & \\
\hline
\end{array}$$

The Quotient is to be a decimal of the *eighth* order,

∴ the result is ·00000357.

101. Next observe that, if the divisor be a *decimal* expression, *we can in every case change it into an Integer* by a process which we shall now explain.

If we multiply a decimal expression

by 10, the effect is to move the point one place to the right ;
by 100, the effect is to move the point two places to the right;
by 1000, the effect is to move the point three places to the right ;

and so on.

For instance, 123·456 × 10 = 124·56,
and 123·456 × 100 = 12345·6.

The reason is obvious,

for $123{\cdot}456 \times 10 = \frac{123456}{1000} \times 10 = \frac{123456}{100} = 1234{\cdot}56$,
and $123{\cdot}456 \times 100 = \frac{123456}{1000} \times 100 = \frac{123456}{10} = 12345{\cdot}6$.

Hence we can transform any Divisor into an Integer by multiplying it by 10, 100, 1000, ... according as the Divisor is a decimal of the first, second, third .... order.

For example, if the Divisor be ·000492, and we multiply it by 1000000, we transform it into the Integer 492.

Now, we may multiply a Divisor by any number, if we multiply the Dividend by the same number.

For instance, if the Divisor be 8 and the Dividend 32, we may multiply each by 10,

so that the Divisor becomes 80, and the Dividend 320 ;

and whether we divide 32 by 8, or 320 by 80, the Quotient will be the same number, that is, 4.

102. We can now lay down a general Rule for Division of Decimals.

If the Divisor be a decimal, change it into an Integer by removing the point a sufficient number of places to the right, and also remove the point in the Dividend the same number of places to the right. Divide as in the case of integers. Then, if the Dividend be an integer, the Quotient will be an integer, and if the Dividend be a decimal, the Quotient will be a decimal of the same order.

The process will be better understood from the following examples.

Ex. (1). Divide ·625 by ·025.

$$\cdot625 \div \cdot025 = \frac{\cdot625}{\cdot025} = \frac{625}{025} = \frac{625}{25}$$

$$\begin{array}{r} \cdot25\ )\ 625\ (\ 25 \\ 50 \\ \hline 125 \\ 125 \\ \hline \end{array}$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

$\therefore$ the Quotient required is 25.

Ex. (2). Divide 108·997 by 2·3.

$$108\cdot997 \div 2\cdot3 = \frac{108\cdot997}{2\cdot3} = \frac{1089\cdot97}{23\cdot} = \frac{1089\cdot97}{23}$$

$$\begin{array}{r} 23\ )\ 1089\cdot97\ (\ 4739 \\ 92 \\ \hline 169 \\ 161 \\ \hline 89 \\ 69 \\ \hline 207 \\ 207 \\ \hline \end{array}$$

Here the Quotient is a decimal of the *second* order, because the Dividend is a decimal of the second order;

∴ the Quotient required is 47·39.

Ex. (3). Divide ·625 by ·00025.

$${\cdot}625 \div {\cdot}00025 = \frac{{\cdot}625}{{\cdot}00025} = \frac{62500{\cdot}}{00025{\cdot}} = \frac{62500}{25}.$$

$$\begin{array}{l} 25\ )\ 62500\ (\ 2500 \\ 50 \\ \overline{125} \\ 125 \\ \overline{\ 00} \end{array}$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

∴ the Quotient required is 2500.

Ex. (4). Divide ·00169 by 1·3.

$${\cdot}00169 \div 1{\cdot}3 = \frac{{\cdot}00169}{1{\cdot}3} = \frac{0{\cdot}0169}{{\cdot}13} = \frac{{\cdot}0169}{13}.$$

$$\begin{array}{l} 13\ )\ {\cdot}0169\ (\ 13 \\ \phantom{13\ )\ {\cdot}0}13 \\ \phantom{13\ )\ {\cdot}0}\overline{39} \\ \phantom{13\ )\ {\cdot}0}39 \end{array}$$

Here the Quotient is a decimal of the *fourth* order, because the Dividend is a decimal of the fourth order;

∴ the Quotient is ·0013.

Ex. (5). Divide 625 by ·25.

$$625 \div {\cdot}25 = \frac{625}{{\cdot}25} = \frac{62500}{25}.$$

$$\begin{array}{l} 25\ )\ 62500\ (\ 2500 \\ 50 \\ \overline{125} \\ 125 \\ \overline{\ 00} \end{array}$$

Here the Quotient is an *Integer*, because the Dividend is an Integer;

$$\therefore 625 \div \cdot 25 = 2500.$$

These are cases of *exact* division, that is, when, on the process of division being carried out, *there is no remainder.*

Examples. (xlv.)

Divide

(1) 1·296 by ·108.
(2) 17·28 by ·0012.
(3) ·00169 by 1·3.
(4) 2921 by ·23.
(5) 15633·0062 by 362·9.
(6) 1 by ·0001.
(7) ·03096 by ·000072.
(8) ·7644 by ·0052.
(9) ·0000615228 by 307.
(10) 746·44808 by 7·58.
(11) ·24294591 by 36·9.
(12) 63987·42 by ·000073.
(13) ·26986365 by 3500.
(14) 26986·14 by ·00009.
(15) ·00131053 by ·0065.
(16) 617325 by ·00025.
(17) ·830676 by ·000231.
(18) ·00019517 by 673.
(19) 1·0191 by ·00079.
(20) 2078·61 by 579.
(21) 241·16047 by ·527.
(22) ·65220834 by ·00854.
(23) 4700460·66583 by ·00518963.

103. We next take the following example:

Divide 347 by ·64.

$$\text{Here } 347 \div \cdot 64 = \frac{347}{\cdot 64} = \frac{34700\cdot}{64\cdot} = \frac{34700}{64}$$

and we proceed thus:

$$\begin{array}{r@{}l} 64\ )\ 34700 & \ (\ 542 \\ 320 & \\ \hline 270 & \\ 256 & \\ \hline 140 & \\ 128 & \\ \hline 12 & \end{array}$$

We have then the Quotient 542, and Remainder 12.

If we wish to carry on the division further, we may do so by placing a decimal point at the end of the Dividend, and affixing as many zeros as we please, observing that all the figures which will come after those already in the Quotient will be decimals.

The operation, completed from the outset, will stand thus:

$$\begin{array}{l}
64\ )\ 34700\cdot0000\ (\ 542\cdot1875 \\
320 \\
\overline{270} \\
256 \\
\overline{140} \\
128 \\
\overline{120} \\
64 \\
\overline{560} \\
512 \\
\overline{480} \\
448 \\
\overline{320} \\
320 \\
\overline{}
\end{array}$$

Examples. (xlvi.)

Divide

(1) 7·45 by ·32.

(2) 14·327 by 12·8.

(3) 43·26 by 12·5.

(4) 7432·976 by ·225.

(5) 1·2 by 625.

(6) ·217 by 1250.

104. The student is now to observe that, by employing Short Division, the example just worked out may be put in a very concise form. Thus, taking up the work at the point where we have to divide 34700 by 64, we proceed thus:

$$\begin{array}{r|l}
8 & 34700\cdot0 \\ \hline
8 & 4337\cdot5000 \\ \hline
 & 542\cdot1875 \text{ Quotient.}
\end{array}$$

So, also, if we have to divide 43672·509 by 36, we proceed thus:

$$\begin{array}{r|l}
4 & 43672\cdot50900 \\ \hline
9 & 10918\cdot12725 \\ \hline
 & 1213\cdot12525 \text{ Quotient.}
\end{array}$$

Again, to divide ·0000013932 by 32, we proceed thus·

$$\begin{array}{r|l} 4 & \underline{\cdot 0000013932} \\ 8 & \underline{\cdot 0000003483000} \\ & \cdot 0000000435375 \text{ Quotient.} \end{array}$$

NOTE.—Division by 10, 100, 1000...is effected by moving the decimal place in the Dividend one, two, three...places to the left.

Thus 24·6 ÷ 10 = 2·46.
·47 ÷ 100 = ·0047.

Examples. (xlvii.)

Employ Short Division in finding the Quotient when we divide

(1) 426·478 by 16.   (2) ·07849782 by 72.
(3) 362·47 by ·025.   (4) ·00007263 by 4·5.
(5) 42·007437 by ·24.   (6) ·00463 by 50.
(7) 2·4715 by ·000016.   (8) 9000 by ·00036.
(9) ·001 by 100.   (10) ·001001001 by 2000.

N. B.—The process of Division may often be shortened by multiplying the Dividend and Divisor by a number which will transform the Divisor into a power or a multiple of 10; thus, if we have to divide 24·46927151 by 12·5, we multiply both by 8.

Then $\frac{24\cdot46927151}{12\cdot5} = \frac{195\cdot75417208}{100} = 1\cdot9575417208.$

105. In the examples hitherto given the cases are all those of *exact* division.

In all cases we may proceed with the division till there is no remainder, or till certain figures in the Quotient recur again and again in the same order.

We shall give an example of this recurrence of figures in Art. 106, but first we must observe that we often require to find the Quotient *up to a certain place of decimals.*

For example, suppose we have to find the Quotient arising from the division of 2·47 by ·37, to four places of decimals.

$$2{\cdot}47 \div {\cdot}37 = \frac{2{\cdot}47}{{\cdot}37} = \frac{247}{37}.$$

$$\begin{array}{l}
37\ )\ 247{\cdot}0000\ (\ 6{\cdot}6756 \\
\quad\quad\ 222 \\
\quad\quad\ \overline{250} \\
\quad\quad\ 222 \\
\quad\quad\ \overline{280} \\
\quad\quad\ 259 \\
\quad\quad\ \overline{210} \\
\quad\quad\ 185 \\
\quad\quad\ \overline{250} \\
\quad\quad\ 222 \\
\quad\quad\ \overline{}
\end{array}$$

Hence, the Quotient, correct to four places of decimals, is 6·6756.

Examples. (xlviii.)

Find the Quotient to three places of decimals when we divide

(1)	42·5 by ·0023.	(2)	·197 by ·79.
(3)	37·9 by 409.	(4)	27100 by ·00313.
(5)	·0269 by ·281.	(6)	229 by ·007.

106. If we continue the division further in the example given in Art. 105, we find the figures 756 coming again and again in the same order in the Quotient, so that the Quotient is 6·6756756756 . . . without any termination.

Let us now take this example.

Divide 90 by ·0011.

$$\text{Here } 90 \div {\cdot}0011 = \frac{90}{{\cdot}0011} = \frac{900000}{11}$$

$$\begin{array}{r}
11\ \underline{)\ 900000} \\
81818
\end{array}$$

Up to this point the Quotient is an Integer: but, if we proceed further with the division, we shall obtain a decimal expression: thus, if we affix two more zeros, preceded by a decimal point, to the dividend, we shall have

$$\begin{array}{r}
11\ \underline{)\ 900000{\cdot}00} \\
81818{\cdot}18
\end{array}$$

If we carry on the division to any extent, we shall have the two figures 18 coming again and again in the same order. A decimal of this kind is called *Periodic*, *Circulating*, or *Recurring*.

107. The extent of the Period is denoted by placing a dot over the *first*, and another dot over the *last* of the figures in it.

Thus 18 denotes a decimal *of an order such that it can be represented by no finite index*, since it runs on ·18181818 . . . to an infinite number of figures.

So also, $6{\cdot}\dot{7}5\dot{6}$ stands for 6·756756756 .. ..
$\cdot\dot{0}4\dot{7}$ stands for ·047047047 .. ..
$\cdot4\dot{3}7\dot{2}$ stands for ·4372372372 .. ..
$26{\cdot}04\dot{7}\dot{9}$ stands for 26·04797979 .. ..
$\cdot0002\dot{6}$ stands for ·000266666 .. ..

108. A Vulgar Fraction may be converted into a Decimal Fraction by the following process:

Reduce the fraction to its lowest terms, and then find the Quotient resulting from the division of the numerator by the denominator by the rule for division of decimals.

Thus, to reduce $\frac{3}{8}$ to a decimal, we proceed thus:

```
8 ) 3·000
    -----
     ·375
```

$\therefore \frac{3}{8} = {\cdot}375.$

Again, to reduce $\frac{47}{32}$ to a decimal, we proceed thus:

```
32 ) 47·00000 ( 1·46875
     32
     ---
     150
     128
     ---
      220
      192
      ---
       280
       256
       ---
        240
        224
        ---
         160
         160
         ---
```

$\therefore \frac{47}{32} = 1{\cdot}46875.$

Or, we might work by Short Division, thus:

$$\begin{array}{r|l} 4 & 47\cdot00 \\ \hline 8 & 11\cdot75 \\ \hline & 1\cdot46875 \end{array}$$

Again, to reduce $\frac{1}{7}$ to a decimal, we proceed thus:

$$\begin{array}{r|l} 7 & 1\cdot00000000 \\ \hline & \cdot14285714\ldots. \end{array}$$

$$\therefore \tfrac{1}{7} = \cdot\dot{1}4285\dot{7}.$$

109. *To show that, when a Vulgar Fraction is reduced to a Decimal, either the operation must terminate or the figures of the Quotient must recur in the same order.*

Consider the operation by which such a fraction as $\frac{1}{7}$ is reduced to a decimal. The only remainders that can occur are 0, 1, 2, 3, 4, 5, 6. If the remainder 0 should occur, the division terminates: if not, we can only have *six* different remainders, and when any of these occurs a second time, we must have a recurrence of the former remainders in the same order.

When a fraction in its lowest terms is reduced to a decimal and produces a recurring decimal, the *extreme* limit of the number of places in the *period* of the recurring decimal is one less than the denominator.

Thus $\frac{1}{7}$ produces a recurring decimal of 6 places.
$\frac{2}{19}$ produces a recurring decimal of 18 places.
$\frac{3}{29}$ produces a recurring decimal of 28 places.

110. When a Vulgar Fraction is in its lowest terms it can only be expressed as an Exact Decimal when the denominator is composed of factors, each of which is one of the numbers 2 and 5.

Thus $\frac{3}{8}$ can be expressed as an exact decimal because $8 = 2\times2\times2$.

$\frac{3}{20}$ can be expressed as an exact decimal because $20 = 2\times2\times5$.

$\frac{4}{125}$ can be expressed as an exact decimal because $125 = 5\times5\times5$.

The reason for this is, that no Vulgar Fraction can be expressed as an Exact Decimal unless it can be transformed to one which has 10, or some power of 10, for its denominator. Now, no number can by multiplication be made a power of 10 unless it be composed of factors each of which is 2 or 5.

Thus 8 can be made into a power of 10 by multiplying it by 5 × 5 × 5.

125 can be made into a power of 10 by multiplying it by 2 × 2 × 2.

40 can be made into a power of 10 by multiplying it by 5 × 5.

$$\text{Hence } \frac{3}{8} = \frac{3}{2\times2\times2} = \frac{3\times5\times5\times5}{2\times2\times2\times5\times5\times5} = \frac{375}{1000} = \cdot375.$$

$$\frac{7}{125} = \frac{7}{5\times5\times5} = \frac{7\times2\times2\times2}{5\times5\times5\times2\times2\times2} = \frac{56}{1000} = \cdot056.$$

$$\frac{9}{40} = \frac{9}{2\times2\times10} = \frac{9\times5\times5}{2\times2\times10\times5\times5} = \frac{225}{1000} = \cdot225.$$

But such numbers as 7, 12, 30, cannot be made into powers of 10 by multiplication, and hence $\frac{3}{7}$, $\frac{5}{12}$, $\frac{11}{30}$, cannot be reduced to exact decimals.

It may also be remarked that, when a Vulgar Fraction in its lowest terms is reduced to an exact decimal, the order of that decimal is expressed by the greatest number of times that either of the factors 2 or 5 occurs in the denominator.

Examples. (xlix.)

Convert into decimals the following vulgar fractions:

(1) $\frac{7}{20}$. (2) $\frac{11}{25}$. (3) $\frac{6}{7}$. (4) $\frac{1}{99}$.
(5) $\frac{1}{999}$. (6) $\frac{1}{41}$. (7) $\frac{11}{21}$. (8) $\frac{13}{60}$.
(9) $\frac{17}{1375}$. (10) $\frac{129}{55}$.

CONTRACTIONS IN MULTIPLICATION AND DIVISION OF DECIMALS.

111. When the number of decimal places is great the figures obtained by the ordinary mode of multiplication are often unnecessarily numerous. Thus, in multiplying 62·37416 by 2·34169 by the ordinary method, there would be ten places of decimals in the product, while for all practical purposes three or four are quite enough.

Ex. Multiply 62·37416 by 2·34169 so as to retain only 4 places of decimals.

ORDINARY METHOD.		CONTRACTED METHOD.	
62·37416		62·37416	
2·34169		96143·2	
56	136744	1247483	= 623741 × 2 + 1
374	24496	187122	= 62374 × 3
623	7416	24950	= 6237 × 4 + 2
24949	664	624	= 623 × 1 + 1
187122	48	374	= 62 × 6 + 2
1247483	2	56	= 6 × 9 + 2
146·0609	467304	146·0609	

By comparing the contracted method with the ordinary method, the reason of the preceding operation will be readily understood.

Since the product of any order of units by units is of the same order as the figure multiplied, the units' figure of the multiplier is written under the place to be retained. For convenience, the other figures are written in an inverted order. Now (Art. 99) 4, a decimal of the *third* order, multiplied by 3, a decimal of the *first* order, will give a decimal of the *fourth* order; also, 7, a decimal of the *second* order, multiplied by 4, a decimal of the *second* order, will give a decimal of the *fourth* order, etc., etc.

Now, to the product of 2 and 1, 1 must be added: since, if 6 had not been rejected, there would have been 1 to carry; then the other figures are multiplied in the usual way. Next, multiply 4 by 3 and set down 2 under the 3, and multiply the other figures by 3 in the usual way.

Next, multiply 7 by 4, and to the product add 2: since, if 416 had not been rejected the product would have approximated to 2 thousand, etc.

Hence we have the following rule:

Write the Multiplier with the order of its figures reversed under the Multiplicand, so that the units' figure may be under that figure of the Multiplicand which is the lowest decimal to be retained in the Product. Then multiply by each figure of the Multiplier, neglecting all the figures of the Multiplicand to the right of it, except to find what is to be carried, and carrying one more when the rejected part of any product is 5 or greater than 5. Arrange the partial products so that their right-hand figures may stand in the same vertical column. Their sum will be the product required. From this product cut off the desired number of decimal places.

112. When the divisor consists of several figures, the work will be much shortened by cutting off a figure from the divisor at each successive step of the division, instead of annexing a figure to the dividend. Care must be taken to increase each product by what would have been carried if the figure or figures had not been cut off.

Ex. (1). Divide 3·784169 by 2·716418 correct to three places of decimals.

$$\begin{array}{r@{\ )\ }r@{\ (\ }l}
2716\underset{\cdot}{4}\underset{\cdot}{1}\underset{\cdot}{8} & 3784169 & 1393 \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{2716418} & \\ \cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{1067751} & \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{814925} & \\ \cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{252826} & \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{244478} & \\ \cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{8348} & \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{8149} & \\ \cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{199} &
\end{array}$$

By comparing the units of the highest order in the divisor with the units of the same order in the dividend, it is evident that there must be one figure to the left of the point in the quotient; hence the answer is 1·393.

Ex. (2). Divide 763·14163 by 21·3642 correct to four places of decimals.

ORDINARY METHOD.	CONTRACTED METHOD.
213642) 76314163 (357205	213642) 76314163 (357205
640926	640926
122215 6	122215
106821 0	106821
15394 63	15394
14954 94	14955
439 690	439
427 284	427
12 40600	12
10 68210	11
1 72390	1

Here the figures of the quotient are 357205, and by comparing the 2 tens of the divisor with the 76 tens of the dividend, it is plain there must be 2 places to the left of the point; hence the quotient is 35·7205.

From considering these cases we have the following rule:

Compare the left-hand figure of the divisor with the units of the same order in the dividend, and thus determine the position of the point in the quotient. Then divide as in Ex. (1), dropping a figure from the right of the divisor at each step of the division.

Note.—Care should be taken to mark the figures dropped by placing a dot or other mark beneath them.

Examples. (1.)

(1) ·863541 × ·10983 to five places of decimals.
(2) ·053407 × ·047126 to six places of decimals.
(3) 3·141592 × 52·7438 to four places of decimals.
(4) 325·701428 × ·7218393 to three places of decimals.
(5) 3·1729432 × 8·316259 to four places of decimals.
(6) 2·3748 ÷ 1·4736 to three places of decimals.
(7) 31·47 ÷ 839·27656 to four places of decimals.
(8) 252070·520751 ÷ 591·57 to three places of decimals.
(9) 73·64 ÷ ·43232 to four places of decimals.
(10) 6·5555 ÷ 7·06249 to three places of decimals.

RECURRING DECIMALS.

113. *Pure* Recurring Decimal Fractions are those in which the period commences immediately after the decimal point:

Thus $\cdot\dot{3}$, $\cdot\dot{2}\dot{7}$, $\cdot\dot{0}42\dot{9}$ are pure recurring decimals.

Mixed Recurring Decimal Fractions are those in which one or more figures precede the period:

Thus $\cdot 2\dot{3}$, $\cdot 24\dot{2}\dot{7}$, $\cdot 35\dot{0}42\dot{9}$ are mixed recurring decimals.

114. *To find the Vulgar Fraction which is equivalent to a given Pure Recurring Decimal.*

Ex. (1). Find the Vulgar Fraction equivalent to $\cdot\dot{3}$.

The decimal = ·333....
From 10 times the decimal, or 3·333....
take the decimal, or ·333....

Then 9 times the decimal = 3·000....
∴ the decimal = $\frac{3}{9} = \frac{1}{3}$.

Ex. (2). Find the Vulgar Fraction equivalent to $\cdot\dot{2}4\dot{7}$.

The decimal = ·247247....
From 1000 times the decimal, or 247·247....
take the decimal, or ·247....

Then 999 times the decimal = 247·000....
∴ the decimal = $\frac{247}{999}$.

Ex. (3). Find the Vulgar Fraction equivalent to $\dot{0}42\dot{3}$.

The decimal = ·04230423....
From 10000 times the decimal, or 423·0423....
take the decimal, or ·0423....

Then 9999 times the decimal = 423·0000....
∴ the decimal = $\frac{423}{9999} = \frac{47}{1111}$.

Examples. (li.)

Convert into Vulgar Fractions in their lowest terms:

(1) $\cdot\dot{6}$. (2) $\cdot\dot{2}\dot{7}$. (3) $\cdot\dot{0}4\dot{5}$. (4) $\cdot\dot{3}12\dot{3}$.

(5) $\cdot\dot{0}07\dot{2}$. (6) $\cdot\dot{4}02\dot{3}$. (7) $\cdot\dot{0}005\dot{4}$. (8) $\cdot\dot{0}000\dot{9}$.

115. Hence we deduce the following rule for reducing a Pure Recurring Decimal to a Vulgar Fraction:

Take one of the periods to form the numerator, and for the denominator the number formed by repeating 9 as many times as there are figures in the period.

$$\text{Thus } \cdot\dot{7} = \tfrac{7}{9}.$$
$$\cdot\dot{0}\dot{5} = \tfrac{5}{99}.$$
$$\cdot\dot{4}32\dot{7} = \tfrac{4327}{9999}.$$

116. *To find the Vulgar Fraction which is equivalent to a given Mixed Recurring Decimal.*

Ex. (1). Find the Vulgar Fraction equivalent to $\cdot 2\dot{3}\dot{7}$.

$$\begin{array}{rr}
\text{The decimal} = \cdot 23737\ldots\ldots & \\
\text{From 1000 times the decimal, or} & 237\cdot 37\ldots\ldots \\
\text{take 10 times the decimal, or} & 2\cdot 37\ldots\ldots \\ \hline
\text{Then 990 times the decimal} = & 235\cdot 00\ldots\ldots
\end{array}$$
$$\therefore \text{ the decimal} = \tfrac{235}{990} = \tfrac{47}{198}.$$

Ex. (2). Find the Vulgar Fraction equivalent to $\cdot 04\dot{7}2\dot{6}$.

$$\begin{array}{rr}
\text{The decimal} = \cdot 04726726. & \\
\text{From 100000 times the decimal, or} & 4726\cdot 726\ldots\ldots \\
\text{take 100 times the decimal, or} & 4\cdot 726\ldots\ldots \\ \hline
\text{Then 99900 times the decimal} = & 4722\cdot 000\ldots\ldots
\end{array}$$
$$\therefore \text{ the decimal} = \tfrac{4722}{99900} = \tfrac{787}{16650}.$$

Ex. (3). Find the Vulgar Fraction equivalent to $3\cdot 1\dot{4}$.

$$\begin{array}{rr}
\text{The decimal} = 3\cdot 1444\ldots\ldots & \\
\text{From 100 times the decimal, or} & 314\cdot 44\ldots \\
\text{take 10 times the decimal, or} & 31\cdot 44\ldots\ldots \\ \hline
\text{Then 90 times the decimal} = & 283\cdot 00\ldots\ldots
\end{array}$$
$$\therefore \text{ the decimal} = \tfrac{283}{90}.$$

Examples. (lii.)

Convert into Vulgar Fractions in their lowest terms:

(1) $\cdot 4\dot{2}\dot{5}$. (2) $\cdot 47\dot{5}\dot{9}$. (3) $4\cdot 25\dot{3}$. (4) $\cdot 004\dot{2}\dot{6}$.
(5) $53\cdot 00\dot{2}4\dot{3}$. (6) $7\cdot 2\dot{0}1\dot{1}$. (7) $2\cdot 53\dot{0}\dot{6}$.

117. Hence we deduce the following rule for reducing a Mixed Recurring Decimal to a Vulgar Fraction:

Form the Numerator by taking from the figures up to the end of the first period the figures that precede the first period; and form the Denominator by setting down 9 as many times as there are figures in the period, and affixing 0 as many times as there are figures between the decimal point and the first period.

$$\text{Thus}\quad \cdot 2\dot{4}\dot{5} = \frac{245-2}{990} = \frac{243}{990}.$$

$$\cdot 004\dot{7}\dot{3} = \frac{473-4}{99000} = \frac{469}{99000}.$$

$$4\cdot\dot{5} = \frac{45-4}{9} = \frac{41}{9}$$

$$7\cdot 34\dot{5} = \frac{7345-734}{900} = \frac{6611}{900}.$$

118. The method of performing arithmetical operations with Recurring Decimals will be best explained by taking the operations separately.

I. Addition.

Find the sum of $3\cdot 4\dot{9}$, $4\cdot\dot{0}4\dot{7}$, and $\cdot 1\dot{4}\dot{6}3$.

First make the decimals all of the same order, thus:

$$3\cdot 499\dot{9},\ 4\cdot 0\dot{4}7\dot{0},\ \cdot 14\dot{6}\dot{3}.$$

Then, since the periods consist of 1, 3, 2 figures respectively, and the L. C. M. of 1, 3, and 2 is 6, carry on all the decimals *six* places further, thus:

$$\begin{array}{r} 3\cdot 4999999999 \\ 4\cdot 0470470470 \\ \cdot 1463636363 \\ \hline 7\cdot 69\dot{3}4106\dot{8} \end{array}$$

II. Subtraction.

Here we proceed on the same principle as in Addition.

Thus to subtract $5\cdot 24\dot{7}$ from $8\cdot\dot{0}5\dot{9}$:

$$\begin{array}{r} 8\cdot 059059 \\ 5\cdot 247777 \\ \hline 2\cdot 81\dot{1}2\dot{8} \end{array}$$

In both operations some care is requisite in observing what figure would be carried on if the columns omitted were taken into account.

III. In Multiplication and Division the recurring decimals should be converted into vulgar fractions, and when the product or quotient of these fractions has been found, it may be converted into a decimal.

Thus, $4{\cdot}\dot{5} \times 3{\cdot}\dot{7} = \frac{45-4}{9} \times \frac{37-3}{9} = \frac{41}{9} \times \frac{34}{9} = \frac{1394}{81}$,

and ${\cdot}0\dot{5} \div {\cdot}04\dot{2} = \frac{5}{90} \div \frac{38}{900} = \frac{5}{90} \times \frac{900}{38} = \frac{5 \times 10}{38} = \frac{25}{19}$.

We may then, if it be required, convert $\frac{1394}{81}$ and $\frac{25}{19}$ into decimals by the process explained in Art. 108.

Examples. (liii.)

Find the value of the following expressions:

(1) $2{\cdot}\dot{5}7 + {\cdot}\dot{0}4\dot{3} + 13{\cdot}\dot{2}$.   (2) $14{\cdot}\dot{7}\dot{6}2 + 3{\cdot}\dot{5}4\dot{9} + 2{\cdot}20\dot{4}$.

(3) $15{\cdot}0\dot{2}\dot{5} - 13{\cdot}\dot{2}4\dot{7}$.   (4) ${\cdot}02\dot{4}\dot{6} - {\cdot}00\dot{3}9\dot{7}$.

(5) $3{\cdot}\dot{7} \times 5{\cdot}\dot{4}\dot{9}$.   (6) ${\cdot}007\dot{2} \times {\cdot}\dot{4}\dot{5}$.

(7) $3{\cdot}\dot{4} \div 4{\cdot}0\dot{9}$.   (8) ${\cdot}0\dot{7}\dot{4} \div {\cdot}5\dot{9}$.

119. When vulgar and decimal fractions are combined in the same expression, it may *usually* be simplified in the neatest and easiest way by reducing the vulgar fractions to a decimal form.

Thus, if we have to find the sum of $476\frac{1}{4}$, $13\frac{3}{8}$, and 10·375, we should proceed thus:

$$\begin{aligned} 476\tfrac{1}{4} &= 476{\cdot}25 \\ 13\tfrac{3}{8} &= 13{\cdot}375 \\ & 10{\cdot}375 \\ \hline \text{Sum } & 500{\cdot}000 \end{aligned}$$

Examination Papers.

I.

(1) Show that any decimal is multiplied by 1000 by removing the decimal point in the multiplicand three places towards the right.

(2) Enunciate the general rules for the division of decimals.

In cases when the division does not terminate, explain how to determine the place of the decimal point in the quotient.

(3) Which of the following statements is more nearly correct? $\frac{10}{9\cdot009} = 1\cdot11$ or $\frac{10}{1\cdot11} = 9\cdot009$.

(4) How many times can ·0087 be taken from 2·291? What fraction will the remainder be of the former?

(5) Whence does it appear that a vulgar fraction may always be reduced either to a terminated or a circulating decimal?

Calculate the limits of the error made in taking $\frac{355}{113}$ as an approximate value of 3·1415926 to seven places of decimals.

II.

(1) Explain what vulgar fractions can be expressed as finite decimals.

Which of the following fractions can be thus expressed?

$$\frac{5}{32},\ \frac{77}{1100},\ \frac{1820}{2912},\ \frac{231}{288},\ \frac{79}{405},\ \frac{91}{560}.$$

(2) If a pound of sugar cost ·0093125 of $8, find the value of ·0625 of 16 barrels of 200 pounds each.

(3) Whether is 3·714535 more accurately represented by 3·715 or $3\cdot71\dot{4}$, and why?

(4) What vulgar fraction is equivalent to the sum of 14·4 and 1·44 divided by their difference?

(5) Find a decimal which shall not differ from $\frac{5}{7}$ by a ten-thousandth.

III.

(1) What are the advantages and disadvantages of working with decimals instead of vulgar fractions?

(2) If a business produces an annual return of $6,000, and of three partners one has ·475 and another $\cdot3\dot{8}$ share of the profits, how much money falls to the share of the third partner?

(3) A man who owns $\frac{3}{5}$ of a steamboat sells $\cdot\dot{7}$ of his share for $1,400; what decimal part of the boat does he still own, and what was the boat worth?

(4) A man paid $120 for a horse; for a buggy $$36\frac{5}{16}$ more than $\cdot\dot{3}$ of the cost of the horse; for harness $\cdot\dot{1}8\dot{5}$ of the cost of horse and buggy. Find his entire outlay.

(5) The product of three vulgar fractions is $\frac{4}{7}$; two of them are expressed by the decimals, $\cdot\dot{6}\dot{3}$ and $\cdot13\dot{6}$ by what fraction will the third one be expressed?

IV.

(1) How do the Decimals differ from Vulgar Fractions?

(2) A storekeeper buys 140 yards of cloth at \$·36 per yard. In selling he uses a measure which is $\frac{1}{36}$ of a yard too short, and charges \$·50 per yard. What is his net gain?

(3) One vessel contains a mixture of 18 pints of brandy and 7 of water; another contains 34 pints of brandy and 13 of water. If the strength of the first mixture is represented by 423, what number will represent that of the second?

(4) Write in figures four millions and four, and ten billions ninety thousand and seven hundred quadrillionths. Express in words 74000306·000060000007.

(5) A piece of cloth was said to contain 84 yards, but it was found that the so-called yard measure with which it was measured was $\cdot 0208\dot{3}$ of a yard too short; what was the correct length of the cloth?

V.

(1) When a vulgar fraction is changed to a decimal, explain how many figures there will be in the decimal if it does not repeat; if it is a repeating decimal explain when it will consist of a part which does not repeat, and show how many figures there will be in this part.

(2) The French metre is 39·371 inches in length. Express the length of 25 metres as a fraction of an English mile, there being 5280 feet in it and 12 inches in a foot.

(3) If a steamer makes a passage from New York to Liverpool (say 2700 miles) in 230 hours, and a train goes from London to Edinburgh (say 405 miles) in 18 hours; how much does the one go faster than the other? Give answer in miles and decimal of a mile.

(4) Given that the sum of the divisor and quotient is 7·5; and that the divisor is $\frac{3}{7}$ of the quotient; also that the remainder is $\frac{29}{27}$ of the divisor. Find the dividend.

(5) Divide \$448.71$\frac{1}{3}$ among A, B, and C, so as to give B \$46.70 less than A, and \$34.59 more than C.

VI.

(1) What vulgar fractions must be represented by mixed repetends, and what by pure repetends?

(2) Show that no recurring decimal can have more places in the period than there are units in the denominator less one.

(3) A man spent \$2.50 more than $\cdot 7\dot{9}$ of his money at one time, and $\$1\cdot\dot{1}\dot{5}$ less than $\frac{960}{1441}$ of the remainder at another, and now has $\$2\cdot\dot{6}0\dot{9}$; how much had he at first?

(4) Simplify $16\left\{\frac{1}{5}-\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}-\frac{1}{7}\cdot\frac{1}{5^7}+\dots\right\}-\frac{4}{239}$.

(5) Simplify $\frac{1}{10^3}\times\left\{1-\frac{3}{10^2}+\frac{3\times 4}{1\times 2}\times\frac{1}{10^4}+\frac{3\times 4\times 5}{1\times 2\times 3}\cdot\frac{1}{10^6}\right\}$

XIII. Square Root.

120. When a number is multiplied by itself, the result is called the SQUARE of the number. Thus 144 is the square of 12, and 225 is the square of 15.

The symbol ^{2} placed over a number expresses the square of the number: thus 5^2 denotes the square of 5.

121. The SQUARE ROOT of a given number is that number whose square is equal to the given number.

Thus the square root of 144 is 12, because the square of 12 is 144.

The symbol $\surd$, placed before a number denotes that the square root of that number is to be taken: thus $\surd 25$ is read "the square root of 25."

122. A number which has an Integer for its square root is called a PERFECT SQUARE.

123. For Perfect Squares not greater than 100 we know the square roots, thus we know that the square root of 81 is 9; and for many Perfect Squares greater than 100 we know the square roots by experience, as, for instance, we know that the square root of 169 is 13, and the square root of 400 is 20, and the square root of 10000 is 100. But we have rules for finding the Square Root of any number, as we shall now explain.

First, suppose we have to find the Square Root of 1225.

We draw a line separating the two figures on the right from the other two, thus:

12|25.

The figures 12 make what is called the *first period.*

The figures 25 make what is called the *second period.*

We then take the nearest perfect square not greater than 12, that is 9, and place it under the 12 and put its square root, that is 3, as the first figure of the square root we have to find, thus

$$\begin{array}{l} 12|25\ (\ 3 \\ 9 \end{array}$$

We subtract 9 from 12, and annex to the remainder 3 *the second period* 25, to make a dividend, and we double the first figure of the root, and set down the result as the first term of a divisor; thus our process up to this point will stand thus:

$$\begin{array}{r|l} & 12|25\ (\ 3 \\ & 9 \\ \hline 6 & 325 \end{array}$$

Now we shall have to annex another figure to the 6, and we must therefore reckon the 6 as *six tens*, or 60, and then we seek the number of times 60 is contained in 325, and this being *five* times, we set down 5 as the second figure of the root, and annex 5 to the 6, so that our process up to this point will stand thus:

$$\begin{array}{r|l} & 12|25\ (\ 35 \\ & 9 \\ \hline 65 & 325 \end{array}$$

We then multiply 65 by 5, and set the product down under the 325; and subtracting the product from the 325, we have no remainder, and we conclude that 35 is the square root of 1225, the full process being:

$$\begin{array}{r|l} & 12|25\ (\ 35 \\ & 9 \\ \hline 65 & 325 \\ & 325 \\ \hline \end{array}$$

∴ 35 is the root required.

Next, to find the Square Root of 622521

Drawing a line to mark off the two figures on the right, and another line to mark off the next two figures, our process for finding the first two figures of the root will be the same as that explained in the first example, and it will stand thus:

$$\begin{array}{r|l}
 & 62|25|21\ (\ 78 \\
 & 49 \\ \hline
148 & 1325 \\
 & 1184 \\ \hline
 & \ \ 14121
\end{array}$$

We now annex to the remainder the *third* period 21, and we double the part of the root already found, 78, and set down the result 156 as a partial divisor, and proceed, as before, to divide 14121 by 1560, and annex the quotient 9 to the root and to the divisor; and multiplying 1569 by 9 we set the product under the 14121: thus our process in full will be

$$\begin{array}{r|l}
 & 62|25|21\ (\ 789 \\
 & 49 \\ \hline
148 & 1325 \\
 & 1184 \\ \hline
1569 & \ \ 14121 \\
 & \ \ 14121 \\ \hline
\end{array}$$

∴ 789 is the root required.

NOTE.—In practice, instead of dividing 1325 by 140, it is usual to divide 132 by 14, and instead of dividing 14121 by 1560, to divide 1412 by 156. The quotient thus obtained is, however, sometimes too great, as will be seen in the next examples.

We now give two examples in which the first period has only *one* figure, which must always be the case when the proposed square has an *odd* number of figures in it.

To find the Square Root of 189475225.

Marking off the figures by pairs, commencing from the right, we have

	1|89|47|52|25 (13765
	1
23	89
	69
267	2047
	1869
2746	17852
	16476
27525	137625
	137625

NOTE.—In dividing 89 by 20 the quotient is 4, but if we added this to complete the divisor, it would become 24, which, being multiplied by 4, would give 96, a number larger than 89.

To find the Square Root of 39601.

	3|96|01 (199
	1
23	296
	261
389	3501
	3501

NOTE I.—The division of 296 by 20 illustrates the remarks made on the last example.

NOTE II.—The second remainder, 35, is greater than the divisor, 29, a result not uncommon in this operation.

Examples. (lv.)

Find the Square Roots of

(1) 196.
(2) 529.
(3) 1024.
(4) 5625.
(5) 88209.
(6) 119025.

(7) 106929. (8) 751689.
(9) 193600. (10) 697225.
(11) 36372961. (12) 22071204.
(13) 550183936. (14) 5256250000.
(15) 4124961. (16) 546121000000.
(17) 32239684. (18) 191810713444.

124. *To find the Square Root of a Decimal Fraction.*

When the given number has an *even* number of decimal places, we proceed to find the Square Root as if the number were an integer, and mark off in the root a number of decimal places equal to *half the number in the square.*

Thus, if the square be a decimal of the *sixth* order, the root will be a decimal of the *third* order.

For example, to find the Square Root of 5·322249.

```
         5·|32|22|49 ( 2·307
         4
    43 | 132
       | 129
       | -----
    46 |  322
```

Since 46 is not contained in 32, we annex an 0 to the divisor, and also to the root, and bring down the next period thus:

```
4607 | 32249
     | 32249
```

Examples. (lvi.)

Find the Square Roots of

(1) 16·81. (2) 281·9041. (3) ·9025.
(4) ·2601. (5) ·0625. (6) ·000729.
(7) 17242·3161. (8) 1·002001. (9) 44415·5625.
(10) 18947·5225.

125. In finding the Square Root of a Decimal Fraction we must be careful to make the decimal such that the index of its order is an *even* number.

Thus, if we have to find the Square Root of ·4, we change the decimal into an equivalent decimal of the *second*, *fourth*, *sixth*....order, thus, ·40, ·4000, ·400000,....

This is done in order that the denominator of the equivalent fraction may be a perfect square, which is the case in the fractions

$$\frac{40}{100},\ \frac{4000}{10000},\ \frac{400000}{1000000}\ldots$$

but not in the fractions

$$\frac{4}{10},\ \frac{400}{1000},\ \frac{40000}{100000}\ldots$$

Also, since for every *pair* of figures in the square we have *one* figure in the root, we shall have to take a number of figures in the decimal part of the square double the number of decimal places we are to have in the root.

Suppose, for example, we have to find the Square Root of ·144 to *four* places of decimals.

We must have *eight* decimal places in the square, thus, ·14400000, and we mark off these and proceed as in the extraction of the root of whole numbers, the root being a decimal of the *fourth* order, thus:

$$\begin{array}{r|r}
 & \cdot 14|40|00|00\ (\ \cdot 3794\ldots \\
 & 9 \\ \hline
67 & 540 \\
 & 469 \\ \hline
749 & 7100 \\
 & 6741 \\ \hline
7584 & 35900 \\
 & 30336 \\ \hline
 & 5564
\end{array}$$

NOTE.—The Square Root of a decimal of an *odd* order is a non-terminating decimal.

Examples. (lvii.)

Extract to four places of decimals the Square Roots of

(1) 20. (2) 30. (3) ·9. (4) ·121.

(5) ·169. (6) ·016. (7) ·00064. (8) ·00121.

(9) 16·245. (10) $\cdot\dot{9}$. (11) $\cdot\dot{2}\dot{5}$. (12) $42\cdot0\dot{3}$.

126. If we have to find the Square Root of a Vulgar Fraction we can always, by multiplication, make the denominator a perfect square, if it be not already so, multiplying the numerator by the same number.

We then find the Square Root of the denominator, and find, exactly or approximately, the square root of the numerator, and make the results respectively the denominator and numerator of a fraction, which is the root required, exactly or approximately.

Ex. (1). $\sqrt{\dfrac{25}{36}} = \dfrac{\sqrt{25}}{\sqrt{36}} = \dfrac{5}{6}.$

Ex. (2). $\sqrt{\dfrac{2}{3}} = \sqrt{\dfrac{2 \times 3}{3 \times 3}} = \dfrac{\sqrt{6}}{\sqrt{9}} = \dfrac{\sqrt{6}}{3}.$

We can now extract the square root of 6 to, say, *three* places of decimals, thus:

$$\begin{array}{r|l}
 & 6\cdot|00|00|00\ (\,2\cdot449\ldots. \\
 & 4 \\
\hline
44 & 200 \\
 & 176 \\
\hline
484 & 2400 \\
 & 1936 \\
\hline
4889 & 46400 \\
 & 44001 \\
\hline
 & 2399
\end{array}$$

$$\therefore \sqrt{\frac{2}{3}} = \frac{2\cdot449\ldots}{3} = \cdot816\ldots.$$

Or, we might have reduced $\frac{2}{3}$ to a decimal, thus: ·666666 . . ., and then have extracted the square root of this decimal.

Ex. (3). $\sqrt{8\dfrac{17}{64}} = \sqrt{\dfrac{529}{64}} = \dfrac{\sqrt{529}}{\sqrt{64}} = \dfrac{23}{8} = 2\dfrac{7}{8}.$

Ex. (4). To find the Square Root of $\dfrac{1\cdot28}{12\cdot5}$.

Here we can reduce the fraction to lower terms;

$$\text{Thus, } \sqrt{\frac{1\cdot28}{12\cdot5}} = \frac{\sqrt{\cdot64}}{\sqrt{6\cdot25}} = \frac{\cdot8}{2\cdot5} = \cdot32.$$

127. An integer can always be changed into a perfect square by multiplying by a number equal to or less than the proposed integer.

For example,

7 is changed into a perfect square if multiplied by 7,
18 is changed into a perfect square if multiplied by 2.

Examples. (lviii.)

Find the Square Roots of

(1) $\frac{36}{49}$. (2) $\frac{64}{121}$. (3) $\frac{289}{625}$.
(4) $\frac{1369}{7569}$. (5) $\frac{15129}{182329}$. (6) $5\frac{1}{16}$.
(7) $5\frac{19}{25}$. (8) $3\frac{22}{169}$. (9) $65\frac{64}{81}$.
(10) $38\frac{11}{25}$. (11) $17\frac{16}{25}$. (12) $11\frac{37}{49}$.

and find to four places of decimals the Square Roots of

(13) $\frac{5}{8}$. (14) $\frac{10}{24}$. (15) $6\frac{2}{5}$.
(16) $9\frac{1}{2}$. (17) $76\frac{14}{17}$.

XIV. Cube Root.

128. When a number is multiplied by itself twice, the result is called the CUBE of the number. Thus 27 is the cube of 3, and 216 is the cube of 6.

129. The CUBE ROOT of a given number is that number whose cube is equal to the given number.

Thus the Cube Root of 343 is 7, because the cube of 7 is 343.

The symbol $\sqrt[3]{\ }$, placed before a number, denotes that the cube root of that number is to be taken; thus $\sqrt[3]{125}$ is read "the cube root of 125."

130. A number which has an integer for its cube root is called a PERFECT CUBE.

The numbers, less than 1000, which are perfect cubes should be committed to memory; they are

1, 8, 27, 64, 125, 216, 343, 512, 729;

and the Cube Roots of these numbers are respectively

1, 2, 3, 4, 5, 6, 7, 8, 9.

131. To find the Cube Root of a perfect cube, greater than 1000, we proceed by a rule which we shall now explain.

Ex. To find the Cube Root of 91125.

	4		91|125
			64
12	5	4800	27125
		625	
		5425	27125

First divide the number 91125 into two periods by drawing a line marking off *three* figures on the right.

Then take the nearest perfect cube not greater than 91, which is 64, and set down its cube root, which is 4, in a line with 91125, and some way to the left. This is the first figure of the root.

Then subtract 64 from 91, and to the remainder attach the second period, 125.

Now place three times the first figure of the root, 12, to the extreme left, and three times the square of the first figure of the root, 48, with two zeros annexed to it, just on the left of the 27125.

Divide 27125 by 4800, and set the quotient, 5, midway between 12 and 4800. Then read 12 5 as 125; multiply this by 5; put the result, 625, under the 4800; add it to the 4800; this gives 5425; multiply this by 5; put the result, which is 27125, under the first remainder; subtract, and as there is no remainder, the process is complete, and the root is 45.

Examples. (lix.)

Find the Cube Roots of

(1) 4096.
(2) 32768.
(3) 74088.
(4) 493039.
(5) 614125.
(6) 262144.
(7) 39304.
(8) 389017.
(9) 614125.
(10) 970299.
(11) 59319.
(12) 250047.

Next, let us take the case in which the cube root has three figures, and extract the cube root of 428661064.

	7		428|661|064
			343
21	5	14700	85661
		1075	
		15775	78875
		25	
225	4	1687500	6786064
		9016	
		1696516	6786064

We separate the number 428661064 into three periods, and take the nearest perfect cube not greater than 428, which is 343, and we set down its cube root, which is 7. We then subtract 343 from 428, and annex to the remainder 661, the second period.

Then we set down three times 7, which is 21, and three times the square of 7, which is 147, and add two zeros to it.

Then we divide 85661 by 14700, which gives the quotient 5, and this we put down midway between 21 and 14700.

Then we multiply 215 by 5, which gives 1075; we add this to 14700; we multiply the result, 15775, by 5; and subtract the product, 78875, from 85661; and to the remainder we annex the third period, 064.

We then set down three times 75, which is 225,
and three times the square of 75, which is 16875.

N. B.—This last result can be obtained by setting the square of 5, the second figure of the root, under the second divisor, and adding the three numbers coupled by the bracket.

We then annex two zeros to 16875 and repeat the process explained above to find 4, the third figure of the cube root, which is in this case 754.

Next, take the case in which the root has *four* figures and find the Cube Root of 14832537993.

	2		14|832|537|993
			8
6	4	1200	6832
		256	
		1456	5824
		16	
72	5	172800	1008537
		3625	
		176425	882125
		25	
735	7	18007500	126412993
		51499	
		18058999	126412993

Hence the root required is 2457.

NOTE.—In dividing 6832 by 1200, the quotient is 5, but if we took this for the second figure of the root we should find that the addition of 5 times 65, or 325, to 1200 would give 1525, and this multiplied by 5 would give 7625, a number too large to be subtracted from 6832.

Examples. (lx.)

Find the Cube Roots of

(1) 14706125.
(2) 149721291.
(3) 28934443.
(4) 300763000.
(5) 2097152.
(6) 5735339.
(7) 99252847.
(8) 1092727.
(9) 16777216.
(10) 194104539.
(11) 84027672.
(12) 130323843.
(13) 322828856.
(14) 354894912.
(15) 700227072.
(16) 134217728.
(17) 122615327232.
(18) 673373097125.

132. *To extract the Cube Root of a Decimal Fraction.*

In order that a Decimal Fraction may be a Perfect Cube, it must be of the 3rd, 6th, 9th.... order, the index of the order being some multiple of 3.

We then proceed in the following way:

Ex. (1). To find the Cube Root of ·343.

$$\sqrt[3]{\cdot 343} = \sqrt{\frac{343}{1000}} = \frac{7}{10} = \cdot 7.$$

Ex. (2). To find the Cube Root of ·039304.

$$\sqrt[3]{\cdot 039304} = \sqrt{\frac{39304}{1000000}} = \frac{34}{100} = \cdot 34.$$

Ex. (3). To find the Cube Root of ·012812904.

$$\sqrt[3]{\cdot 012812904} = \sqrt{\frac{12812904}{1000000000}} = \frac{234}{1000} = \cdot 234.$$

133. To extract the cube root of an integer or decimal expression to a particular place of decimals, we must take *three times the number* of decimal places in the expression.

Thus, to find the cube root of 4·23 accurately to three places of decimals we extract the cube root of 4·230000000, making the given expression a decimal of the *ninth* order. In working this example we find the cube root of 4230000000, *regarded as a whole number*, and mark off three decimal places in the result.

134. The Cube Root of a *Vulgar Fraction* may be found by taking the roots of the numerator and denominator, or by reducing the fraction to a decimal of the 3rd, 6th, 9th. . . .order, and proceeding as in Art. 133.

Examples. (lxi.)

Find the Cube Roots of

(1) ·389017. (2) ·048228544. (3) 27054·036008.
(4) $\frac{1331}{1728}$. (5) $\frac{250}{686}$. (6) $5\frac{13}{343}$. (7) $405\frac{28}{125}$.

and find to three places of decimals the Cube Roots of

(8) 5. (9) 576. (10) ·121861281.
(11) 15·926972504. (12) $\frac{5}{9}$. (13) $\frac{3}{4}$.
(14) $\frac{1}{3}$. (15) $7\frac{3}{5}$. (16) $3\frac{1}{5}$.

135. The *fourth* root of a number is found by taking the square root of the square root of the number.

Thus $\sqrt[4]{4096} = \sqrt{64} = 8$.

The *sixth* root of a number is found by taking the cube root of the square root of the number.

Thus $\sqrt[6]{64} = \sqrt[3]{8} = 2$.

Examples. (lxii.)

Find the Fourth Roots of

(1) 531441. (2) 4100625. (3) 1575·2961 ;

and the Sixth Roots of

(4) 4826809. (5) 24794911296. (6) 282429·536481.

COMMERCIAL ARITHMETIC.

XV. On English, Canadian, and United States Currencies.

136. Having explained the principles and processes of Pure Arithmetic, we proceed to show how they are applied to commercial affairs.

Measures of Money.

4 farthings are equivalent to 1 penny.
12 pence are equivalent to 1 shilling.
20 shillings are equivalent to 1 pound.

The symbol £ placed before or over a number denotes *pounds*.
......*s.*...... after........................*shillings*.
......*d.*...... after........................*pence*.

Thus £14 5*s.* 7*d.*, or $\overset{£}{14}\ \overset{s.}{5}\ \overset{d.}{7}$, stands for fourteen pounds, five shillings, and seven pence.

Since 1 farthing is one-fourth of a penny,
2 farthings are one-half of a penny,
3 farthings are three-fourths of a penny.

Hence the symbol ¼*d.* is placed for 1 farthing,
..........½*d.*.............2 farthings or a halfpenny,
..........¾*d.*.............3 farthings.

The symbol *q.*, placed after a number, is sometimes used to denote farthings; thus, 3*q.* stands for three farthings.

137. We call £14 a *simple* quantity, and £14 5*s.* 7*d.* a *compound* quantity, because the former is expressed with reference to a *single* unit, while the latter is expressed with reference to *three different* units.

138. The unit in Canadian and United States currencies is called a Dollar. The *tenth* part of this unit is called a Dime; the *tenth* part of the dime is called a Cent; and the *tenth* of the cent is called a Mill. We may conceive the unit, then, to be divided into *ten* equal parts, each of these parts into *ten* other equal parts, and so on. Hence Canadian and United States currencies are based on the *Decimal System of Notation*, and, therefore, all operations in these currencies are performed by means of the rules in Decimal Fractions. It is to this circumstance that they owe their great simplicity.

TABLE OF CANADIAN AND UNITED STATES COINS.

CANADIAN COINS.	UNITED STATES COINS.
Gold.	*Gold.*
British Sovereign, worth $4.86⅔.	Double Eagle, or... $20
	Eagle, or $10
British Half-Sovereign.	Half Eagle, or..... $5
	Three Dollar Piece.
	Quarter Eagle, or.. $2½
	Dollar.
Silver.	*Silver.*
	Dollar.
50-cent piece, answers to	Half dollar.
25-cent piece, answers to	Quarter dollar.
20-cent piece (no longer coined).	
10-cent piece, answers to	Dime.
	Nickel.
5-cent piece, answers to	5-cent piece.
	3-cent piece.
Bronze.	*Bronze.*
1 cent.	1 cent.
Mill, not coined.	Mill, not coined.

Ex. (1). $251, 7 cents, 3 mills

$$
\begin{aligned}
&= \$(251 + \tfrac{0}{10} + \tfrac{7}{100} + \tfrac{3}{1000}) \\
&= \$(251 + \tfrac{0}{1000} + \tfrac{70}{1000} + \tfrac{3}{1000}) \\
&= \$(251 + \tfrac{73}{1000}) \\
&= \$\ 251{\cdot}073.
\end{aligned}
$$

Ex. (2). $\$55{\cdot}923$

$$
\begin{aligned}
&= \$(55 + \tfrac{9}{10} + \tfrac{2}{100} + \tfrac{3}{1000}) \\
&= \$(55 + \tfrac{92}{100} + \tfrac{3}{1000}) \\
&= \$\ 55 + 92 \text{ cents} + 3 \text{ mills} \\
&= \$\ 55,\ 92 \text{ cents},\ 3 \text{ mills}.
\end{aligned}
$$

The English gold coinage consists of $\frac{11}{12}$ pure metal and of $\frac{1}{12}$ alloy.

The gold and silver coinage of the United States consists of $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy.

The silver coin in Canada and England is $\frac{37}{40}$ pure metal and $\frac{3}{40}$ copper.

Gold and silver thus alloyed are called *standard*. The gold or silver before it is coined is called *bullion*.

The term *carat* is employed to denote the fineness of gold. Perfectly pure gold is said to be 24 carats fine ; a mixture of 18 parts pure gold and six parts of some other metal, is said to be 18 carats fine. This latter is termed jewellers' gold.

REDUCTION OF MONEY.

139. The expression 5*s.* 7*d.* stands for a sum of money which is made up of five shillings and seven pence. Now, since one shilling is equivalent to twelve pence, five shillings are equivalent to sixty pence ; and therefore five shillings and seven pence are equivalent to sixty-seven pence.

The process by which we change the *compound* expression 5*s.* 7*d.* into the equivalent *simple* expression 67*d.* is arranged thus :

$$
\begin{array}{cc}
s. & d. \\
5 & 7 \\
12 & \\
\hline
\multicolumn{2}{c}{67d.}
\end{array}
$$

and we describe the process thus : *We change the 5 shillings into pence by multiplying by 12, and add to the product the 7 pence.*

Again, to change the compound expression £4 7*s*. $10\frac{1}{2}$*d*. into an equivalent number of farthings, we proceed thus:

£	*s*.	*d*.
4	7	$10\frac{1}{2}$
20		
87*s*.		
12		
1054*d*.		
4		
4218*q*.		

First we change £4 to shillings, and add 7*s*., making 87*s*.;
then..........87*s*. to pence,..........10*d*., ... 1054*d*.;
then........1054*d*. to farthings,........ 2*q*., ... 4218*q*.

Examples. (lxiii.)

Reduce to farthings

(1) $3\frac{1}{4}$*d*.; $7\frac{1}{2}$*d*.; 9*d*.; $11\frac{3}{4}$*d*.
(2) 2*s*. 3*d*.; 5*s*. $7\frac{1}{2}$*d*.; 12*s*. $9\frac{3}{4}$*d*.; 17*s*. $7\frac{1}{4}$*d*.
(3) £3 12*s*.; £5; £2 17*s*. $6\frac{1}{2}$*d*.; £17 4*s*. $5\frac{3}{4}$*d*.

Reduce to pence

(4) 6*s*.; 4*s*. 10*d*.; 7*s*. 10*d*.; 8*s*. 9*d*.; 13*s*. 7*d*.
(5) £4; £5 2*s*. 4*d*.; £17 14*s*. 5*d*.; £58 13*s*. 11*d*.
(6) £174 10*s*.; £432 15*s*. 10*d*.; £1274 17*s*. 9*d*.

140. The converse operation, by which we express a *simple* quantity in terms of an equivalent *compound* quantity, will be best explained by the following examples.

Ex. (1). Nine farthings will be expressed as pence and farthings if we divide 9 by 4 (since 4 farthings = 1 penny), set down the quotient as pence and the remainder as farthings, thus: 9 farthings $= \frac{9}{4}$ pence $= 2\frac{1}{4}$*d*.

Ex. (2). Again, 33 pence will be expressed as shillings and pence if we divide 33 by 12 (since 12 pence = 1 shilling), set down the quotient as shillings, and the remainder as pence, thus: 33 pence $= \frac{33}{12}$ shillings = 2*s*. 9*d*.

Ex. (3). Also, 75 shillings $= \frac{75}{20}$ pounds = £3 15*s*.

Ex. (4.) To express 4275639 farthings in terms of £ *s. d.*

	farthings.
4	4275639
12	1068909*d.* and 3 farthings over.
20	89075*s.* and 9 pence over.
	£4453 and 15 shillings over.

∴ 4275639 farthings = £4453 15*s.* 9¾*d.*

These methods of expressing a given sum of money in another, but equivalent, form are included in the word *Reduction.*

Examples. (lxiv.)

Reduce to pence and farthings the following numbers of farthings:

(1) 57. (2) 173. (3) 197.

Reduce to shillings, pence, and farthings the following numbers of farthings:

(4) 357. (5) 479. (6) 747.

Reduce to £ *s. d.* the following numbers of farthings:

(7) 4238. (8) 376289. (9) 542380.

141. The copper coins in use in Great Britain are the Farthing, the Halfpenny, and the Penny.

The silver coins in use are the Crown (5*s.*), the Half-crown (2*s.* 6*d.*), the Florin (2*s.*), the Shilling, the Sixpence, the Fourpenny piece (or Groat), and the Threepenny piece.

The gold coins in use are the Sovereign or Pound, and the Half-sovereign. The Guinea (21*s.*) and the Half-guinea (10*s.* 6*d.*) are not in use, but reference is frequently made to them.

COMPOUND ADDITION.

142. In adding compound expressions together, we follow the principles which regulate the process of Addition in the case of pure numbers.

Thus, in adding sums of money we arrange them so that the pounds stand under pounds in vertical columns, shillings under shillings, pence under pence, and farthings under farthings. For example, if we have to add together 4*s.* $3\frac{1}{4}$*d.*, 3*s.* $3\frac{1}{2}$*d.*, 5*s.* 4*d.*, and 17*s.* $9\frac{3}{4}$*d.*, we arrange them thus:

	s.	*d.*
	4	$3\frac{1}{4}$
	3	$3\frac{1}{2}$
	5	4
	17	$9\frac{3}{4}$
£1	10	$8\frac{1}{2}$

Adding the columns of farthings, we find its sum to be 6 farthings, and this being equivalent to 1 penny and 2 farthings, we place $\frac{1}{2}$ under the column of farthings, and carry on 1 for addition to the column of pence.

The sum of the column of pence, increased by 1, we find to be 20 pence, and this being equivalent to 1 shilling and 8 pence, we place 8 under the column of pence and carry on 1 for addition to the column of shillings.

The sum of the columns of shillings, increased by 1, we find to be 30 shillings, and this being equivalent to 1 pound and 10 shillings, we place 10 under the columns of shillings, and set down 1 pound by itself on the left hand.

Again, if we have to add together £26 4*s.* $9\frac{3}{4}$*d.*, £32 12*s.* $7\frac{1}{4}$*d.*, £245 0*s.* 2*d.*, £7 15*s.* $8\frac{1}{2}$*d.*, and 4*s.* $8\frac{3}{4}$*d.*, we arrange them thus:

£	*s.*	*d.*
26	4	$9\frac{3}{4}$
32	12	$7\frac{1}{4}$
245	0	2
7	15	$8\frac{1}{2}$
0	4	$8\frac{3}{4}$
£311	18	$0\frac{1}{4}$

Adding the column of farthings, we find its sum to be 9 farthings, and this being equivalent to 2 pence and 1 farthing, we place $\frac{1}{4}$ under the column of farthings, and carry on 2 for addition to the column of pence.

The sum of the column of pence, increased by 2, we find to be 36 pence, and this being equivalent to 3 shillings, we place 0 under the column of pence, and carry on 3 for addition to the column of shillings.

The sum of the columns of shillings, increased by 3, we find to be 38 shillings, and this being equivalent to 1 pound and 18 shillings, we place 18 under the columns of shillings, and carry on 1 for addition to the columns of pounds.

The sum of the columns of pounds, increased by 1, we find to be 311, which we place under those columns, and the sum is complete.

Examples. (lxv.)

Perform the operation of addition on the following sums of money.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(1)	3	5	2	(2)	5	8	3	(3)	6	8	7	(4)	7	6	8
	4	6	8		7	9	6		4	6	3		5	8	4
	7	9	3		3	4	9		8	9	10		9	6	0
	2	4	10		6	5	2		9	7	6		7	4	11
	4	9	2		9	0	4		4	3	0		2	6	10

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(5)	3	4	$3\frac{1}{2}$	(6)	4	7	$5\frac{3}{4}$	(7)	7	8	$4\frac{3}{4}$	(8)	6	2	$5\frac{1}{4}$
	2	5	$4\frac{1}{4}$		6	8	$9\frac{1}{2}$		6	9	2		5	3	$2\frac{3}{4}$
	7	6	$8\frac{1}{2}$		9	5	$2\frac{1}{4}$		5	2	$7\frac{1}{2}$		7	8	$4\frac{1}{2}$
	6	9	$6\frac{1}{4}$		8	7	$4\frac{1}{2}$		6	3	9		8	9	1
	4	7	$9\frac{3}{4}$		5	9	$6\frac{3}{4}$		7	5	$4\frac{1}{2}$		5	3	$3\frac{1}{2}$

(9)			(10)			(11)			(12)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
16	19	4	26	9	2	17	9	10	21	11	3
14	13	2	13	0	11	61	11	4	37	5	9
67	8	10	9	16	4	18	5	9	4	6	2
42	5	8	67	17	8	28	14	7	17	17	7
12	7	9	24	19	2	21	3	7	39	18	5
15	10	4	39	15	3	93	14	6	47	11	10

	£	s.	d.		£	s.	d.		£	s.	d.
(13)	16	3	7½	(14)	35	8	9½	(15)	143	17	9½
	78	13	9¼		76	10	4¾		876	11	2¼
	21	17	4¾		25	8	9		972	9	10
	5	3	7		4	0	7½		397	4	3½
	36	11	4¼		36	12	5¼		674	4	10½
	42	8	2½		42	5	9¾		538	9	6¼

COMPOUND SUBTRACTION.

143. The process of subtracting one compound quantity from another is founded on the principles explained in Art. 20, and the following example will supply all that is necessary to make the method clear:

	£	s.	d.
From	27	5	2¼
Take	13	17	4½
	£13	7	9¾

Arranging the columns as in addition, we reason thus: we cannot take 2 farthings from 1 farthing, and we therefore add 4 farthings to the 1 farthing, making 5 farthings, and taking 2 farthings from 5 farthings we obtain as a remainder 3 farthings, which we set down under the column of farthings.

We then add, by way of compensation, 1 penny to the 4 pence in the lower line. We have then to take 5 pence from 2 pence, and as we cannot do this, we add 12 pence to the 2 pence, making 14 pence, and taking 5 pence from 14 pence, we obtain as a result 9 pence, which we place under the column of pence.

We then add, by way of compensation, 1 shilling to the 17 shillings in the lower line. We have then to take 18 shillings from 5 shillings, and as we cannot do this, we add 20 shillings to the 5 shillings, making 25 shillings, and taking 18 shillings from 25 shillings, we obtain as a result 7 shillings, which we place under the column of shillings.

Finally we add, by way of compensation, 1 pound to the 13 pounds, and we take 14 pounds from 27 pounds, obtaining as a remainder 13 pounds, which we place under the column of pounds.

Examples. (lxvi.)

		£	s.	d.		£	s.	d.
(1)	From	94	12	7	take	58	9	2.
(2)	"	75	9	6	"	47	8	8.
(3)	"	58	13	4	"	49	14	5.
(4)	"	276	17	5½	"	37	19	7¼.
(5)	"	1247	5	10¼	"	1246	11	8½.
(6)	"	3000	10	7½	"	2998	13	11¾.
(7)	"	199	0	0¼	"	198	19	10½.
(8)	"	80609	5	2¾	"	79089	12	5¼.
(9)	"	44005	7	9¼	"	7896	10	2½.
(10)	"	30704	0	5	"	29484	0	6½.

COMPOUND MULTIPLICATION.

144. To multiply a compound expression, as £4 8*s.* 9¾*d.*, by a number, as 9, is equivalent to taking the sum of nine expressions, each equal to £4 8*s.* 9¾*d.* Instead of writing these expressions one under the other, and finding their sum by the process of addition, we obtain the required result by multiplying each of the four quantities composing the expression separately by 9, calculating the value of each result as in addition, setting down part of those results under the several columns, and carrying on part as in addition, thus:

£	s.	d.
4	8	9¾
		9
£39	19	3¾

The process may be more fully explained thus:

9 times 3 farthings = 27 farthings = 6¾*d.*: set down ¾ under the column of farthings, and carry on 6 to the pence.

9 times 9 pence = 81 pence, and 6 pence added gives 87 pence = 7*s.* 3*d.*: set down 3 under the column of pence, and carry on 7 to the shillings.

9 times 8 shillings = 72 shillings, and 7 shillings added gives 79 shillings = £3 19*s.*: set down 19 under the column of shillings, and carry on 3 to the pounds.

9 times 4 pounds = 36 pounds, and 3 pounds added gives 39 pounds, which is set down under the pounds.

145. When the multiplier can be split up into factors, each of which is not greater than 12, we multiply the compound expression first by one of the factors, and then multiply the product by another of the factors, as in the case of Simple Multiplication.

Thus, if we have to multiply £12 4*s.* 7½*d.* by 15, we multiply first by 5, and the product by 3, thus:

£	*s.*	*d.*	
12	4	7½	
		5	
61	3	1½	Product by 5.
		3	
£183	9	4½	Product by 15.

Again, to multiply £17 14*s.* 9*d.* by 180, we may proceed thus:

£	*s.*	*d.*	
17	14	9	
		10	
177	7	6	Product by 10.
		6	
1064	5	0	Product by 60.
		3	
£3192	15	0	Product by 180.

Examples. (lxvii.)

Find the value of

(1) 4 things at 7*s.* 3*d.* each.
(2) 5 at 14*d.*
(3) 6 at 7½*d.*
(4) 7 at 9*s.* 6*d.*
(5) 8 at 2*s.* 4*d.*
(6) 10 at 2*s.* 2½*d.*
(7) 11 at £2 1*s.* 4*d.*
(8) 12 at £1 4*s.* 3*d.*
(9) 14 at 17*s.* 6*d.*
(10) 15 at 7*s.* 10½*d.*
(11) 16 at 27*s.*
(12) 18 at 17*s.* 6*d.*
(13) 20 at £5 11*s.* 4*d.*
(14) 21 at 5*s.* 7½*d.*
(15) 22 at £5 11*s.* 4*d.*
(16) 24 at £4. 7*s.* 2*d.*
(17) 25 at 4*s.* 6*d.*
(18) 27 at 5*s.* 11½*d.*
(19) 28 at 2*s.* 8*d.*
(20) 30 at £1 12*s.*
(21) 33 at £1 2*s.*
(22) 35 at £1 2*s.* 6*d.*

146. When the multiplier cannot be split up into factors, we may proceed as in the following examples:

Ex. (1). To multiply £17 12*s.* $9\frac{1}{4}$*d.* by 79.

	£	*s.*	*d.*	
	17	12	$9\frac{1}{4}$	
			10	
	176	7	$8\frac{1}{2}$	Product by 10.
			7	
	1233	13	$11\frac{1}{2}$	Product by 70.
Multiplying 1st line by 9	158	14	$11\frac{1}{4}$	Product by 9.
Adding last two results	£1393	8	$10\frac{3}{4}$	Product by 79.

Ex. (2). To multiply £3 17*s.* $9\frac{1}{2}$*d.* by 3296.

	£	*s.*	*d.*	
	3	17	$9\frac{1}{2}$	
			10	
	38	17	11	Product by 10.
			10	
	388	19	2	Product by 100.
			10	
	3889	11	8	Product by 1000
			3	
	11668	15	0	Product by 3000
Multiplying 5th line by 2	777	18	4	" by 200.
Multiplying 3rd line by 9	350	1	3	" by 90.
Multiplying 1st line by 6	23	6	9	" by 6.
Adding last four results	£12820	1	4	Product by 3296.

147. The following is a method by which the process of multiplying a compound quantity by a number greater than 1000 is somewhat shortened. We take as an illustration the example just worked. The process is so simple that no verbal explanation is necessary.

2 | 3296

d. 1648
29664 the result of multiplying the top line by 9.

12 | 31312

s. 2609 and 4*d.*
23072 } the result of multiplying the top line by 17.
3296

20 | 5864,1

£ 2932 and 1*s.*
9888 the result of multiplying the top line by 3.

£12820 1*s.* 4*d.*

Examples. (lxviii.)

Find the value of

(1) 29 things at 4*s.* 6*d.* each.
(2) 39 at 12*s.* 6½*d.*
(3) 47 at 1*s.* 6½*d.*
(4) 71 at 1*s.* 8*d.*
(5) 89 at 6*s.* 8*d.*
(6) 123 at 5*s.* 6½*d.*
(7) 145 at £1 3*s.* 2*d.*
(8) 2154 at £7 1*s.* 3*d.*
(9) 3210 at £1 18*s.* 6¾*d.*
(10) 2175 at £2 15*s.* 4½*d.*
(11) 3684 at £2 6*s.* 9¼*d.*

COMPOUND DIVISION.

148. The process of dividing a compound quantity by a number is based upon the principles explained in the case of Simple Division, as will be seen from the following examples:

Ex. (1). To divide £13 17*s.* 1½*d.* by 9.

£ *s.* *d.*
9) 13 17 1½
£1 10 9½ Quotient.

We reason thus:

£13 divided by 9 gives £1 as quotient and £4 remainder;
£4 = 80 shillings, and 17 shillings added gives 97 shillings.
97*s.* divided by 9 gives 10*s.* as quotient and 7*s.* remainder;
7*s.* = 84 pence, and 1 penny added gives 85 pence.
85*d.* divided by 9 gives 9*d.* as quotient and 4*d.* remainder;
4*d.* = 16 farthings, and 2 farthings added gives 18 farthings.
18*q.* divided by 9 gives 2*q.* as quotient and no remainder.

Ex. (2). To divide £51 15*s.* 5*d.* by 35.

$$\text{The factors of 35 are}\left\{\begin{array}{r|rrr} & \pounds & s. & d. \\ 5 & 51 & 15 & 5 \\ \cline{2-4} 7 & 10 & 7 & 1 \\ \cline{2-4} \end{array}\right. \\ \begin{array}{rrr} \pounds 1 & 9 & 7 \end{array}\ \text{Quotient.}$$

Ex. (3). To divide £53 15*s.* 8*d.* by 112.

$$\text{The factors of 112 are}\left\{\begin{array}{r|rrr} & \pounds & s. & d. \\ 4 & 53 & 15 & 8 \\ \cline{2-4} 4 & 13 & 8 & 11 \\ \cline{2-4} 7 & 3 & 7 & 2\frac{3}{4} \\ \cline{2-4} & & 9 & 7\frac{1}{4} \end{array}\right.\ \text{Quotient.}$$

Ex. (4). To divide £119232 1*s.* 10½*d.* by 3465.

$$\begin{array}{rcrl}
 & & \pounds\quad\ s.\quad\ d. & \\
3465 &) & 119232\quad 1\quad 10\frac{1}{2} & (\ \pounds 34 \\
 & & 10395 & \\ \cline{3-3}
 & & 15282 & \\
 & & 13860 & \\ \cline{3-3}
 & & 1422 & \\
 & & 20 & \\ \cline{3-3}
3465 &) & 28441 & (\ 8s. \\
 & & 27720 & \\ \cline{3-3}
 & & 721 & \\
 & & 12 & \\ \cline{3-3}
3465 &) & 8662 & (\ 2d. \\
 & & 6930 & \\ \cline{3-3}
 & & 1732 & \\
 & & 4 & \\ \cline{3-3}
3465 &) & 6930 & (\ 2q. \\
 & & 6930 & \\ \cline{3-3}
\end{array}$$

∴ the Quotient is £34 8*s.* 2½*d.*

Examples. (lxix.)

I. Divide

(1) £1 3*s*. 7½*d*. by 3.
(2) £39 7*s*. 6*d*. by 7.
(3) £11 3*s*. 6*d*. by 12.
(4) £43 12*s*. 8*d*. by 11.
(5) £6 2*s*. 11*d*. by 10.
(6) £22 11*s*. 6*d*. by 12.

II. Divide

(1) £98 11*s*. 9*d*. by 54.
(2) £13 7*s*. 9*d*. by 63.
(3) £29 14*s*. 0*d*. by 108.
(4) £15 8*s*. by 132.
(5) £3 9*s*. 4½*d*. by 45.
(6) £43 12*s*. 8*d*. by 44.

III. Divide

(1) £167 19*s*. 2*d*. by 145.
(2) £40 8*s*. 4¼*d*. by 241.
(3) £453 11*s*. 9¼*d*. by 365.
(4) £40669 2*s*. 1*d*. by 9652.
(5) £93 1*s*. 2¼*d*. by 291.
(6) £139 3*s*. 6*d*. by 117.

149. One quantity is contained in another of the same kind as often as the measure of the first is contained in the measure of the second, the same unit of measurement being taken in both cases.

Ex. (1). How many times is 1*s*. 1*d*. contained in 16*s*. 3*d*.?

1*s*. 1*d*. = 13*d*.; and 16*s*. 3*d*. = 195*d*.
Now 13 is contained 15 times in 195;
∴ 13*d*. is contained 15 times in 195*d*.

Ex. (2). How many times is £4 3*s*. 2*d*. contained in £87 6*s*. 6*d*.?

£4 3*s*. 2*d*. = 998*d*.; and £87 6*s*. 6*d*. = 20958*d*.
Now 20958 ÷ 998 = 21;
∴ £4 3*s*. 2*d*. is contained in £87 6*s*. 6*d*. 21 times.

Examples. (lxx.)

(1) How many times is £346 16*s*. contained in £34680?

(2) .... £5 11*s*. 4*d*. .... £122 9*s*. 4*d*.?

(3) .... £1 12*s*. 6*d*. .... £68 5*s*.?

(4) .... £17 12*s*. 9¼*d*. .... £1393 8*s*. 10¾*d*.?

(5) Among how many persons must £641 14*s*. 11¼*d*. be divided, so that the share of each may be £2 15*s*. 6¾*d*.?

(6) Divide £17 into an equal number of sovereigns, half-sovereigns, half-crowns, shillings, and sixpences.

FRACTIONAL MULTIPLICATION AND DIVISION OF MONEY.

150. Ex. (1). Find the value of $\frac{3}{4}$ of 14*s*. 8*d*.

$$\tfrac{1}{4}\text{ of }14s.\ 8d. = \frac{14s.\ 8d.}{4} = 3s.\ 8d.$$
$$\therefore \tfrac{3}{4}\text{ of }14s.\ 8d. = 3 \times 3s.\ 8d. = 11s.$$

It is immaterial whether we divide by 4, and then multiply the quotient by 3, or first multiply by 3, and then divide the product by 4; thus:

$$\tfrac{3}{4}\text{ of }14s.\ 8d. = \frac{3 \times 14s.\ 8d.}{4} = \frac{44s.}{4} = 11s.$$

Ex. (2). Find the value of $\frac{2}{3}$ of $\frac{5}{7}$ of £43 4*s*. 6*d*.

$$\tfrac{2}{3}\text{ of }\tfrac{5}{7}\text{ of }£43\ 4s.\ 6d. = \tfrac{10}{21}\text{ of }£43\ 4s.\ 6d.$$
$$= \frac{10 \times £43\ 4s.\ 6d.}{21}$$
$$= 10 \times £2\ 1s.\ 2d. = £20\ 11s.\ 8d.$$

Ex. (3). What is the value of $2\frac{3}{7}$ of 14*s*. 9*d*.?

$$2\tfrac{3}{7}\text{ of }14s.\ 9d. = \tfrac{17}{7}\text{ of }177d.$$
$$= \frac{17 \times 177d.}{7} = \frac{3009d.}{7} = 429\tfrac{6}{7}d. = £1\ 15s.\ 9\tfrac{6}{7}d.$$

NOTE.—To find the value of $\frac{3}{5} \times$ 2*s*. 9*d*., we extend the meaning of the sign × (as explained in Art. 71), and replace it by the word *of*.

$$\text{Thus } \tfrac{3}{5} \times 2s.\ 9d. = \tfrac{3}{5}\text{ of }2s.\ 9d. = \frac{8s.\ 3d.}{5} = 1s.\ 7\tfrac{4}{5}d.$$

Ex. (4). Divide 4*s*. 2*d*. by $\frac{5}{8}$.

$$4s.\ 2d. \div \tfrac{5}{8} = 4s.\ 2d. \times \tfrac{8}{5}$$
$$= \tfrac{8}{5}\text{ of }4s.\ 2d. = 8 \times 10d. = 6s.\ 8d.$$

Ex. (5). Divide £4 3*s*. 9*d*. by $2\frac{2}{3}$.

$$£4\ 3s.\ 9d. \div 2\tfrac{2}{3} = £4\ 3s.\ 9d. \div \tfrac{8}{3}$$
$$= \tfrac{3}{8}\text{ of }£4\ 3s.\ 9d. = \frac{£12\ 11s.\ 3d.}{8} = £1\ 11s.\ 4\tfrac{7}{8}d.$$

Examples. (lxxi.)

Find the value of

(1) $\frac{3}{4}$ of 4*s*. 9*d*.

(2) $\frac{5}{8}$ of 7*s*. 2*d*.

(3) $\frac{5}{16}$ of a guinea.

(4) $\frac{2}{7}$ of 3*s*. 6*d*.

(5) $\frac{1}{2}$ of $\frac{3}{4}$ of 4*s*. 10*d*.

(6) $\frac{2}{3}$ of $\frac{4}{9}$ of £83 16*s*. 3*d*.

(7) $9\frac{1}{3}$ of 1*s*. $1\frac{1}{2}$*d*.

(8) $5\frac{13}{15}$ of half a crown.

(9) $2\frac{3}{5}$ of £5 2*s*. 6*d*.

(10) $\frac{27}{56}$ of £99 14*s*.

(11) £60 1*s*. 8*d*. ÷ $\frac{7}{9}$. (12) £2 6*s*. 9*d*. ÷ $1\frac{1}{3}$.
(13) £53 15*s*. 8*d*. ÷ $6\frac{10}{17}$. (14) £36 2*s*. 9*d*. ÷ $4\frac{1}{5}$.

NOTE.—If we have to multiply a compound expression by a mixed number, it is not always necessary to turn the mixed number into an improper fraction, as we did in Ex. (3), but we can frequently effect the multiplication more neatly by multiplying first by the fractional part and then by the whole number, and adding the two results.

Thus, to multiply £427 12*s*. 9*d*. by $5\frac{2}{3}$.

	£	*s*.	*d*.	
	427	12	9	
			$5\frac{2}{3}$	
3	855	5	6	the result of multiplying the top line by 2
	285	1	10	.... $\frac{2}{3}$
	2138	3	9	 5
	£2423	5	7	

Examples. (lxxii.)

Multiply

(1) £245 13*s*. 4*d*. by $5\frac{3}{4}$. (2) £439 18*s*. 3*d*. by $7\frac{1}{3}$.
(3) £4214 15*s*. 2*d*. by $6\frac{5}{7}$. (4) £8629 12*s*. 8*d*. by $3\frac{5}{8}$.
(5) £7258 17*s*. 6*d*. by $2\frac{3}{5}$. (6) £4372 19*s*. 4*d*. by $6\frac{3}{8}$.

XVI. On Measures.

151. MEASURES OF TIME.

1 second is written 1 sec., or $1^{s.}$
60 seconds make 1 minute, written 1 min., or $1^{m.}$
60 minutes make 1 hour, written 1 hr., or $1^{h.}$
24 hours make 1 day, written 1 da., or $1^{d.}$
7 days make 1 week, written 1 wk.

In rough calculations a year is taken to consist of 365 days.
In rough calculations a month is taken to consist of 30 days.

A *Lunar* Month, or the time between two new moons, is rather more than $29\frac{1}{2}$ days.

The 12 months into which we divide the year are called *Calendar* Months: they are of variable length, for

7 of them contain 31 days, 4 contain 30 days, and February has 28 days (and in Leap-year 29).

The names of the 4 months which have 30 days are given in the old verse:

Thirty days have September,
April, June, and November,

To find whether a particular year is a Leap-year, we divide the number of the year by 4; if no remainder be left, the year is Leap-year, but to correct an error in our present Calendar, the *centuries* which are not exactly divisible by 400, as 1900, 2100...are to be taken as common years, and not as leap-years.

Examples. (lxxiii.)

Reduction.

(1) Reduce 6 hr. 17 min. 25 sec. to seconds; $17^{h.}\ 0^{m.}\ 43^{s.}$ to seconds.

(2) Reduce 3 yr. 143 d. 16 hr. to seconds; 1 yr. 13 d. 0 hr. 4 min. to minutes.

(3) Reduce 48567 min. to days; 23567 sec. to hours.

(4) Reduce 742392 sec. to days; 174296 sec. to weeks.

(5) Find the number of days, reckoning from noon of the one to noon of the other, between the following days in the year 1872:

1st February and 29th May; 4th July and 2nd December; 3rd January and 15th October; 24th February and 23rd June.

Also between 25th December, 1872, and 25th May, 1873.

Addition.

	hr.	min.	sec.
(6)	14	21	37
	17	13	32
	9	47	43
	12	53	54
	22	17	50

	da.	hr.	min.
(7)	23	15	16
	57	12	38
	13	17	43
	24	22	7
	16	5	58

	wk.	da.	hr.
(8)	4	3	16
	2	5	17
	3	6	9
	10	4	13
	4	2	19

	yr.	da.	hr.
(9)	3	137	15
	4	243	6
	1	56	7
	6	135	12
	7	85	9

	hr.	min.	sec.
(10)	14	43	13
	32	36	40
	10	12	53
	16	38	47
	2	52	8

	da.	hr.	min.	sec.
(11)	42	14	30	31
	65	22	19	42
	74	11	42	15
	24	18	58	57
	43	3	29	48

Subtraction.

	hr.	min.	sec.
(12)	7	14	26
	4	19	38

	da.	hr.	min.
(13)	123	16	4
	39	22	17

	wk.	da.	hr.
(14)	4	6	18
	3	6	20

	yr.	da.	hr.
(15)	3	147	14
	2	213	17

	yr.	da.	hr.
(16)	4	45	16
	2	78	19

	da.	hr.	min.	sec.
(17)	14	1	0	13
	8	15	23	27

(18) Multiply 13 hr. 14 min. 43 sec. by 35;
17 hrs. 13 min. 39 sec. by 43.

(19) Divide 15 wks. 5 dys. 17 hrs. 26 min. by 49;
14 hrs. 56 min. 41 sec. by 73.

152. MEASURES OF LENGTH.

12 inches	make 1 foot, usually written	1 ft.,
3 feet	1 yard,	1 yd.
$5\frac{1}{2}$ yards	1 pole,	1 po.
40 poles	1 furlong	1 fur.
8 furlongs	1 mile	1 mi.
3 miles	1 league	1 lea.

Hence 1 furlong = 220 yards, and 1 mile = 1760 yards.

Cloth Measures.

$2\frac{1}{4}$ inches make 1 nail.
4 nails........1 quarter.

4 quarters make 1 yard.
5 quarters......1 ell.

Ex. (1). Reduce 3 mi. 5 fur. 17 po. 4 yd. 1 ft. 3 in. to inches.

mi.	fur.	po.	yd.	ft.	in.
3	5	17	4	1	3

8

29 fur.
40

1177 po.
$5\frac{1}{2}$

$588\frac{1}{2}$ the result of dividing 1177 by 2.
5889

$6477\frac{1}{2}$ yd.
3

$19433\frac{1}{2}$ ft.
12

233205 inche

Ex. (2). Reduce 47293 yards to poles.

$$\begin{aligned} 47293 \text{ yd.} &= (47293 \div 5\tfrac{1}{2}) \text{ poles.} \\ &= (47293 \div \tfrac{11}{2}) \text{ poles.} \\ &= (47293 \times \tfrac{2}{11}) \text{ poles.} \end{aligned}$$

We may proceed thus:

$$\begin{array}{r|l} & 47293 \text{ yards} \\ & \quad\;\; 2 \\ \hline 11 & 94586 \text{ half-yards} \\ \hline & \;\, 8598 \text{ poles, and 8 half-yards over.} \end{array}$$

∴ 47293 yd. = 8598 po. 4 yd.

Examples. (lxxiv.)

Reduction.

(1) Reduce 3 yd. 2 ft. to inches; 4 mi. 3 fur. 4 po. to feet.

(2) Reduce 7 mi. 14 po. $3\frac{1}{2}$ yd. to inches; 27 po. $4\frac{1}{4}$ yd. to inches.

(3) Reduce 74325 yd. to poles; 2423694 in. to furlongs.

(4) Reduce 723964 ft. to miles; 829764332 in. to miles.

Addition.

	yd.	ft.	in.
(5)	4	2	7
	19	1	9
	5	2	10
	23	2	8
	35	1	6
	17	2	4

	mi.	fur.	po.
(6)	13	4	20
	43	3	9
	56	2	13
	4	7	32
	16	3	15
	19	5	11

	fur.	po.	yd.
(7)	2	19	2
	4	25	$2\frac{1}{3}$
	6	11	$3\frac{1}{2}$
	5	23	4
	3	0	$1\frac{1}{3}$
	1	21	$1\frac{3}{4}$

Subtraction.

	yd.	ft.	in.
(8)	134	2	7
	59	1	11

	mi.	fur.	po.
(9)	235	0	19
	184	5	24

	fur.	po.	yd.
(10)	5	23	$1\frac{1}{2}$
	4	27	4

(11) Multiply 7 yd. 2 ft. 9 in. by 11; 16 mi. 5 fur. 7 po. by 56.

(12) Multiply 32 po. 3 yd. 1 ft. by 57; 36 mi. 3 fur. 6 po. $3\frac{1}{2}$ yd. by 49.

(13) Divide 25 yd. 1 ft. 8 in. by 4; 17 mi. 3 fur. 7 po. by 27.

(14) Divide 14 po. 2 yd. 1 ft. 8 in. by 32; 11 mi. 7 fur. 7 po. by 55.

153. Measures of Surface.

144	square inches make	1 square foot, written	1 sq. ft.
9	square feet	1 square yard	1 sq. yd.
$30\frac{1}{4}$	square yards	1 square pole	1 sq. po.
40	square poles	1 rood	1 ro.
4	roods	1 acre	1 ac.

Hence 1 acre = 4840 square yards.
640 acres = 1 square mile.

Land surveyors make use of a Chain 22 yards in length, divided into 100 equal parts, called Links.

The square of 22 is 484, and therefore 10 Square Chains make an Acre.

Note.—The Square Inch is a square whose side is an inch in length.

Ex. (1). How many square inches are there in 3 ac. 2 ro. 27 po. 27 sq. yd. 7 sq. ft. 23 sq. in.?

ac.	ro.	po.	sq. yd.	sq. ft.	sq. in.
3	2	27	27	7	23

4

14 ro.
40

587 po.
$30\frac{1}{4}$

$146\frac{3}{4}$ the result of the division of 587 by 4.
17637

$17783\frac{3}{4}$ sq yd.
9

160054
$6\frac{3}{4}$ the result of multiplying $\frac{3}{4}$ by 9.

$160060\frac{3}{4}$ sq. ft.
144

640263
640240
160060
108 the result of multiplying $\frac{3}{4}$ by 144.

23047771 sq. in.

Ex. (2). Reduce 74237 sq. yards to poles.

$$\begin{aligned} 74237 \text{ sq. yd.} &= (74237 \div 30\tfrac{1}{4}) \text{ poles} \\ &= (74237 \div \tfrac{121}{4}) \text{ poles} \\ &= (74237 \times \tfrac{4}{121}) \text{ poles.} \end{aligned}$$

We may proceed thus:

$$\begin{array}{r} \\ \\ 121\left\{\begin{array}{l} 11 \\ \\ 11 \end{array}\right. \\ \\ \end{array} \begin{array}{|r l} 74237 & \text{yards} \\ 4 & \\ \hline 296948 & \text{quarter-yards.} \\ \hline 26995 & \text{and 3 quarter-yards over.} \\ \hline 2454 & \text{po. and 1 parcel of 11 quarter-yards over.} \end{array}$$

The remainder is (11 + 3) quarter-yards, or 14 quarter-yards, or 3½ yd.

∴ 74237 sq. yd. = 2454 po. 3½ sq. yd.

Examples. (lxxv.)

Reduction.

(1) Reduce 5 ac. 3 ro. 17 po. 13 sq. yd. 6 sq. ft. 15 sq. in. to square inches.

(2) Reduce 7 ac. 13 po. 5 sq. yd. 3 sq. ft. to square inches.

(3) Reduce 250 ac. to square yards, and 73 sq. yd. to square inches.

(4) Reduce 5239 sq. in. to square yards, and 15376 sq. yd. to acres.

(5) Reduce 34729 sq. yd. to poles, and 562934 sq. in. to square poles.

Addition.

	ac.	ro.	po.
(6)	47	2	13
	72	1	24
	89	2	32
	4	2	23
	27	3	8
	42	2	5

	sq. yd.	sq. ft.	sq. in.
(7)	19	7	42
	27	5	52
	32	8	124
	5	2	72
	21	6	98
	56	3	135

	ac.	ro.	po.	sq. yd.
(8)	46	2	16	22
	17	3	14	13
	7	1	39	14
	24	2	15	19
	12	0	17	22
	4	1	9	16

Subtraction.

	ac.	ro.	po.		sq. yd.	sq. ft.	sq. in.		ac.	ro.	po.
(9)	57	2	30	(10)	42	8	124	(11)	16	2	0
	29	3	34		36	8	139		14	3	24

	ac.	ro.	po.		sq. yd.	sq. ft.	sq. in.		ac.	ro.	po.
(12)	247	1	14	(13)	39	7	12	(14)	245	3	19
	243	3	24		32	8	134		178	3	23

(15) Multiply 5 ac. 3 ro. 24 po. by 15 ; 17 ac. 2 ro. 13 po. by 53.

(16) Divide 7 ac. 2 ro. 18 po. by 21 ; 29 ac. 2 ro. 37 po. by 71.

154. MEASURES OF SOLIDITY.

1728 cubic inches make 1 cubic foot, written 1 cub. ft.
27 cubic feet make 1 cubic yard, written 1 cub. yd.

A Cube is a solid figure contained by six equal squares. Hence a cubic inch is a six-sided figure, each of whose sides is a square inch. The lines that form the boundaries of the sides are called the Edges of the Cube.

Examples. (lxxvi.)

Reduction.

(1) Reduce 7 cub. yd. 13 cub. ft. to cubic feet; 25 cub. yd. 5 cub. ft. 143 cub. in. to cubic inches ; 14 cub. yd. 1374 cub. in. to cubic inches.

(2) Reduce 74325 cub. in. to cubic feet ; 439284 cub. in. to cubic yards.

(3) Reduce $5\frac{1}{4}$ cub. yd. to cubic inches ; 3 cub. yd. $5\frac{3}{4}$ cub. ft. to cubic inches.

Addition.

	cub. yd.	cub. ft.	cub. in.		cub. yd.	cub. ft.	cub. in.		cub. yd.	cub. ft.	cub. in.
(4)	57	13	572	(5)	43	7	1638	(6)	528	16	432
	32	25	493		26	22	472		237	19	583
	46	19	374		19	16	1384		764	10	1359
	76	8	587		45	13	427		446	0	1275
	4	26	1249		26	5	1286		729	11	346
	52	14	1324		33	18	275		852	5	1473

Subtraction.

	cub. yd.	cub ft.	cub. in.
(7)	47	17	543
	38	23	726

	cub. yd.	cub. ft.	cub. in.
(8)	247	19	1274
	239	18	1368

	cub. yd.	cub. ft.	cub. in.
(9)	527	0	0
	499	10	256

(10) Multiply 26 cub. yd. 5 cub. ft. 49 cub. in. by 27; 472 cub. yd. 17 cub. ft. 238 cub. in. by 53.

(11) Divide 78 cub. yd. 13 cub. ft. 252 cub. in. by 12; 472 cub. yd. 0 cub. ft. 1416 cub. in. by 59.

155. MEASURES OF CAPACITY.

2 pints make 1 quart, written 1 qt.
4 quarts ....1 gallon, ...1 gall.
2 gallons ....1 peck, ...1 pk.
4 pecks .....1 bushel ....1 bus.
8 bushels....1 quarter ....1 qr.

Examples. (lxxvii.)

Reduction.

(1) Reduce 3 pk. 1 gall. 3 pt. to pints, and 214 qrs. 3½ bus. to pints.

(2) Reduce 4234 pt. to quarters, and 3047 gall. to quarters.

Addition.

	gall.	qt.	pt.
(3)	4	3	1
	3	2	1½
	12	3	0
	14	0	1½
	5	2	1

	bush.	pk.	gall.
(4)	4	3	1
	5	2	1½
	1	3	1
	4	2	1½
	3	1	0

	qr.	bush.	pk.
(5)	42	5	3
	27	7	2
	64	3	1
	49	6	2
	12	4	0

Subtraction.

	gall.	qt.	pt.
(6)	5	2	0
	4	3	1

	bus.	pk.	gall.
(7)	6	3	0
	5	3	1

	qr.	bus.	pk.
(8)	36	7	2
	29	7	3

(9) Multiply 5 qr. 3 bus. 2 pk. by 63, and 15 qr. 2 bus. 1 pk. by 73.

(10) Divide 13 gall. 1 pt. by 15, and 348 qr. 0 bus. 1 pk. by 43.

156. TROY WEIGHT.

24 grains	make 1 pennyweight,	written 1 dwt.
20 pennyweights	make 1 ounce,	written 1 oz.
12 ounces	make 1 pound,	written 1 lb.

Chiefly used for weighing gold, silver, and jewels.

Examples. (lxxviii.)

Reduction.

(1) In 27 ounces of gold how many grains are there?

(2) Reduce 7 lb.; 14 lb. 3 oz.; 25 lb. 9 oz. 5 dwt. to pennyweights.

(3) Reduce 3 lb. 10 oz. 7 dwt. 5 gr.; 7 lb. 4 oz. 17 dwt. 15 gr. to grains.

(4) Reduce 3145 gr. to ounces; 42672 gr. to lb.

(5) Reduce 72469 gr. to lb.; 3246 dwt. to lb.

Addition.

(6)	lb.	oz.	dwt.
	21	2	12
	27	9	4
	3	8	17
	14	3	19
	7	6	8

(7)	oz.	dwt.	gr.
	7	13	21
	4	6	19
	6	17	23
	2	9	5
	8	16	13

(8)	lb.	oz.	dwt.	gr.
	13	8	6	14
	12	4	17	8
	5	10	13	0
	42	7	15	21
	12	11	19	23

Subtraction.

(9)	oz.	dwt.	gr.
	6	19	13
	3	14	16

(10)	lb.	oz.	dwt.
	37	8	6
	29	10	13

(11)	lb.	oz.	dwt.	gr.
	35	9	8	22
	34	11	15	23

(12) Multiply 7 lb. 5 oz. 9 dwt. by 12; 6 lb. 8 oz. 19 dwt. by 21.

(13) Multiply 10 oz. 16 dwt. 23 gr. by 37; 3 lb. 7 oz. 10 dwt. 21 gr. by 41.

(14) Divide 16 lb. 4 oz. 16 dwt. by 8; 7 lb. 10 oz. 17 dwt. 7 gr. by 15.

(15) Divide 9 oz. 17 dwt. 8 gr. by 37; 15 lb. 8 oz. 9 dwt. 12 gr. by 63.

157. AVOIRDUPOIS WEIGHT.

16 drachms	make 1 ounce,	written 1 oz.
16 ounces	..... 1 pound,	.... 1 lb.
14 pounds	..... 1 stone,	.... 1 st.
25 pounds	..... 1 quarter,	.... 1 qr.
4 quarters	..... 1 hundredweight or 1 cental	.. 1 cwt.
20 hundredweight	.... 1 ton.	

The pound Avoirdupois contains 7000 grains Troy.
The pound Troy contains 5760 grains Troy.

NOTE.—In Great Britain 28 pounds make 1 quarter.

Examples. (lxxix.)

Reduction.

(1) Reduce 11 cwt. to oz.; 17 lb. to dr.; 5 tons to lb.
(2) Reduce 6 tons 7 cwt. to oz.; 15 tons 2 qr. to lb.
(3) Reduce 3 cwt. 6 lb. 5 oz. to dr.; 3 tons 15 cwt. 7 lb. to lb.
(4) Reduce 4763 oz. to cwt.; 3749 lb. to tons.
(5) Reduce 7432 oz. to cwt.; 247294 dr. to cwt.

Addition.

	lb.	oz.	dr.
(6)	3	3	9
	19	8	6
	7	10	13
	14	5	7
	8	15	14

	qr.	lb.	oz.
(7)	3	16	8
	4	7	12
	16	19	5
	8	20	13
	12	5	9

	cwt.	qr.	lb
(8)	13	2	24
	11	3	5
	29	1	19
	16	2	9
	17	0	7

Subtraction.

	lb.	oz.	dr.
(9)	16	13	5
	14	11	12

	qr.	lb.	oz.
(10)	17	13	3
	14	15	11

	cwt.	qr.	lb.
(11)	19	1	4
	17	3	18

	tons.	cwt.	qr.
(12)	37	19	2
	29	19	3

	cwt.	qr.	lb.
(13)	16	0	3
	15	3	25

	tons.	cwt.	qr.	lb.
(14)	74	15	1	13
	39	16	3	25

(15) Multiply 17 cwt. 23 lb. 14 oz. by 7; 4 cwt. 17 lb. by 45.

(16) Multiply 6 cwt. 3 qr. 5 lb. by 23; 10 oz. 9 dr. by 37.

(17) Divide 14 cwt. 2 qr. 8 lb. by 12; 32 tons 15 cwt. 1 qr. by 40.

(18) Divide 16 cwt. 3 qr. 9 lb. by 65; 37 tons 4 cwt. 3 qr. 7 lb. by 17.

158. APOTHECARIES' WEIGHT.

1. *Measures of Weight.*

437½ grains make 1 ounce,
16 ounces make 1 pound.

The grain is the same as the grain Troy.

The ounce is the same as the ounce Avoirdupois.

This is the table given in the British Pharmacopœia. The Avoirdupois ounce and pound are taken in preference to the ounce and pound Troy of the old table, because the former are used by wholesale dealers in drugs and medicines. In prescribing, many physicians still employ the scruple (℈) of 20 grains and the drachm (ʒ) of 60 grains.

159. 2. *Measures of Capacity.*

60 minims	make	1 fluid drachm,	written	fl dr.
8 fluid drachms		1 fluid ounce,	"	fl oz.
20 fluid ounces		1 pint,	"	O.
8 pints		1 gallon,	"	C.

Note.—O is a contraction for *Octavus* or *eight*, and C for *Congius*, a Roman liquid measure.

The relation of the measures of capacity to those of weight in these tables is given by the definition that

1 Minim is the measure of ·91 Grain of Water.

The connection may be better remembered by the old rhyme:—

A Pint of Water
Weighs a Pound and a Quarter.

160. Multiplication of Compound Quantities when the multiplier contains a fraction. (See page 128.)

Examples. (lxxx.)

Multiply

(1) 3 cwt. 2 qr. 12 lb. by $3\frac{2}{3}$. (2) 6 lb. 5 oz. 4 dr. by $2\frac{1}{5}$.
(3) 4 mi. 3 fur. 10 po. by $18\frac{1}{5}$. (4) 15 yd. 2 ft. 3 in. by $43\frac{1}{7}$.
(5) 37 ac. 3 ro. 8 po. by $4\frac{5}{12}$. (6) 25 ac. 2 ro. 15 po. by $29\frac{3}{5}$.
(7) 27 sq. yd. 7 sq. ft. 36 sq. in. by $2\frac{5}{6}$.

161. Division of Compound Quantities when the divisor contains a fraction. (See page 128.)

Examples. (lxxxi.)

Divide

(1) 5 cwt. 2 qr. 11 lb. by $2\frac{3}{4}$. (2) 7 lb. 4 oz. 14 dr. by $11\frac{1}{3}$.
(3) 7 mi. 2 fur. 12 po. by $4\frac{5}{12}$. (4) 17 yd. 1 ft. 3 in. by $5\frac{1}{2}$.
(5) 25 ac. 2 ro. 12 po. by $4\frac{3}{7}$. (6) 14 ac. 3 ro. 8 po. by $8\frac{2}{9}$.
(7) 107 sq. yd. 4 sq. ft. 132 sq. in. by $18\frac{2}{5}$.

162. XVII. Fractional Measures.

Ex. (1). How many shillings and pence are there in $\frac{5}{8}$ of a pound?

$$\begin{aligned} \tfrac{5}{8} \text{ of a pound} &= \tfrac{5}{8} \text{ of 20 shillings.} \\ &= \tfrac{5\times 20}{8} \text{ shillings.} \\ &= \tfrac{100}{8}s. \\ &= 12s.\ 6d. \end{aligned}$$

Ex. (2). Find the value of $\frac{3}{7}$ of £15 5*s*. 8*d*.

$$\begin{aligned} \tfrac{3}{7} \text{ of £15 } 5s.\ 8d. &= 3 \text{ times } \tfrac{1}{7} \text{ of £15 } 5s.\ 8d. \\ &= 3 \text{ times £2 } 3s.\ 8d. \\ &= \text{£6 } 11s. \end{aligned}$$

Or thus:

$$\begin{array}{r|rrr} & \text{£} & s. & d. \\ & 15 & 5 & 8 \\ & & & 3 \\ \hline 7 & 45 & 17 & 0 \\ \hline & \text{£6} & 11 & 0 \end{array}$$

Ex. (3). Find the value of $2\frac{3}{4}$ of $\frac{3}{22}$ of 5 acres.

$$\begin{aligned} 2\tfrac{3}{4} \text{ of } \tfrac{3}{22} \text{ of 5 acres} &= \tfrac{11}{4} \text{ of } \tfrac{3}{22} \text{ of 5 acres.} \\ &= \tfrac{11\times 3}{4\times 22} \text{ of 5 acres.} \\ &= \tfrac{3}{8} \text{ of 5 acres.} \\ &= \tfrac{15}{8} \text{ ac.} \\ &= 1 \text{ ac. } 3 \text{ ro. } 20 \text{ po.} \end{aligned}$$

Examples. (lxxxii.)

Find the value of the following:

(1) $\frac{2}{3}$ of £1; $\frac{5}{8}$ of £2 10*s*.; $\frac{3}{7}$ of £5 18*s*. 5*d*.

(2) $\frac{4}{5}$ of a mile; $\frac{3}{16}$ of an acre; $\frac{5}{6}$ of a cwt.

(3) $2\frac{4}{5}$ of £54 9*s*. 8*d*.; $3\frac{3}{14}$ of half-a-guinea; $\frac{2}{3}$ of $3\frac{3}{8}$ of a mile.

(4) $\frac{4}{5}$ of $\frac{1}{7}$ of $1\frac{5}{19}$ of $1\frac{3}{8}$ of 2470 guineas; $\frac{2}{3}$ of $\frac{1}{4}$ of $4\frac{1}{2}$ guineas.

(5) $\frac{3}{5}$ of £1 + $\frac{2}{3}$ of 1*s* + $\frac{5}{8}$ of 16*s*. 4*d*.

(6) $\frac{3}{24}$ of £1 + $\frac{4}{5}$ of 2*s*. 6*d*. + $\frac{3}{7}$ of a guinea.

(7) $\frac{3}{4}$ of 5 ac. 3 ro. + $\frac{5}{8}$ of 7 ac. 2 ro. 20 po. + $\frac{3}{5}$ of 3 ro. 15 po.

(8) $\frac{3}{73}$ of a year + $\frac{9}{56}$ of a week + $\frac{7}{12}$ of an hour.

(9) $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of a furlong + $\frac{3}{8}$ of a yard.

(10) $\frac{3}{4}$ of 2 cwt. 3 qr. + $\frac{3}{7}$ of 5 cwt. 3 qr. 14 lb. + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.

163. The following are examples of an operation which is the *converse* of that just explained.

Ex. (1). Express 14*s.* 7*d.* as the fraction of £5.

14*s.* 7*d.* = 175*d.*, and £5 = 1200*d.*

Now 1*d.* = $\frac{1}{1200}$ of 1200*d.*

∴ 175*d.* is $\frac{175}{1200}$ of 1200*d.*;

Hence the fraction required is $\frac{175}{1200}$, or $\frac{35}{240}$, or $\frac{7}{48}$.

Ex. (2). Express 6 lbs. 5 oz. avoird. as the fraction of 3 lb. 12 oz.

6 lb. 5 oz. = 101 oz., and 3 lbs. 12 oz. = 60 oz.;

∴ the fraction required is $\frac{101}{60}$.

Ex. (3). Express $\frac{2}{3}$ of 5*s.* 9*d.* as the fraction of 4*s.* 7*d.*

5*s.* 9*d.* = 69*d.*, and 4*s.* 7*d.* = 55*d.*

∴ 5*s.* 9*d.* is $\frac{69}{55}$ of 4*s.* 7*d.*

∴ $\frac{2}{3}$ of 5*s.* 9*d.* is $\frac{2}{3}$ of $\frac{69}{55}$ of 4*s.* 7*d.*

∴ the fraction required is $\frac{2 \times 69}{3 \times 55}$ or $\frac{46}{55}$.

Ex. (4). Express $\frac{3}{7}$ of $2\frac{4}{5}$ of 5 ac. 3 ro. as the fraction of $\frac{3}{5}$ of 14 ac. 2 ro.

5 ac. 3 ro. = 23 roods, and 14 ac. 2 ro. = 58 roods;

∴ fraction required is ($\frac{3}{7}$ of $\frac{14}{5}$ of 23) ÷ ($\frac{3}{5}$ of 58);

or $\frac{3 \times 14 \times 23 \times 5}{7 \times 5 \times 3 \times 58}$, or $\frac{2 \times 23}{58}$, or $\frac{23}{29}$.

NOTE.—There are several modes of demanding the operation explained in the foregoing examples. Thus the demand,

Express 3 shillings as the fraction of 6 shillings,

may be put in the following terms:

(1) Reduce 3 shillings to the fraction of 6 shillings.
(2) What part of 6 shillings is 3 shillings?
(3) What fraction of 6 shillings is 3 shillings?
(4) If 6 shillings be the unit, what is the measure of 3 shillings?

Examples. (lxxxiii.)

(1) Express $1\frac{3}{4}$*d.* as the fraction of 6*s.* $8\frac{1}{2}$*d.*
(2) Express £10 5*s.* 4*d.* as the fraction of £11 6*s.* 5*d.*
(3) Express 5*s.* 6*d.* as the fraction of a guinea.
(4) Reduce 9*s.* $10\frac{1}{2}$*d.* to the fraction of 13*s.* $2\frac{1}{4}$*d.*
(5) Reduce 2 days 3 hrs. 5 min. to the fraction of a week.

(6) Reduce 2 roods 20 poles to the fraction of an acre.

(7) What fraction is 8 lb. 1 oz. 19 dwt. 9 gr. of 13 lb. 7 oz. 5 dwt. 15 gr. ?

(8) What part of 2 qr. 10 lb. 7 oz. 9 dr. is 1 qr. 7 oz. 13 dr.?

(9) What fraction of 4 lb. 1 oz. 8 dwt. 15 gr. is 1 lb. 1 oz. 9 dwt. 15 gr.?

(10) If the unit of measurement be $2\frac{1}{3}$ yd., what is the measure of $2\frac{1}{2}$ feet ?

(11) If the unit of measurement be 5 inches, what is the measure of $\frac{5}{374}$ of a mile ?

(12) What fraction of 2 ac. 37 po. is 3 ac. 2 ro. 1 po.?

XVIII. Decimal Measures.

164. REDUCTION OF DECIMALS.

Ex. (1). How many shillings and pence are there in ·375 of a pound?

$$\begin{aligned} \cdot 375 \text{ of } \pounds 1 &= (\cdot 375 \times 20)s. \\ &= 7\cdot 5s. \\ \text{and } \cdot 5 \text{ of } 1s. &= (\cdot 5 \times 12)d. \\ &= 6d. \\ \therefore\ \cdot 375 \text{ of } \pounds 1 &= 7s.\ 6d. \end{aligned}$$

The operation is performed more briefly thus:

$$\begin{array}{r} \pounds\cdot 375 \\ 20 \\ \hline s.\ 7\cdot 500 \\ 12 \\ \hline d.\ 6\cdot 000 \end{array}$$

Ex. (2). Find the value of 3·16875 of £1.

$$\begin{array}{r} \pounds 3\cdot 16875 \\ 20 \\ \hline s.\ 3\cdot 37500 \\ 12 \\ \hline d.\ 4\cdot 50000 \\ 4 \\ \hline q.\ 2\cdot 00000 \end{array}$$

$$\therefore\ \pounds 3\cdot 16875 = \pounds 3\ 3s.\ 4\tfrac{1}{2}d.$$

Ex. (3). Find the value of ·4256 of 12*s.* 8*d.*

·4256 of 12*s.* 8*d.* = ·4256 of 152*d.* = (·4256 × 152)*d.*

$$\begin{array}{r} \cdot4256 \\ 152 \\ \hline 8512 \\ 21280 \\ 4256 \\ \hline 64\cdot6912 \end{array}$$

∴ value required is 64·6912*d.*

Ex. (4). Multiply 27 ac. 3 ro. 14 po. by ·235.

$$\begin{array}{rll} \text{ac.} & \text{ro.} & \text{po.} \\ 27 & 3 & 14 \\ 4 & & \\ \hline 111 & \text{ro.} & \\ 40 & & \\ \hline 4454 & \text{po.} & \\ \cdot235 & & \\ \hline 22270 & & \\ 13362 & & \\ 8908 & & \end{array}$$

$$\begin{array}{r|ll} 40 & 1046\cdot690 & \\ \hline 4 & 26 & 6\cdot69 \text{ po.} \\ \hline \end{array}$$

6 ac. 2 ro. 6·69 po.

Ex. (5). Find the value of $\cdot2\dot{5}$ of £1.

$\cdot2\dot{5}$ of £1 = $\frac{25-2}{90}$ of £1 = $\frac{23}{90}$ of £1 = $\frac{460}{90}$*s.* = 5*s.* $1\frac{1}{3}$*d.*

Or thus:

$$\begin{array}{r} £\cdot255\dot{5}\ldots. \\ 20 \\ \hline s.\ 5\cdot111\dot{1}\ldots. \\ 12 \\ \hline d.\ 1\cdot333\dot{3}\ldots. \end{array}$$

∴ value required is 5*s.* $1\frac{1}{3}$*d.*

Examples. (lxxxiv.)

Find the value of

(1) ·625 of £1.
(2) £15·275.
(3) £·009765.
(4) ·9375 of a cwt.
(5) ·046875 of 1 lb. avoir.
(6) 2·003125 of £8.
(7) ·425 of 3*s.* 4*d.*
(8) 2·46875 of £1 3*s.*
(9) $\cdot 8\dot{3}$ of 5*s.*
(10) $4\cdot 1\dot{3}$ of 12*s.* 6*d.*
(11) ·35 of 2 qr. 14 lb.
(12) 2·125 of $3\frac{1}{2}$ guineas.
(13) 2·1372 of 2 tons 5 cwt.
(14) $5\cdot 24\dot{7}$ of £5 2*s.* 6*d.*
(15) ·45 of £3 10*s.* + ·75 of 4*s.* 8*d.* + 3·245 of 3*s.* 4*d.*
(16) ·7 of £1 + $\cdot\dot{8}$ of 7*s.* 6*d.* − 2·45 of 1*s.* 8*d.*
(17) $\cdot\dot{2}8571\dot{4}$ of £3 3*s.* + $\cdot\dot{1}4285\dot{7}$ of £3 17*s.* + $\cdot\dot{3}\dot{4}$ of 16*s.* 6*d.*

165. The following examples illustrate the operation which is the *converse* of that already explained.

Ex. (1). Express 5*s.* 6*d.* as the decimal of £1.

$$5s.\ 6d. = 66d., \text{ and } £1 = 240d.;$$
$$\therefore 5s.\ 6d. = \tfrac{66}{240} \text{ of } £1.$$
$$\text{Now } \tfrac{66}{240} = \tfrac{11}{40} = \cdot 275;$$
$$\therefore 5s.\ 6d. = \cdot 275 \text{ of } £1.$$

Or more briefly thus:

$$\begin{array}{r|l} 12 & 6\cdot 0 \quad d. \\ \hline 20 & 5\cdot 5 \quad s. \\ \hline & \cdot 275 \quad £. \end{array}$$

Where we first express 6*d.* as the decimal of a shilling, *i.e.*, ·5, and then express 5·5*s.* as the decimal of a pound, *i.e.*, ·275.

Ex. (2). Express £7 15*s.* $10\frac{1}{2}d.$ as the decimal of £1.

$$\begin{array}{r|l} 4 & 2\cdot 0 \\ \hline 12 & 10\cdot 5 \\ \hline 20 & 15\cdot 875 \\ \hline & £\,7\cdot 79375 \end{array}$$

Ex. (3). Express £3 5*s.* 9*d.* as the decimal of £5 7*s.* 6*d.*

£	*s.*	*d.*	£	*s.*	*d.*
3	5	9	5	7	6
20			20		
65			107		
12			12		
789			1290		

Now $\frac{789}{1290} = \frac{263}{430} = \frac{26\cdot3}{43} = \cdot611\ldots\ldots$

∴ £3 5*s.* 6*d.* is ·611......of £5 7*s.* 6*d.*

Ex. (4). Express $\frac{2}{3}$ of 5*s.* 9¼*d.* as the decimal of $\frac{3}{5}$ of 6*s.* 2*d.*

5*s.* 9¼*d.* = 277*q.*, and 6*s.* 2*d.* = 296*q.*

∴ $\frac{2}{3}$ of 5*s.* 9*d.* is $\dfrac{\frac{2}{3} \times 277}{\frac{3}{5} \times 296}$ of $\frac{3}{5}$ of 6*s.* 2*d.*

Now $\dfrac{\frac{2}{3} \times 277}{\frac{3}{5} \times 296} = \dfrac{2 \times 277 \times 5}{3 \times 3 \times 296} = \dfrac{1385}{1332} = 1\cdot039\ldots\ldots$

Examples. (lxxxv.)

(1) Express 6 cwt. 2 qr. 7 lb. as the decimal of a ton.

(2) Express 12 grains as the decimal of a lb. troy.

(3) What decimal of 10 guineas is £1 19*s.* 4½*d.* ?

(4) Express $\frac{3}{5}$ of 14*s.* 4*d.* as the decimal of £1.

(5) Reduce 3·45 of half a guinea to the decimal of 2*s.* 6*d.*

(6) Express $\frac{2}{3}$ of 2 qr. 14 lb. as the decimal of a cwt.

(7) Express $4\frac{2}{5}$ of 7 oz. 4 dwt. as the decimal of a pound troy.

(8) Reduce $3\frac{7}{8}$ of $1\frac{1}{4}$ of 5 cwt. 2 qr. 21 lb. to the decimal of a ton.

(9) What decimal of a pound troy is $\frac{2}{3}$ of a dwt. ?

(10) Reduce $3\frac{3}{4}$ guineas to the decimal of £2 15*s.*

(11) Reduce 2*s.* 6*d.* to the decimal of $\frac{5}{12}$ of £1.

(12) Express 18*s.* 4½*d.* as the decimal of £1000.

(13) Reduce £24·25 + 3·4125*s.* + 9·25*d.* to the decimal of £10.

(14) Express $\cdot4\dot{3}$ of 8*s.* 3*d.* as the decimal of $\cdot0\dot{1}$ of £9.

(15) Express $\cdot0\dot{4}$ of £[illegible] 5*s.* + $\cdot\dot{2}\dot{3}$ of 3*s.* 9*d.* as the decimal of $\cdot\dot{2}4\dot{5}$ of £4 3*s.* 3*d.*

EXAMINATION PAPERS.

I. MEASURES OF TIME.

(1) A sidereal day is less than a solar day by 3 minutes 56 seconds; in how many days will the difference amount to 24 hours?

(2) If Sirius, one of the brightest of the fixed stars, which is probably 592200 times farther from the earth than the sun, were suddenly extinguished, for how long would it appear to shine to the inhabitants of the earth, supposing the sun's mean distance from the earth to be 91713000 miles, and that light from the sun reaches the earth in 8 min. 18 sec.?

(3) The exact length of the year being 365 days 5 hrs. 48 min. 49·7 sec., and computing time as at present, find the error in 12000 years.

(4) The *Globe* newspaper of Monday, 18th June, 1877, bears the number 8505. Supposing the paper to have been published every week day without intermission, and numbered consecutively, give the day of the week, month, and year when No. 1 was published.

(5) There was a full moon on June 26, 1858, at 9 hrs. 13 min. a.m. The interval between successive full moons has since been on the average 29 days 12 hrs. 47 min. 30 sec.; how many full moons happened until December 31, 1873, and when did the last take place within that period?

II. MEASURES OF LENGTH.

(1) Reduce 9 mi. 7 fur. 39 per. 5 yds. 1 ft. 9 in. to inches, and show that the work is correct by changing it to miles, etc.

(2) The fore-wheel of a carriage, which is 11 ft. in circumference, makes 718 revolutions more than the hind one in going 7 miles; find the circumference of the hind-wheel.

(3) A train, which travels at the uniform rate of 66 ft. a second, leaves Toronto for Montreal at 6.25 a.m.; when will it reach Montreal, the distance being 333 miles? At what distance from Montreal will it meet a train which leaves Montreal for Toronto at 8 a.m., and travels one-third faster than it does?

(4) From Ephesus to Cunaxa, Xenophon, with the army of Cyrus, marched 16050 stadia of 202 yards 9 inches each in 93 days. Find the average length of a day's march in miles and yards.

(5) How many strokes of his legs must a person on a bicycle give in going 26 miles, supposing each wheel to have a circumference of 3¼ yards, and that 2 strokes turn the wheel once round?

III. MEASURES OF SURFACE.

(1) If the magnitude of the lineal unit be given, what are the corresponding units of area and volume? Exemplify when the lineal unit is 12 inches.

(2) If a halfpenny piece be one inch in diameter, how many can be laid in rows touching each other on a table which is 7 feet 6 inches long and 3 feet 4 inches wide; and what is their amount?

(3) Divide 17 ac. 2 ro. 38 per. 19 yds. 7 ft. 45 in. among A, B, and C, giving to B as much again as to A, and to C ¾ of what A and B got.

(4) If 68 bales of linen contain 67048 yards, and each bale contains 34 pieces, and each piece the same number of yards, how many yards are there in each piece?

(5) If the pressure of the atmosphere at the surface of the earth, when the barometer stands at 30 inches, be 15 lbs. on the square inch, what is the pressure in pounds on the surface of the human body, supposing it to be 15 square feet? What would be the difference of the pressure when the barometer stands at 29 inches?

IV. MEASURES OF CAPACITY.

(1) What will 2 bushels 3 pecks 3 quarts of strawberries amount to at 12½ cents per quart?

(2) A laborer dug 130 rods 4 yards 2½ feet of ditching at $2½ per rod, for which he is to take $100 in cash and wheat at 87½ cents per bushel. To what quantity of wheat will he be entitled?

(3) A grocer exchanged 29 gal. 3 qt. 1 pt. of brandy, at 43¾ cents per gallon, for rye at 31¼ cents per bushel. What quantity of rye did he thus attain?

(4) I wish to put 111 bu. 2 pk. 4 qt. of grain into bags that shall contain 2 bu. 1 pk. 4 qt. each; how many bags will be required?

(5) A farmer had a field of corn consisting of 129 rows, and each row contained 95 hills, and each hill had on an average 4½ ears of corn. If it takes 8 ears of corn to make a quart, what is the produce of the field worth at 45 cents per bushel?

V. MEASURES OF WEIGHT.

(1) If John buy, by Avoirdupois weight, 12 lb. of opium at $37\frac{1}{2}$ cents per ounce, and sell by Troy weight at 40 cents per ounce, should he gain or lose by so doing, and how much?

(2) A person purchases goods at the rate of $1.80 per pound, Troy weight, and sells them again by Avoirdupois weight; at what rate must he sell per ounce so as exactly to reimburse himself?

(3) By multiplying a certain weight by a whole number the result is 8 lbs. 20 grains Avoirdupois weight, and by multiplying the same weight by another whole number the result is 8 lbs. 11 oz. 16 dwt. 16 grs. Find the weight.

(4) A row of cent pieces is laid from Toronto to Hamilton. Find their weight, the distance being 39 miles 1 fur. 1 per. 9 in.

(5) Find the value of 500 times the difference between an eighty-fourth part of $2\frac{1}{2}$ cwt. and a thirtieth part of 1 cwt. 0 qr. 3 lb. (28 lbs. to the quarter).

XIX. Practice.

166. Practice is the name given to a method by which we find the cost of any number of articles of the *same* kind when the price of one is given, or the cost of any quantity of goods of *mixed* denominations, when the cost of a single unit of any denomination is given.

I. SIMPLE PRACTICE.

When the articles are of the *same* kind or denomination.

Ex. (1). Suppose I have to find the cost of 2478 articles at 3*s.* 4*d.* each.

Knowing that 3*s.* 4*d.* is one-sixth part of £1, I reason thus: If the articles had cost £1 each, the total cost would have been £2478;

$\therefore$ as they cost $\frac{1}{6}$ of £1 each, the cost will be £$\frac{2478}{6}$, or £413.

The process may be written thus:

$$\begin{array}{l|l} 3s.\ 4d. \text{ is } \frac{1}{6} \text{ of } £1 & £2478 = \text{cost of the articles at } £1 \text{ each.} \\ \hline & £413 = \text{cost of the articles at } 3s.\ 4d. \text{ each.} \end{array}$$

Ex. (2). Find the cost of 2897 articles at £2 12*s*. 9*d*. each.

	£	*s*.	*d*.	
£2 is 2 × £1	2897	0	0	= cost at £1 each.
10*s*. is ½ of £1	5794	00	0	= ...... £2 ....
2*s*. is ⅕ of 10*s*.	1448	10	0	= ...... 10*s*. ...
8*d*. is ⅓ of 2*s*.	289	14	0	= ..... 2*s*. ....
1*d*. is ⅛ of 8*d*.	96	11	4	= ...... 8*d*. ....
	12	1	5	= ...... 1*d*. ....
	£7640	16	9	= cost at £2 12*s*. 9*d*. each

Note.—A shorter method would be to take the parts thus:

10*s*. = ½ of £1; 2*s*. 6*d*. = ¼ of 10*s*.; 3*d*. = $\frac{1}{10}$ of 2*s*. 6*d*.

Ex. (3). Find the cost of 425 articles at £2 18*s*. 4*d*. each.

Since £2 18*s*. 4*d*. is the difference between £3 and 1*s*. 8*d*. (which is $\frac{1}{12}$ of £1), the shortest course is to find the cost at £3 each, and to *subtract from it* the cost at 1*s*. 8*d*. each, thus:

	£	*s*.	*d*.	
£3 is 3 × £1	425	0	0	= cost at £1 each.
1*s*. 8*d*. is $\frac{1}{12}$ of £1	1275	0	0	= ...... £3 ......
	35	8	4	= ...... 1*s*. 8*d*. each.
	£1239	11	8	= cost at £2 18*s*. 4*d*. each.

Ex. (4). A bankrupt pays 6*s*. 7½*d*. in the pound; what is the dividend on a debt of £362 15*s*.?

	£	*s*.	*d*.	
5*s*. is ¼ of £1	362	15	0	= amount of debt.
1*s*. is ⅕ of 5*s*.	90	13	9	= amount at 5*s*. in the £.
6*d*. is ½ of 1*s*.	18	2	9	= .... 1*s*. ....
1½*d*. is ¼ of 6*d*.	9	1	4·5	= .... 6*d*. ....
	2	5	4·125	= .... 1½*d*. ....
	£120	3	2·625	= amount at 6*s*. 7½*d*. in £.

Note.—Shorter thus:

4*s*. = ⅕ of £1; 2*s*. 6*d*. = ⅛ of £1; 1½*d*. = $\frac{1}{20}$ of 2*s*. 6*d*.

Ex. (5). Find the cost of 784½ articles at £2 12*s*. 10*d*. each.

The cost at £1 each is £784 10*s.*

	£	*s.*	*d.*	
	784	10	0	= cost at £1 each.
			2	
10*s.* is $\frac{1}{2}$ of £1	1569	0	0	= ...... £2 .....
2*s.* is $\frac{1}{5}$ of 10*s.*	392	5	0	= ...... 10*s* .....
10*d.* is $\frac{1}{12}$ of 10*s.*	78	9	0	= ...... 2*s.* .....
	32	13	9	= ...... 10*d.* ....
	£2072	7	9	= cost at £2 12*s.* 10*d* each.

167. The fractions of a Unit, which have for their numerator *Unity*, are called ALIQUOT PARTS of the unit Thus 5*s.*, being $\frac{1}{4}$ of £1, is an aliquot part of a pound. and 5 lb., being $\frac{1}{5}$ of 1 qr., is an aliquot part of a quarter'

Examples. (lxxxvi.)

Find the cost of the following articles:

(1) 4321 at £1 17*s.* $3\frac{3}{4}$*d.*
(2) 2175 at £2 15*s.* $4\frac{1}{3}$*d.*
(3) $3768\frac{1}{2}$ at £1 7*s.* $4\frac{1}{2}$*d.*
(4) 4276 at £12 11*s.* $5\frac{1}{2}$*d.*
(5) 5783 at £14 9*s.* $6\frac{1}{3}$*d.*
(6) $3689\frac{2}{3}$ at £16 12*s.* 9*d.*
(7) 7483 at £22 13*s.* $2\frac{1}{2}$*d.*
(8) What is the dividend on £4234 10*s.* at 5*s.* 6*d.* in the £?
(9) What is the dividend on £4975 at 3*s.* $4\frac{1}{2}$*d.* in the £?
(10) What is the dividend on £3729 18*s.* 6*d.* at 7*s.* $9\frac{3}{4}$*d.* in the £?

II. COMPOUND PRACTICE.

Ex. (1). When we have to find the cost of a quantity of goods of *mixed* denomination (as 14 cwt. 3 qr. 17 lb.), the cost of a single unit of one of the denominations being given (as £3 7*s.* 6*d.* per cwt. of 112 lbs.), we proceed thus:

	£	*s.*	*d.*	
14 cwt. is 14 × 1 cwt.	3	7	6	= cost of 1 cwt.
			2	
	6	15	0	
			7	
2 qr. is $\frac{1}{2}$ of 1 cwt.	47	5	0	= cost of 14 cwt.
1 qr. is $\frac{1}{2}$ of 2 qr.	1	13	9	= ...... 2 qr.
14 lb. is $\frac{1}{2}$ of 1 qr.		16	$10\frac{1}{2}$	= ...... 1 qr.
2 lb. is $\frac{1}{7}$ of 14 lb.		8	$5\frac{1}{4}$	= ...... 14 lb.
1 lb. is $\frac{1}{2}$ of 2 lb.		1	$2\frac{13}{28}$	= ...... 2 lb.
			$7\frac{13}{56}$	= ...... 1 lb.
	£50	5	$10\frac{25}{56}$	= cost of 14 cwt. 3 qr. 17 lb.

Ex. (2). What is the rent of 12 ac. 3 ro. 26 poles at £3. 5*s*. an acre?

	£.	*s*.	*d*.	
12 ac. is 12 × 1 ac.	3	5	0	= the rent of 1 acre.
2 ro. is ½ of 1 ac.	39	0	0	= ....12 ac.
1 ro. is ½ of 2 ro.	1	12	6	= .... 2 ro.
20 po. is ½ of 1 ro.		16	3	= .... 1 ro.
5 po. is ¼ of 20 po.		8	1·5	= ....20 po.
1 po. is ⅕ of 5 po.		2	0·375	= .... 5 po.
			4·875	= .... 1 po.
	£41	19	3·75	= the rent of 12a. 3r. 26p.

NOTE.—When the divisor is any number less than 12 (except 7), it is desirable to employ decimals instead of vulgar fractions to express the result of the division after the line of pence.

Examples. (lxxxvii.)

(1) 5 ac. 3 ro. 4 po. 4½ yd. at £10 per rood.
(2) 12 cwt. 3 qr. 22 lb. 12 oz. at £3 18*s*. 2*d*. per cwt.
(3) 10 ac. 3 ro. 26 po. at £2 18*s*. 10⅔*d*. per acre.
(4) 6 tons 12 cwt. 3 qr. 10½ lb. at £3 14*s*. 8¼*d*. per cwt.
(5) 63 cwt. 3 qr. 17½ lb. at 12 guineas per cwt.
(6) 29 ac. 3 ro. 5 po. at 100 guineas per acre.
(7) 16 oz. 6 dwt. 20 gr. at £3 17*s*. 6*d*. per oz.
(8) 25 ac. 1 ro. 10 po. at £42 2*s*. 4*d*. per acre.
(9) 13 cwt. 3 qr. 17 lb. at £22 8*s*. per cwt.
(10) 319 cwt. 3 qr. 16 lb. at £2 12*s*. 6*d*. per cwt.

Invoices and Accounts.

168. An INVOICE is a statement in detail, sent by a Seller to the Buyer at the time the goods are delivered to the Buyer, of the quantity, description, and price of the goods.

An ACCOUNT is a statement sent by the Seller to the Buyer at the end of a term of credit, showing the totals and dates of each Invoice and the sum total of the whole.

Each separate article or amount in an Invoice or an Account is called an **Item.**

A **Detailed Account** is a full statement, sent by the Seller to the Buyer at the end of a term of credit, showing the dates of delivery, the quantities, description, prices, and sum total of the goods delivered by the Seller to the Buyer during that term of credit.

When an account has been made out it is *rendered*, *i.e.*, sent in to the Buyer.

Specimen of an Invoice.

Toronto, June 20, 1889.

John Smith, Esq.,

Bought of J. Jones & Co., 21 Front St.

	$	cts.
5 lbs. of Tea.........at 75 cts.	3	75
8 lbs. of Loaf Sugar..at 12½ cts.	1	00
2½ lbs. of Butter....at 30 cts.	0	75
	5	50

Specimen of an Account.

Toronto, July 21, 1889.

John Smith, Esq.,

To J. Jones & Co., 21 Front St.

1889		$	cts.
June 20	To Goods, as per invoice	5	50
June 23	To “ “	7	80
July 3..	To “ “	3	60
July 12.	To “ “	2	27
		19	17

Specimen of a Detailed Account.

Toronto, July 21, 1889.

John Smith, Esq.,

To J. Jones & Co., 21 Front Street.

1889		$	cts.
June 20	5 lbs. of Tea ....at 75 cts.	3	75
" 20	8 lbs. of Loaf Sugar..at 12½ cts.	1	00
" 20	2½ lbs. of Butter.....at 30 cts.	0	75
" 23	1 bbl. of Flour ......at $6.	6	00
" 23	18 lbs. of Cheese.....at 10 cts.	1	80
July 3	12 lbs. of Biscuit.....at 15 cts.	1	80
" 3	6 jars of Pickles.....at 30 cts.	1	80
" 12	1 gal. of Coal Oil ....at 37 cts.	0	37
" 12	8 lbs. of Sugar ......at 11 cts.	0	88
" 12	8½ lbs. of Raisins....at 12 cts.	1	02
		19	17

Examples. (lxxxviii.)

Make out Invoices of the following sales, supplying names and dates of your own selection :

(1) 100 yds. of broadcloth at $3.25 per yard ; 2500 yards of sheeting at 12 cts. per yard ; 3000 yds. of prints at 18 cts. per yard ; 300 yds. of French silk at $1.75 per yard.

(2) 5 lbs. of black tea at 70 cts.; 2¼ lbs. of green tea at 90 cts.; 15½ lbs. of lump sugar at 12 cts.; 17 lbs. of brown sugar at 9 cts.; 7½ lbs. of raisins at 20 cts.; 4 lbs. of currants at 13 cts.

Make out Accounts of the following sales, supplying names and dates of your own selection :

(3) 39½ yds. of Brussels carpet at $1.50; 62¾ yds. of Kidderminster carpet at $1.10; 27 yds. of matting at 23 cts.; 34½ yds. of drugget at 65 cts.; 43½ yds. of India matting at 18 cts.

(4) 23 yds. of black silk at $2.15; 17 yds. of ribbon at 23 cts.; 13½ yds. of silk velvet at 25 cts.; 1½ doz. pairs of stockings at 45 cts. a pair; 5 pairs of gloves at $1.25; 18 yds. of muslin at 17 cts.

(5) 6 pairs of blankets at $5.50; 12½ yds. of merino at 45 cts.; 15¾ yds. of cloth at $3.25; 5⅓ yds. of flannel at 30 cts.; 2 counterpanes at $4.25 each; 25¼ yds. of calico at 15 cts.

XX. Problems.

169. The *Unitary Method*, which is rapidly displacing the Rule of Three, will be gradually explained in this and the succeeding Sections.

Ex. (1). If 23 bullocks cost \$483, what is the cost of 1 bullock.

Since 23 bullocks cost \$483,
1 bullock will cost $\$\frac{483}{23}$, or \$21.

Ex. (2). If 7 men do a piece of work in 12 days, how long will it take 1 man to do it?

Since 7 men can do the work in 12 days,
1 man can do the work in (7 × 12) days, or 84 days.

Ex. (3). If 28 men do a piece of work in 42 days, in how many days can 21 men do it?

Time for 28 men to do the work = 42 days.
" 1 man " " = 28 × 42 days.
" 21 men " " = $\frac{28 \times 42}{21}$ days.
= 56 days.

Ex. (4). If 75 men finish a piece of work in 12 days, how many men will finish it in 20 days?

In 12 days the work is done by 75 men.
In 1 day the work is done by (12 × 75) men.
In 20 days the work is done by $\frac{12 \times 75}{20}$ men, or 45 men.

Ex. (5). A bankrupt's debts are \$2520, and his assets (that is, the value of his property) are \$1890. What can he pay in the dollar?

In the place of \$2520, he can pay \$1890.
In the place of \$1 he can pay $\$\frac{1890}{2520}$, or $\$\frac{3}{4}$, or 75 cents;
∴ he pays 75 cents in the dollar.

Ex. (6). A bankrupt's debts are £4264, and he pays 12*s.* 6*d.* in the pound. What are his assets?

That which he has to meet a debt of £1 is $12\frac{1}{2}$*s.*
That which he has to meet a debt of £4264 is $(4264 \times 12\frac{1}{2})$*s.*;
∴ his assets are $\frac{4264 \times 25}{2}$*s.*, or £2665.

Ex. (7). If 27 men can do a piece of work in 14 days, working 10 hours a day, how many hours a day must 12 men work to do the same in 45 days?

Since 27 men can do the work in (14×10) hours, or 140 hours,

1 man can do the work in (27×140) hr.

$\therefore$ 12 men can do the work in $\frac{27 \times 140}{12}$ hr., or 315 hr.

Now 315 hours have to be distributed equally over 45 days;

$\therefore$ the number of hours they work each day $= \frac{315}{45}$, or 7.

Ex. (8). If 7 lbs. of tea cost \$5.60, what will be the cost of 12 lbs.?

Since 7 lb. of tea cost \$5.60,

1 lb. of tea costs $\$\frac{5.60}{7}$, or 80 cents.

$\therefore$ 12 lb. of tea cost 12×80 cts., or \$9.60.

Ex. (9). If 9 horses can plough 46 acres in a certain time, how many acres can 12 horses plough in the same time?

Since 9 horses can in the given time plough 46 ac.,

1 horse can in the given time plough $\frac{46}{9}$ ac.

$\therefore$ 12 horses can in the given time plough $\frac{12 \times 46}{9}$ ac.,

or $61\frac{1}{3}$ ac.

Ex. (10). If 15 horses can plough a certain quantity of land in five days, how many horses will be required to plough it in three days?

In 5 days the land can be ploughed by 15 horses;

In 1 day the land can be ploughed by (5×15) horses;

In 3 days the land can be ploughed by $\frac{5 \times 15}{3}$, or 25 horses.

NOTE I.—In simple questions of this kind we have a *supposition* and a *demand.* Each contains two kinds of things; in the *supposition* the magnitudes of *both* kinds are given; in the *demand* a magnitude of one kind is given, and the appropriate corresponding magnitude of the other kind has to be found. The first line of the solution contains the magnitudes of the supposition so arranged that *at the end of the line we have that kind of thing of which the magnitude is required in the demand.*

Thus in Ex. (10) the order of the supposition is changed, and the magnitude, 15 horses, put at the end of the line, because we have to find how many horses will be required in the demand.

Examples. (lxxxix.)

(1) If a man walk 62 miles in 4 days, in how many days will he walk 93 miles?

(2) If 12 men reap a field in 4 days, in what time will 32 men reap it?

(3) If 350 acres of land cost $61250, what will 273 acres cost?

(4) How many men can perform in 12 days a piece of work which 15 men can perform in 20 days?

(5) The rent of 17 acres is $297, what is the rent of 86 acres?

(6) If a man walk 116 miles in 8 days, how far will he walk in 14 days?

(7) A farmer sells a flock of 270 sheep at $240 a score, what does he get for them?

(8) A servant's wages being $108 per annum, how much ought she to receive for 7 weeks?

(9) A clerk's salary is £191 12*s.* 6*d.* per annum; what ought he to receive for 60 days' service?

(10) A ship performs a voyage in 63 days, sailing at the rate of 6 knots an hour; how long would it take her if she sailed at the rate of 7 knots an hour?

(11) A bankrupt's effects are worth $860, and his debts are $4300; what does he pay in the dollar?

NOTE II.—To one of the magnitudes in a supposition there is a corresponding magnitude of the same kind in the demand, and these magnitudes must be expressed in units of the same denomination.

Ex. A man walks 1 m. 1 fur. 7 po. in 20 minutes; how long will he take to walk 41 m. 2 fur. 12 po.?

Here 1 m. 1 fur. 7 po. = 367 poles,
and 41 m. 2 fur. 12 po. = 13212 poles.

Then he walks 367 poles in 20 minutes;

he walks 1 pole in $\frac{20}{367}$ min.;

he walks 13212 poles in $\frac{13212 \times 20}{367}$ min., or 720 min.

Examples. (lxxxix.)—*Continued.*

(12) If 3 bushels of wheat be worth \$3.50, what is the worth of 43 qr. 6 bus. ?

(13) If 15 yards of silk cost \$6.75, how much will 20 yd. 1 ft. cost ?

(14) If 3 cwt. 3 qr. cost \$27, what will be the cost of 2 cwt ?

(15) If 2 cwt. 3 qr. 7 lb. cost £5 17*s.* $8\frac{1}{2}$*d.*, what is the cost of 9 cwt. ?

170. *Problems involving Fractions.*

Ex. If $\frac{3}{7}$ of an estate be worth \$1500, what is the value of $\frac{4}{5}$ of the estate ?

Since $\frac{3}{7}$ of the estate is worth \$1500,
$\frac{1}{7}$ of the estate is worth $\$\frac{1500}{3}$;
$\therefore$ the estate is worth $\$\frac{7 \times 1500}{3}$ or \$3500.
Hence $\frac{4}{5}$ of the estate is worth $\$\frac{4 \times 3500}{5}$ or \$2800.

Examples. (xc.)

(1) If $\frac{3}{5}$ of an estate be worth \$7520, what is the value of $\frac{5}{8}$ of the estate ?

(2) A person owns $\frac{3}{8}$ of a ship and sells $\frac{2}{3}$ of his share for \$1260 ; what is the value of the ship ?

(3) If $3\frac{2}{5}$ lb. of tea cost 15*s.* 3*d.*, how much can I buy for £4 3*s.* $10\frac{1}{2}$*d.*

(4) If $\frac{2}{11}$ of a piece of work be done in 25 days, how much will be done in $11\frac{2}{3}$ days ?

(5) A man walks 18 m. 2 fur. 26 po. $3\frac{2}{3}$ yd. in $5\frac{1}{2}$ hours. How long does he take to walk a mile and a half ?

(6) A gentleman possessing $\frac{3}{14}$ of an estate sold $\frac{2}{7}$ of $\dfrac{1}{3\frac{1}{5}}$ of his share for $\$603.12\frac{1}{2}$; what would $\frac{1}{5}$ of $\frac{3}{16}$ of the estate sell for at the same rate ?

(7) If the carriage of 15·5 cwt. of goods for 60 miles cost \$3.10, how far ought 3·25 cwt. be carried for the same money ?

(8) What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sell $\frac{1}{9}$ of $\frac{7}{8}$ of his share for \$1400 ?

(9) When the ounce of gold is worth £3·89, what is the cost of ·04 lb. ?

(10) If the price of candles $8\frac{1}{2}$ inches long be 9*d.* per half-dozen, and that of candles of the same thickness and quality $10\frac{1}{4}$ inches long be 11*d.* per half-dozen, which kind do you advise a person to buy ?

(11) If the carriage of 60 cwt. for 20 miles cost £$14\frac{1}{2}$, what weight can be carried the same distance for £$5\frac{7}{16}$?

COMPLEX PROBLEMS.

171. We now proceed to cases in which the supposition, expressed in the simplest form, contains *more than two magnitudes*, the demand containing the same number of magnitudes, all of which are given, except one, which has to be found.

Ex. (1). If 12 horses can plough 96 acres in 6 days, how many horses will plough 64 acres in 8 days ?

In 6 days 96 acres can be ploughed by 12 horses.

In 1 day 96 acres can be ploughed by 6 × 12 horses.

In 1 day 1 acre can be ploughed by $\frac{6 \times 12}{96}$ horses.

In 8 days 1 acre can be ploughed by $\frac{6 \times 12}{8 \times 96}$ horses.

In 8 days 64 acres can be ploughed by $\frac{64 \times 6 \times 12}{8 \times 96}$ horses.

∴ the number of horses required is 6.

Ex. (2). If 35 bushels of oats last 7 horses for 20 days, how many days will 96 bushels last 18 horses ?

35 bushels last 7 horses for 20 days.

1 bushel lasts 7 horses for $\frac{20}{35}$ days.

1 bushel lasts 1 horse for $\frac{7 \times 20}{35}$ days.

96 bushels last one horse for $\frac{96 \times 7 \times 20}{35}$ days.

96 bushels last 18 horses for $\frac{96 \times 7 \times 20}{18 \times 35}$ days.

∴ the number of days is $21\frac{1}{3}$.

Examples. (xci.)

(1) If 40 acres of grass be mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days ?

(2) If $60 will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work ?

(3) If a regiment of 939 soldiers consume 351 quarters of wheat in 168 days, how many soldiers will consume 1404 quarters in 56 days ?

(4) If two horses eat 8 bushels of oats in 16 days, how many horses will eat 3000 quarters in 24 days?

(5) If a carrier receive $12 for the carriage of 3 cwt. for 150 miles, how much ought he to receive for the carriage of 7 cwt. 3 qr. 14 lb. for 50 miles?

(6) If I pay $1.50 for the carriage of 2 tons for 6 miles, what must I pay for the carriage of 12 tons 17 cwt. for 34 miles?

(7) If 3 men earn $15 in 4 days, what sum will 18 men earn in 16 days.

(8) How many bushels of wheat will serve 72 people 8 days, when 4 bushels serve 6 people 24 days?

(9) If a man travel 150 miles in 5 days when the days are 12 hours long, in how many days of 10 hours each will he travel 500 miles?

(10) If the carriage of goods weighing 5 cwt. 2 qr. 12 lb. for 150 miles come to $15.70, what will be the charge for carrying four waggon-loads of the same, each weighing 7 cwt. 0 qr. 2 lb., the same distance, there being 112 lbs. in the cwt.?

(11) If $120 pay 16 labourers for 6 days, how many labourers at the same rate will $270 pay for 8 days?

(12) If the gas for 5 burners, 5 hours every day, for 10 days, cost $1.20, how many burners may be lighted 4 hours every evening for 15 days at a cost of $21.60.

(13) If a travelling party of three spend $190 in 4 weeks, how long will $475 last a travelling party of five at the same rate?

(14) If it cost $120 to keep two horses for five months, what will it cost to keep three horses for eleven months?

(15) If it cost £29 7*s.* 6*d.* to keep 5 horses for 6 weeks, how long may 3 horses be kept for £20 11*s.* 3*d.*?

(16) If 5 men can reap a field of 12½ acres in 3½ days, working 16 hours a day, in what time can 7 men reap a field of 15 acres, working 12 hours a day?

(17) If 858 men in 6 months consume 234 quarters of wheat, how many quarters will be required for the consumption of 979 men for 3 months and a half?

(18) The wages of 5 men for 6 weeks being $315, how many weeks will 4 men work for $231?

(19) If 7 men mow 22 acres in 8 days, working 11 hours a day, in how many days, working 10 hours a day, will 12 men mow 360 acres?

(20) If 10 horses consume 7 bus. 2 pk. of oats in 7 days, in what time will 28 horses consume 3 qr. 6 bus. at the same rate?

(21) If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day?

(22) If the wages of 29 men for 54 days amount to £80 9*s.* 6*d.*, how many men must work 12 days to receive £407?

(23) What must I pay for the hire of 4 horses for 5 months, if I pay £18 for the hire of 3 horses for a month?

172. *Problems relating to Work done in a certain time.*

NOTE I.—If a man can do a piece of work in 7 hours, the part of the work which he can do in 1 hr. will be represented by $\frac{1}{7}$.

Ex. (1). *A* can do a piece of work in 5 days, and *B* can do it in 12 days. How long will *A* and *B*, working together, take to do the work?

Here $\frac{1}{5}$ represents the part *A* does daily,
and $\frac{1}{12}$ represents the part *B* does daily;
$\therefore \frac{1}{5} + \frac{1}{12}$ represents the part *A* and *B* do daily;
$\therefore$ they do $\frac{17}{60}$ in 1 day;
$\therefore$ they do $\frac{1}{60}$ in $\frac{1}{17}$ day;
$\therefore$ they do the whole work in $\frac{60}{17}$ days, or $3\frac{9}{17}$ days.

Ex. (2). *A* can do a piece of work in 50 days, *B* in 60 days, and *C* in 75 days. In what time will they do it, all working together?

Here $\frac{1}{50} + \frac{1}{60} + \frac{1}{75}$ represents the part they do daily;
$\therefore$ they do $\frac{6+5+4}{300}$, or $\frac{15}{300}$, or $\frac{1}{20}$ daily;
$\therefore$ they do the work in 20 days.

Ex. (3). *A* can reap a field in $4\frac{1}{5}$ days, and *B* can reap it in $5\frac{2}{3}$ days. How long will they take to reap it, working together?

$$A \text{ does } \frac{1}{4\frac{1}{5}} \text{ or } \frac{5}{21} \text{ daily,}$$

$$B \text{ does } \frac{1}{5\frac{2}{3}} \text{ or } \frac{3}{17} \text{ daily,}$$

∴ together they do $\frac{5}{21} + \frac{3}{17}$, or $\frac{148}{357}$ daily;
∴ they do $\frac{1}{357}$ of the work in $\frac{1}{148}$ day;
∴ they do the work in $\frac{357}{148}$ days, or $2\frac{61}{148}$ days.

Ex. (4). *A* and *B* do a piece of work in 4 hours; *A* and *C* in $3\frac{3}{5}$ hours; *B* and *C* in $5\frac{1}{7}$ hours. In what time can *A* do it alone?

A and *B* can do $\frac{1}{4}$ in an hour;
A and *C* can do $\frac{5}{18}$ in an hour;
∴ two men of *A*'s strength, assisted by *B* and *C*, can do $\frac{1}{4} + \frac{5}{18}$ in an hour;
Now *B* and *C* can do $\frac{7}{36}$ in an hour;
∴ two men of *A*'s strength can do $\frac{1}{4} + \frac{5}{18} - \frac{7}{36}$ in an hour,
or $\frac{19}{36} - \frac{7}{36}$, or $\frac{12}{36}$, or $\frac{1}{3}$ in an hour;
∴ *A* can do $\frac{1}{6}$ in an hour;
∴ *A* can do the work in 6 hours.

NOTE II.—If a tap can fill a vessel in 5 hours, the part filled by it in 1 hour will be represented by $\frac{1}{5}$.

Ex. (1). A vessel can be filled by three taps, running separately, in 20, 30, and 40 minutes respectively. In what time will they fill it when they all run at the same time?

They fill $\frac{1}{20} + \frac{1}{30} + \frac{1}{40}$ of the vessel in 1 minute;
∴ they fill $\frac{6+4+3}{120}$, or $\frac{13}{120}$ in 1 minute;
∴ they fill $\frac{1}{120}$ in $\frac{1}{13}$ of a minute;
∴ they fill the vessel in $\frac{120}{13}$ minutes, or $9\frac{3}{13}$ minutes.

Ex. (2). A bath is filled by a pipe in 40 minutes. It is emptied by a waste pipe in an hour. In what time will the bath be full if both pipes be opened at once?

One pipe fills $\frac{1}{40}$ of the bath in a minute.
The other empties $\frac{1}{60}$ of the bath in a minute.
∴ when both are running, $\frac{1}{40} - \frac{1}{60}$ or $\frac{1}{120}$ of the bath is filled in a minute;
∴ the bath is filled in 120 minutes.

Examples. (xcii.)

(1) *A* can do a piece of work in 6 hours; *B* can do it in 9 hours. In what time will they do it if they work together?

(2) *A* can do a piece of work in 35 days ; *B* can do it in 40 days ; *C* can do it in 45 days. In what time will they do it, all working together ?

(3) *A* and *B* can reap a field of wheat in 3 days ; *A* and *C* in $3\frac{1}{2}$ days ; *B* and *C* in 4 days. In what time could they reap it, all working together ?

(4) If three pipes fill a vessel in 6, 8, and 12 minutes respectively, in what time will the vessel be filled when all three are open at once ?

(5) *A* does $\frac{7}{10}$ of a piece of work in 14 days. He then calls in *B*, and they finish the work in 2 days. How long would *B* take to do the whole work by himself ?

(6) A does a piece of work in 3 hours, which is twice the time *B* and *C* together take to do it ; *A* and *C* could together do it in $1\frac{1}{3}$ hours. How long would *B* alone take to do it ?

(7) *A* can do a piece of work in 27 days, and *B* in 15 days ; *A* works at it alone for 12 days, *B* then works alone 5 days, and then *C* finishes the work in 4 days. In what time could *C* have done the work by himself ?

(8) A cistern is filled by two pipes in 18 and 20 minutes respectively, and emptied by a tap in 40 minutes ; what part of it will be filled in 10 minutes when all are opened at the same instant ?

173. *Problems relating to Clocks.*

The minute-hand moves 12 times as fast as the hour-hand, and therefore in 12 minutes the minute-hand gains 11 minute-divisions on the hour-hand.

Ex. (1). Find the time between 3 and 4 o'clock when the hands of a watch are together.

At 3 o'clock there are 15 minute-divisions between the hands ; we have therefore to find how long it will take the minute-hand to gain 15 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 minutes ;
1 minute-division in $\frac{12}{11}$ minutes ;
15 minute-divisions in $\frac{15\times12}{11}$ min. ;

$\therefore$ the time required is $\frac{15\times12}{11}$ min., or $16\frac{4}{11}$ min. past 3.

Ex. (2). At what time between 2 and 3 are the hands of a clock at right angles to each other ?

When the hands are at right angles there is a space of 15 minute-divisions between them.

Hence, since at 2 o'clock there are 10 minute-divisions between the hands, we have to find how long it will take the minute-hand to gain 10 + 15, or 25 minute-divisions on the hour-hand.

The minute-hand gains 11 minute-divisions in 12 minutes ;
1 minute-division in $\frac{12}{11}$ minutes ;
25 minute-divisions in $\frac{25\times12}{11}$ min. ;
∴ the time required is $\frac{25\times12}{11}$ min., or $27\frac{3}{11}$ min. past 2.

Ex. (3). At what *times* between 6 and 7 are the hands of a clock at right angles to each other?

Twice between 6 and 7 this will occur ; first, before the minute-hand has over-taken the hour-hand ; secondly, after the minute-hand has passed the hour-hand.

Now, since at 6 o'clock there are 30 minute-divisions between the hands, we have to find :

First, how long it will take the minute-hand to gain 30 – 15, or 15 minute-divisions on the hour-hand.

Secondly, how long it will take the minute-hand to gain 30 + 15, or 45 minute-divisions on the hour-hand.

The process in each case will be similar to that in the preceding examples, and the results are $16\frac{4}{11}$ min. and $49\frac{1}{11}$ min. past 6.

Ex. (4). Find the time between 7 and 8 o'clock when the hands of a watch are opposite to each other.

When the hands are opposite there is a space of 30 minutes between them, and at 7 o'clock there is a space of 35 minutes between the hands.

Hence in this case we have to find how long it will take the minute-hand to gain a space of 35 – 30, or 5 minutes on the hour-hand.

The process will be similar to that in the preceding examples, and the result is $5\frac{5}{11}$ min. past 7.

Examples. (xciii.)

At what time are the hands of a watch together between the hours of

(1) 4 and 5. (2) 6 and 7. (3) 9 and 10 ?

At what time are the hands of a watch at right angles to each other between

(4) 4 and 5. (5) 7 and 8. (6) 11 and 12 ?

At what time are the hands of a watch opposite to each other between

(7) 1 and 2. (8) 4 and 5. (9) 8 and 9 ?

Examination Papers.

I.

(1) If for a given sum I can have 1200 lbs. carried 36 miles, how many pounds can I have carried 24 miles for the same sum ?

(2) If $\frac{4}{9}$ of a ship be worth $13056, what is the value of $\frac{5}{8}$ of the ship ?

(3) A silver tankard weighs 1 lb. 10 oz. ; what is its value when a dozen spoons, weighing $3\frac{3}{4}$ oz. each, are worth $54 ?

(4) A man spends $61.60 every 35 days, and saves $400 a year. What is his annual income ?

(5) When the income-tax is 6*d*. in the £, a man pays £15 7*s*. 6*d*. What is his income ?

II.

(1) A man's income is reduced from $2720 to $2640.66 when he has paid his income tax. What is his tax on the dollar ?

(2) If 10 horses and 132 sheep can be kept 8 days for $202, what sum will keep 15 horses and 148 sheep for the same time, supposing 5 horses to eat as much as 84 sheep ?

(3) A man receives 75 cents in the dollar of what was due to him and thereby loses $602.10. What was due to him ?

(4) If 15 men can perform a piece of work in 22 days, how many men will finish another piece of work 4 times as large in a fifth part of the time ?

(5) If 72 men dig a trench in 63 days, in how many days will 42 men dig another trench three times as great ?

III.

(1) The wages of A and B together for $7\frac{1}{2}$ days amount to the same sum as the wages of A alone for $12\frac{6}{7}$ days. For how many days will the sum pay the wages of B alone ?

(2) If 100 men can perform a piece of work in 30 days, how many men can perform another piece of work thrice as large in one-fourth of the time?

(3) If 5 men or 7 women can do a piece of work in 37 days, how long will a piece of work twice as great occupy 7 men and 5 women?

(4) Two persons, *A* and *B*, finish a work in 20 days, which *B* by himself could do in 50 days. In what time could *A* finish it by himself? How much more of the work is done by *A* than *B*?

(5) If a cistern when full of water can be emptied in 15 minutes by a pipe, and when empty can be filled by another in 20 minutes; if the cistern be full, in what time can it be emptied by both pipes being opened at the same time?

IV.

(1) *A* and *B* can do a piece of work alone in 15 and 18 days respectively; they work together at it for 3 days, when *B* leaves, but *A* continues, and after three days is joined by *C*, and they finish it together in 4 days. In what time would *C* do the work by himself?

(2) If a man can do treble, and a woman double, the work of a boy in the same time, how long would 9 men, 15 women, and 18 boys take to do double the work which 7 men, 12 women, and 9 boys complete in 250 days?

(3) *A* and *B* walk to meet each other from two places 100 miles distant. *A* walks 6 miles an hour and *B* 4 miles an hour. At what point on the road do they meet, and at what two times are they fifty miles apart from each other?

(4) A watch which is 10 minutes too fast at noon on Monday loses 3 min. 10 sec. daily. What will be the time indicated by the watch at a quarter past ten on the morning of the following Saturday?

(5) A watch set accurately at 12 o'clock indicates 10 min. to 5 at 5 o'clock. What is the exact time when the watch indicates 5 o'clock? If it indicated 10 minutes past 5 at 5 o'clock, what would be the exact time when the hands indicated 5 o'clock?

V.

(1) A labourer agreed to work for 60 days on this condition: that every day he worked he should receive $2, and for every day he was idle, he should pay $1.50 for his board. At the expiration of the time he received $92. How many days did he work?

(2) A piece of work can be done in a day of $11\frac{1}{2}$ hours by 2 men, or 5 women, or 12 boys; in what time could it be done by 1 man, 2 women, and 3 boys together?

(3) A cistern has two supplying pipes, *A* and *B*, and a tap, *C*. When the cistern is empty, *A* and *B* are turned on, and it is filled in 4 hours; then *B* is shut and *C* turned on, and the cistern is quite emptied in 40 hours; when, lastly, *A* is shut and *B* turned on, and in 60 hours afterwards the cistern is again filled. In what time could the cistern be filled by each of the pipes, *A* and *B*, singly?

(4) A clock is set at 12 o'clock on Saturday night, and at noon on Tuesday it is 3 minutes too fast. Supposing its rate regular, what will be the true time when the clock strikes four on Thursday afternoon?

(5) A contractor engages what he considers a sufficient number of men to execute a piece of work in 84 days; but he ascertains that three of his men do, respectively, $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$ less than an average day's work, and two others $\frac{1}{8}$ and $\frac{1}{10}$ more, and in order to complete the work in the 14 weeks, he procures the help of 17 additional men for the 84th day. How much less or more than an average day's work on the part of these 17 men is required?

XXI. Simple Interest.

174. INTEREST is that which is paid by one who borrows money for the use of the money.

The money lent is called the PRINCIPAL.

The Borrower agrees to pay at what is called a certain RATE of interest, which is usually reckoned by the sum paid for the use of \$100 for 1 year. Thus, if I borrow \$500 for 1 year, and agree to pay \$25 for the use of the money, I am said to borrow at the *Rate of 5 per cent. per annum;* that is, I agree to pay \$5 for the use of every \$100 in the loan at the end of the year.

The sum made up of the Principal and Interest added together is called the AMOUNT at the end of the time for which the money is borrowed.

175. The solution of questions relating to Interest depends on precisely the same principles as those explained in the last Section, and it is only because of the necessity of explaining technical terms that there is any occasion to separate this or the succeeding Sections from Section XX.

For, just as we reason about the question:

What must I pay for the hire of 4 horses for 5 months, if I pay \$18 for the hire of 3 horses for a month?

so do we reason about the question:

What must I pay for the use of \$550 for 3 years, if I pay \$5 for the use of \$100 for a year?

Ex. (1). To find the Simple Interest on \$2675 for 3 years, at 8 per cent., we reason thus:

$$\begin{aligned}\text{Interest on } \$100 \text{ for 1 year is } &\$8;\\ \text{on } \$1 \text{ for 1 year is } &\$\tfrac{8}{100};\\ \text{on } \$2675 \text{ for 1 year is } &\$\tfrac{2675\times 8}{100};\\ \text{on } \$2675 \text{ for 3 years is } &\$\tfrac{3\times 2675\times 8}{100}=\$642;\end{aligned}$$

∴ the interest is \$642.

Hence we derive the following rule:

Multiply the Principal by the Rate per cent., and the result by the Time expressed in years, and divide the product by 100.

The process stands thus:

$$\begin{array}{r} \$\\ 2675\\ 8\\ \hline 21400\\ 3\\ \hline \$642.00 \end{array}$$

∴ the interest is \$642.

Ex. (2). Find the interest on \$3200 for 2 years and 7 months at $7\frac{1}{2}$ per cent.

Since 7 months is $\frac{7}{12}$ of a year, the time is $2\frac{7}{12}$ years.

Interest on \$100 for 1 year is \$7.50.

" \$1 for 1 year is $\$\frac{7\cdot 50}{100}$.

" \$3200 for 1 year is $\$\frac{3200 \times 7\cdot 50}{100}$.

" \$3200 for $2\frac{7}{12}$ years is $2\frac{7}{12} \times \$\frac{3200 \times 7\cdot 50}{100}$

$$= \$\frac{31 \times 3200 \times 7\cdot 50}{12 \times 100}$$

$$= \$620\,;$$

∴ the interest is \$620.

Ex. (3). Find the interest on \$101178 from January 28th, 1888, to Sept. 15th, 1888, at 6 per cent.

The number of days between January 28th and Sept. 15th is 231, and 231 days is $\frac{231}{365}$ of a year.

$$\text{Interest} = \frac{101178 \times 6}{100} \times \frac{231}{365} = \$3841\cdot 992.$$

Note I.—In calculating the number of days between two given days of the year, the rule is *to include one of them only* in the calculation. Thus from Jan. 4 to Jan. 9 will be 5 days.

In the preceding example, if we multiply numerator and denominator by 2 we have

$$\frac{101178 \times 2 \times 231}{73000}$$

Hence, in computing the interest for any number of days, we have the following rule :

Multiply the Principal by twice the rate, and the result by the number of days, and divide the product by 73000.

When the Principal is not very large the division is most readily effected by dividing the product by 3, the quotient by 10, and the new quotient by 10, and adding these quotients and the product together, and pointing off five places of decimals.

Thus: Find the interest of \$1000 for 121 days at 8 per cent.

$$\begin{array}{r|r}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{\$1000} \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{16} \\
\cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{16000} \\
\multicolumn{1}{r}{} & \multicolumn{1}{r}{121} \\
\cline{2-2}
3 & 1936000 \\
10 & 645333\frac{1}{3} \\
10 & 64533\frac{1}{3} \\
 & 6453\frac{1}{3} \\
\cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{\$26{\cdot}52320}
\end{array}$$

Since 73000 increased by $\frac{1}{3}$ of itself, $\frac{1}{30}$ of itself, and $\frac{1}{30}$ of itself becomes 100010,

Thus,

$$\begin{array}{r|r}
\frac{1}{3} & 73000 \\
\frac{1}{10} \text{ of } \frac{1}{3} & 24333\frac{1}{3} \\
\frac{1}{10} \text{ of } \frac{1}{30} & 2433\frac{1}{3} \\
 & 243\frac{1}{3} \\
\cline{2-2}
\multicolumn{1}{r}{} & \multicolumn{1}{r}{100010}
\end{array}$$

and considering this as 100000, the reason for the above process is evident.

NOTE. II.—In actual practice the time, when not an exact number of years, is always expressed in *days*, or in *years* and *days*.

Examples. (xciv.)

Find the Simple Interest—

(1) On \$2750 for 6 years at 5 per cent. per annum.
(2) On \$3625 for 4 years at 8 per cent. per annum.
(3) On \$2700 for 6 years at $7\frac{1}{2}$ per cent. per annum.
(4) On \$8825 for $6\frac{1}{2}$ years at 8 per cent. per annum.
(5) On \$1160 for 11 months at 9 per cent. per annum.
(6) On \$9125 for 78 days at 8 per cent. per annum.
(7) On \$5913 from Nov. 23, 1876, to April 7, 1877, at $7\frac{1}{2}$ per cent. per annum.
(8) On £204 17*s*. 7*d*. from Aug. 3 to Jan. 9 at 5 per cent.

176. We have explained how to find the Interest (and Amount) when the Principal, Rate, and Time are given. We shall now explain how to find the Rate, or

Time or Principal, when the other two and also the Interest (or Amount) are given.

Ex. (1). At what Rate per cent. will \$520 amount to \$800.80 in 9 years?

As the rate is the interest on \$100 for 1 year, to find the rate we must find the interest on \$100 for 1 year.

Here interest = \$880.80 – \$520 = \$280.80;

Thus, the interest on \$520 for 9 years is \$280.80;

∴ the interest on \$520 for 1 year is $\$\frac{280\cdot80}{9}$;

" on \$1 for 1 year is $\$\frac{280\cdot80}{520\times9}$;

" on \$100 for 1 year is $\$\frac{100\times280\cdot80}{520\times9} = \$6$;

∴ Rate required is 6 per cent.

Ex. (2). In what time will the Interest on \$360 amount to \$126 at 5 per cent.?

Interest on \$360 for 1 year is $\$\frac{360\times5}{100}$, or \$18.

Then, since \$18 is the interest for 1 year,

\$1 is the interest for $\frac{1}{18}$ year,

\$126 is the interest for $\frac{126}{18}$ years, or 7 years;

∴ Time required is 7 years.

Ex. (3). What Principal will amount to \$980 in 3 years at 7½ per cent.?

Interest on \$100 for 3 years at 7½ per cent. is \$22.50;

∴ \$122.50 is the amount which has for its Principal \$100;

\$1 is the amount which has for its Principal $\$\frac{100}{122\cdot50}$

\$980 is the amount which has for its Principal $\$\frac{980\times100}{122\cdot50}$ or \$800;

∴ Principal required is \$800.

Ex. (4). At what rate will any sum triple itself in 20 years at simple interest?

Here the interest is twice the Principal.

Thus the interest on the Principal for 20 years is 2 × Principal;

∴ interest on the Principal for 1 year is $\frac{2 \times \text{Principal}}{20}$;

interest on \$1 for 1 year is $\frac{2 \times \text{Principal}}{\text{Principal} \times 20}$;

on \$100 for 1 year is $\frac{100 \times 2 \times \text{Principal}}{\text{Principal} \times 20} = 10$;

∴ Rate required is 10 per cent.

Examples. (xcv.)

(1) At what rate will the interest on \$326 for 15 years be \$220.05 ?

(2) In what time will \$700 amount to \$920.50 at 6 per cent.?

(3) What sum will amount to \$1325 in 8 months at 9 per cent.?

(4) The interest on a sum of money for 12 years at $4\frac{1}{2}$ per cent. is \$202.50 ; what is the sum ?

(5) In what time will any sum double itself at 5 per cent., simple interest ?

(6) What must be the rate per cent. that the interest at the end of 16 years 8 months may be equal to seven-eighths of the sum lent ?

(7) A sum of money amounts in ten years at 7 per cent. to \$1275 ; in how many years will it amount to \$1406.25 ?

(8) The sum of \$500 is borrowed at the beginning of the year at a certain rate per cent., and after 9 months \$400 more is borrowed at double the previous rate. At the end of the year the interest on both loans is \$35 ; what is the rate at which the first sum was borrowed ?

(9) In how many days will the interest on £243 6*s.* 8*d.* be £4 0*s.* 10*d.* at $6\frac{1}{4}$ per cent.?

(10) If £556 17*s.* 6*d.* be loaned for 125 days and then amount to £565 18*s.* 9*d.*, what was the rate ?

(11) The interest on \$8000 for one day is \$2 ; find the rate per cent. per annum.

(12) Bought 5000 bushels of wheat at \$1.25 a bushel, payable in 6 months ; I immediately realized for it at \$1.20 cash, and put the money out at interest at 10 per cent. At the appointed time I paid for the wheat ; did I gain or lose by the transaction, and how much ?

(13) The interest on a sum of money at the end of $6\frac{1}{4}$ years is three-eighths of the sum itself ; what rate per cent. was charged ?

(14) A sum of money at simple interest has in $4\frac{1}{2}$ years amounted to \$735, the rate of interest being 5 per cent. per annum ; what was the sum at first, and in how many years *more* will it amount to \$1140 ?

(15) The interest on \$1805, loaned on May 13th, at $5\frac{1}{4}$ per cent. per annum, is \$37·905 ; on what day was the money returned ?

PARTIAL PAYMENTS.

177. A partial payment is the payment of a part of the amount due on a note or bond. When partial payments are made they are endorsed on the note or bond. To compute the interest on such a note proceed according to the following rule:

Compute the interest on the principal to the time of the first payment, and if this payment exceed the interest then due add the interest to the principal, and from the sum take the payment; the remainder will form a new principal, with which proceed as before.

But if the payment be less than the interest, compute the interest on the principal to the time when the sum of the payments shall first equal or exceed the interest due; add the interest to the principal, and from the sum subtract the sum of the payments, and treat the remainder as a new principal.

This rule proceeds on the ground that in all cases the payment should be applied first to the interest due, then to the principal, and that the principal remains unchanged until the sum paid exceeds the accrued interest.

Ex. (1). $4000.

TORONTO, June 1, 1872.

Two years after date I promise to pay William Smith, or order, four thousand dollars, for value received, with interest at 7 per cent.

RICHARD PAYWELL.

On this note were the following endorsements:

Sept. 15, 1872, four hundred and fifty dollars.

Dec. 15, 1872, fifty dollars.

Mar. 1, 1873, five hundred dollars.

Jan. 1, 1874, one thousand dollars.

What remained due June 4, 1874.

Principal on interest June 1, 1872	$4000 00
Interest to Sept. 15, 1872	80 89
Amount	$4080 89
Less 1st payment	450 00
Remainder for a new principal	$3630 89
Interest from Sept. 15 to Dec. 15, 1872, is $63.44, which exceeds the payment.	
Interest from Sept. 15, 1872, to March 1, 1873	117 20
Amount	$3748 09
Less the sum of the 2nd and 3rd payments	550 00
Remainder for a new principal	$3198 09
Interest from March 1, 1873, to Jan. 1, 1874	186 47
Amount	$3384 56
Less payment Jan. 1, 1874	1000 00
Remainder for a new principal	$2384 56
Interest from Jan. 1 to June 4, 1874	70 94
Balance due June 4, 1874	$2455 50

Examples. (xcvi.)

(1) $1500.

HAMILTON, Jan. 1, 1877.

One year after date, we promise to pay S. White, or order, fifteen hundred dollars, with interest. Value received.

GEORGE BROWN & Co.

The following payments were made on this note:

March 16, 1877, $100; June 13, 1877, 400; Sept. 1, 1877, $200.

What was due Jan 1, 1878, interest at 6 per cent.?

(2) $3500.

BELLEVILLE, March 15, 1876.

For value received, we jointly and severally promise to pay Wm. Smith, or order, three thousand five hundred dollars, with interest.

JAMES JONES & Co.

Endorsed as follows:

June 1, 1876, $800; Sept. 1, 1876, $100; Jan. 1, 1877, $1560; March 1, 1877, $300.

What was due May 16, 1877, interest at 6 per cent.

(3) $1200.

TORONTO, Oct. 15, 1859.

One year from date we promise to pay James Smith, or order, twelve hundred dollars, for value received, with interest.

WILDER & SON.

Endorsed as follows:

Oct. 15, $1860, $1000; April 15, 1861, $200.

How much remained due Oct. 15, 1861, interest at 6 per cent.?

XXII. Compound Interest.

178. Compound Interest is that which is paid, not only for the use of the original sum lent, but also for *use of the interest* as it becomes due.

The interest on $500 for 1 year at 4 per cent. is $20.

If then $500 be lent at Compound Interest for 2 years at 4 per cent., the interest for the *first* year is $20.

Now, as the borrower has to pay for the use of this $20, the interest for the *second* year must be calculated on $520.

Hence interest for second year $= \$\frac{520 \times 4}{100} = \$20.80$.

To put the matter in a more simple way, we have supposed *the borrower to retain* the interest due at the end of the first year, but the reasoning will be the same if we suppose *the lender to receive* the interest at the end of the first year, and to put it out immediately at the same rate of interest.

179. We may calculate Compound Interest by the following rule:

Find the interest for the first year: add it to the original principal: call the result the Second Principal: find the *interest on this for the second year: add it to the second principal: call the result the Third Principal: find the interest on this for the third year, and so on.*

Ex. (1). Find the Compound Interest on $7500 for 3 years at 4 per cent.

$7500 is the principal for the *first* year.
4

$300.00

The interest for the *first* year is $300.
Add this to the original principal, $7500.
Then $7800 is the principal for the *second* year.
4

$312.00

The interest for the *second* year is $312.
Add this to the second year's principal, $7800.
Then $8112 is the principal for the *third* year.
4

$324.48

The interest for the *third* year is $324.48.
∴ Compound Interest required is

$300 + $312 + $324.48 = $936.48.

If the *Amount* at Compound Interest be required, add the original principal, $7500, to the Compound Interest, $936.48.

Then amount required = $8436.48.

Ex. (2). What is the Compound Interest of $250 for 2 years, at 7 per cent.?

	$250	Principal for 1st year.
$250 × 0.07 =	17.50	Interest for the 1st year.
	267.50	Principal for 2nd year.
$267.50 × 0.07 =	18·725	Interest for 2nd year.
	$286·225	Amt. at Comp. Int. for 2 yrs.
First Principal	250.00	
	$36.225	Com. Int. for 2 years.

Examples. (xcvii.)

Find the Compound Interest on

(1) \$375 for 3 years at 5 per cent.
(2) \$564 for 4 years at 7 per cent.
(3) \$1154.37 for 4 years at 5 per cent.
(4) \$740 for 5 years at 7 per cent.

NOTE I.—When the Compound Interest is required for $3\frac{1}{2}$ years, it is usual to find the Compound Interest for the whole of the fourth year, and take half the result as the Compound Interest for the half year. This really implies that the interest is paid half-yearly, but the approximation does not differ much from the exact truth.

180. The process for finding the amount of a sum at Compound Interest may be presented in a very brief and neat form as follows:—

If the rate of interest be 4 per cent.,
Amount of \$100 at the end of 1 year is \$104,
of \$1 at the end of 1 year is $\frac{104}{100}$ of \$1.

Hence it follows that:

Amount of *any sum* at 4 per cent. in 1 year = $\frac{104}{100}$ of that sum.

Again,

Amount for *second* year = $\frac{104}{100}$ of amount for the first year;

∴ Amount of *any sum* at 4 per cent. in 2 years
= $\frac{104}{100}$ of $\frac{104}{100}$ of that sum.

Suppose, then, we have to find the amount of \$540 in 3 years at 4 per cent., Compound Interest.

The amount is $\frac{104}{100}$ of $\frac{104}{100}$ of $\frac{104}{100}$ of \$540
$= \$540 \times (1{\cdot}04)^3$
$= \$607{\cdot}426$.

From the above example it will be noticed that the amount of \$1 for a year at 4 per cent. is raised to the power indicated by the number of years for which Compound Interest is to be calculated. Hence we have the following rule:—

To find the sum to which any principal will amount if put out to Compound Interest at a given rate in a given *number of years, find the amount of $1 for a year at the given rate, raise that sum to the power which is denoted by the given number of years, and multiply the result by the number of dollars in the given principal.*

Ex. (1). Find the amount of $850 in three years at 6 per cent., Compound Interest.

$$\begin{aligned}\text{The Amount} &= \$850 \times (1{\cdot}06)^3\\ &= \$850 \times 1{\cdot}191016\\ &= \$1012{\cdot}363.\\ \text{The Compound Interest} &= \$1012{\cdot}36 - \$850.\\ &= \$162{\cdot}36.\end{aligned}$$

NOTE II.—When the number of years is large, the student is recommended to employ the contracted method of multiplication, explained in Art. 111.

Interest may be payable either yearly, half-yearly, or quarterly, or at some other stated period.

In finding the Compound Interest on $2000 in two years, when the interest is payable *half-yearly*, at 5 per cent., we reason thus:

$$\begin{aligned}\text{5 per cent. for a year} &= 2\tfrac{1}{2}\text{ per cent. half yearly, 2 years.}\\ &= \text{4 half years.}\end{aligned}$$

Hence we have to find the Compound Interest on $2000, *for four times of payment, at* $2\frac{1}{2}$ *per cent.*

$$\begin{aligned}\text{The Amount} &= \$2000 \times (1{\cdot}025)^4\\ &= \$2000 \times 1{\cdot}1038127\\ &= \$2207{\cdot}625.\\ \text{The Interest} &= \$2207{\cdot}625 - \$2000\\ &= \$207{\cdot}625.\end{aligned}$$

Ex. (2). What principal will amount to $1012·363 in 3 years at 6 per cent.?

$$\begin{aligned}\text{Principal} \times (1{\cdot}06)^3 &= \$1012{\cdot}363\\ \therefore \text{Principal} &= \$\frac{1012{\cdot}363}{(1{\cdot}06)^3}\\ &= \$850.\end{aligned}$$

Examples. (xcviii.)

(1) What is the Compound Interest on $1000 for 2 years, at 6 per cent., payable half-yearly?

(2) What is the amount of $200 for 3 years, at 6 per cent., payable half-yearly.

(3) Find the Compound Interest on \$675.75 for $3\frac{1}{2}$ years, at 6 per cent. per annum.

(4) A money dealer borrowed \$1000 for 2 years at 6 per cent. interest; and loaned the same in such a manner as to *compound* the interest every 6 months. What profit did he make in 2 years by this proceeding?

(5) Find the difference in Compound Interest on £5000 for 2 years at 4 per cent., according as it is reckoned yearly or half-yearly.

(6) What is the difference between the Compound Interest on \$40000 for 4 years, and on \$80000 for 2 years, the rate in both cases being 5 per cent.?

(7) *A* and *B* lend each \$248 for 3 years at $3\frac{1}{2}$ per cent., one at Simple, the other at Compound Interest; find the difference of the amount of interest which they respectively receive.

(8) What sum at four per cent., Compound Interest, will amount in $2\frac{1}{2}$ years to \$16989·7728.

(9) What sum will amount to \$27783 in 3 years at 5 per cent., Compound Interest.

XXIII. Present Worth and Discount.

181. Suppose *A* owes *B* \$105, to be paid at the end of a year. If *A* be disposed to pay off the debt at once the sum which he ought to pay should be such that, if put out at interest by *B*, it will amount at the end of a year to \$105. Suppose, further, that *B* can put out his money at 5 per cent. interest; then, if he put out \$100 at interest, this is the sum which will amount at the end of a year to \$105.

Hence \$100 is the sum, which *A* ought to pay at once, and this is called the PRESENT WORTH of the debt, and is evidently such a sum as would, if put out to interest for the given time and rate, amount to the debt. The difference between the Debt and the Present Worth, which is in the case under consideration \$5, is called the Discount.

DISCOUNT is therefore the abatement made when a sum of money is paid before it is due and is equal to the interest on the Present Worth of the debt.

Ex. (1). Thus, to find the Present Worth of \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The interest on \$100 for 4 years at 5 per cent. is \$20.

∴ \$120 has for its Present Worth \$100 ;

∴ \$1 has for its Present Worth $\$\frac{100}{120}$;

∴ \$1781.40 has for its Present Worth $\$\frac{1781 \cdot 40 \times 100}{120}$

$= \$1484.50$

∴ Present Worth required is \$1484.50.

Ex. (2). Find the Discount on \$1781.40, due 4 years hence, reckoning interest at 5 per cent.

The Present Worth is \$1484.50, as we have just shown ;

the Discount = \$1781.40 – \$1484.50

= \$296.90.

When the Discount alone is required to be found the following is the solution :

The interest on \$100 for 4 years at 5 per cent. is \$20.

∴ \$120 has for its Discount \$20 ;

∴ \$1 has for its Discount $\$\frac{20}{120}$;

∴ \$1781.40 has for its Discount $\$\frac{1781 \cdot 40 \times 20}{120}$

$= \$296.90.$

Ex. (3). What was the debt of which the Discount for 8 months at 9 per cent. was \$44.46 ?

The interest on \$100 for 8 months at 9 per cent. is \$6.

∴ \$6 is the Discount on \$106 ;

∴ \$1 is the Discount on $\$\frac{106}{6}$;

∴ \$44.46 is the Discount on $\$\frac{44 \cdot 46 \times 106}{6}$

$= \$785.46.$

Ex. (4). The interest on a certain sum of money for two years is \$50, and the Discount for the same time and rate is \$45. Find the sum and the rate per cent. per annum.

Since \$50 is the interest on a sum of money which sum

= (its Present Worth + its Discount)

= (its Present Worth + \$45)

and \$45 is the interest on its Present Worth,

∴ \$5 is the interest on \$45 ;

∴ \$1 is the interest on $\$\frac{45}{5}$;

∴ $50 is the interest on $\$\frac{50 \times 45}{5}$, or $450;
∴ $450 is the sum required.

Again, the interest on $45 for 2 years is $5;
∴ the interest on $45 for 1 year is $\$\frac{5}{2}$;
∴ the interest on $1 for 1 year is $\$\frac{5}{45 \times 2}$;
∴ the interest on $100 for 1 year is $\$\frac{100 \times 5}{45 \times 2} = \$5\frac{5}{9}$;
∴ the Rate is $5\frac{5}{9}$ per cent.

NOTE I.—From the above it will be seen that the discount on any sum is the Present Worth of the interest of that sum for the same time and rate: thus $45 is the Present Worth of $50 for two years at a certain rate per cent.

Ex. (5). If $20 be allowed off a bill of $420 due in 6 months, how much shall be allowed off the same bill due in 12 months?

$20 is the discount off $420 for 6 months;
∴ $20 is the interest on $400 for 6 months;
∴ $40 is the interest on $400 for 12 months;
∴ $40 is the discount off $440 for 12 months;
∴ $\$\frac{40}{440}$ is the discount off $1 for 12 months;
∴ $\$\frac{420 \times 40}{440}$ is the discount off $420 for 12 months.
Now $\$\frac{420 \times 40}{440} = \$38\frac{2}{11}$;
∴ the Discount required is $\$38\frac{2}{11}$.

NOTE II.—The student will observe that the Discount is not proportional to either the time or the rate.

Ex. (6). If $15 be the Interest on $115 for a given time, what should be the Discount off $115 for the same time?

$15 is the interest on $115;
∴ $15 is the discount off $130;
∴ $\$\frac{15}{130}$ is the discount off $1;
∴ $\$\frac{115 \times 15}{130}$ is the discount off $115.
Now $\$\frac{115 \times 15}{130} = \$13\frac{7}{26}$;
∴ the Discount required is $\$13\frac{7}{26}$.

Ex. (7). If \$10 be allowed off a bill of \$110, due 8 months hence, what should be the bill from which the same sum is allowed as 4 months' discount?

\$10 is the discount off \$110 for 8 months;
∴ \$10 is the interest on \$100 for 8 months;
∴ \$10 is the interest on \$200 for 4 months;
∴ \$10 is the discount off \$210 for 4 months;
∴ the sum required is \$210.

Ex. (8). Find the Present Worth of \$842.70 for t years, at 6 per cent., Compound Interest.

The Compound Interest on \$100 for 2 years at 6 per cent. is \$12.36.

∴ \$112.36 has for its Present Worth \$100;
∴ \$1 has for its Present Worth $\$\frac{100}{112 \cdot 36}$;
∴ \$842.70 has for its Present Worth $\$\frac{842 \cdot 70 \times 100}{112 \cdot 36}$;
= \$750.
∴ Present Worth required = \$750.

Examples. (xcix.)

Find the Present Worth of

(1) \$5520, due 4 years hence, at 5 per cent.
(2) \$84.70, due $2\frac{1}{3}$ years hence, at 9 per cent.
(3) \$615, due 1 year 4 months hence, at 7 per cent.
(4) \$1120, due 16 months hence, at 5 per cent.
(5) £618 2*s*. 6*d*., due $3\frac{3}{4}$ years hence, at 4 per cent.

Find the Discount on

(6) \$636, due in 9 months, at 8 per cent.
(7) \$1884.30, due in $3\frac{1}{5}$ years, at 10 per cent.
(8) \$637.50, due in $5\frac{1}{2}$ years, at 5 per cent.
(9) £1165 16*s*. 3*d*., due in $2\frac{1}{2}$ years, at 6 per cent.
(10) £252 19*s*. 3*d*., due in 9 months, at $4\frac{1}{3}$ per cent.

(11) Find the Present Worth of \$6945.75, due 3 years hence, reckoning compound interest at 5 per cent.

(12) Find the Discount on \$245.25, due $1\frac{1}{4}$ years hence, at $5\frac{1}{2}$ per cent. compound interest, payable quarterly.

(13) A tradesman accepts \$19·3125 in payment of a debt of $\$20\frac{287}{960}$, due in 12 months, in consideration of being paid at once. What rate of discount does he allow?

(14) Find the Present Worth of a bill for $1127.10, drawn Jan. 1 at 4 months, and discounted Feb. 20 at 10 per cent. per annum.

(15) The Discount on $275 for a certain time is $25; what is the Discount on the same sum (1) for twice that time, and (2) for half the time?

(16) A tradesman marks his goods with two prices, one for cash and the other for credit of 6 months; what relation should the two prices bear to each other, allowing interest at $7\frac{1}{2}$ per cent.? If the credit price of an article be $33.20, what is the cash price?

(17) If $98 be accepted in present payment of $128, due some time hence, what should be a proper discount off a bill of $128 which has only half the time to run?

(18) A certain sum ought to have $20.80 allowed as 8 months' interest on it; but a bill for the same sum due in 8 months at the same rate should have $20 only allowed off as discount in consideration of present payment. What is the sum and the rate per cent.?

182. The Discount of which we have been treating is called Mathematical Discount, or True Discount, to distinguish it from Practical Discount, of which there are two kinds:

(1) The deduction made by a trader, when an account is paid to him before the time when he proposes to demand payment. It is then calculated as interest on the account. Thus, if a trader gives notice on his bill that he will allow 10 per cent. discount for immediate payment, and if the amount of the bill be $25.50, he deducts $2.55, and the customer pays him $22.95.

(2) The deduction made by a lender of money from the sum which he proposes to lend. Thus, if a borrower binds himself by a bill to pay $100 a year hence, and a discounter advances money on the security of this bill, at the rate of 5 per cent., he gives to the holder of the bill $95 and takes the bill.

True Discount is the Interest on the Present Worth of a debt. Practical Discount is the Interest on the debt itself. Hence Practical Discount is greater than True Discount.

183. Three days, called *Days of Grace*, are always allowed after a bill of exchange or a promissory note is *nominally* due before it is *legally* due. Thus a bill drawn on July 5, for 3 months, would be nominally due on Oct. 5, but legally on Oct. 8. Calendar months are always reckoned so that a bill of 3 months whether drawn on the 28th, 29th, or 30th of Nov., 1876, would be due on the 3rd of March, 1877. The banker or money lender who discounts a note always charges *interest* on the note from the time it is discounted till it is legally due; hence in computing Practical Discount of this nature interest must be calculated for 3 days more than the time the note has to run.

Ex. (1). What would a banker gain by discounting on Sept. 21 a bill of \$318.15, dated July 31, at 4 months at 5 per cent.?

The bill is legally due on Dec. 3.

The number of days from Sept. 21 to Dec. 3 is 73.

The interest on \$318.15 for 73 days at 5 per cent. is \$3·1815.

The mathematical discount is \$3.15.

∴ the banker's gain is \$·0315.

Ex. (2). A merchant wishes to borrow \$96.91 on a bill made on July 5 for 3 months. What must be the face of the bill, interest being reckoned at $8\frac{1}{9}$ per cent.?

Time between July 5 and Oct. 8 is 95 days.

Interest on \$100 for 95 days at $8\frac{1}{9}$ per cent. is $\$2\frac{1}{9}$.

∴ a note for \$100 would produce $\$97\frac{8}{9}$;

∴ a note for $\$\frac{100}{97\frac{8}{9}}$ would produce \$1;

∴ a note for $\$\frac{96.91 \times 100}{97\frac{8}{9}}$ would produce \$96.91.

Now $\$\frac{96.91 \times 100}{97\frac{8}{9}} = \$99.$

∴ the face of the note is \$99.

Examples. (c.)

(1) What is the difference between the true and the bank discount of \$950 for 3 mos. at 7 per cent.?

(2) A bill is drawn for $722.70 on July 17 at 2 months, and discounted on Aug. 11 at 7½ per cent.; how much did the holder receive?

(3) Find the discount charged in discounting a bill for $7850 drawn April 9 at 7 months and discounted June 19th at 10 per cent.

(4) For what sum must a note be drawn on July 3, at 3 months, so that discounted immediately it may produce $501.69, money being worth 7 per cent.?

(5) Find the difference between the true and bank discounts on $5555 at 6 per cent. for 1 year.

Examination Papers.

I.

(1) Explain the difference between Simple and Compound Interest. Find the interest on $25000 for three years at 4 per cent., supposing interest to make capital at the end of each year.

(2) The difference between the Compound and Simple Interest of a certain sum of money for 3 years at 4 per cent. is $3.80. Find the sum.

(3) Find at what rate Simple Interest in two years a sum of money would amount to the same sum as at 4 per cent. Compound Interest.

(4) Find the Compound Interest on $1000 at 3 per cent. per annum for 2 years and 195 days.

(5) A person puts out to interest $8000 at 4 per cent.; he spends annually $300, and adds the remainder of his dividend to his stock. What is he worth at the end of 5 years?

II.

(1) Explain the distinction between true discount and bank discount. Does the creditor or the debtor gain by computing the interest instead of the discount?

(2) Find the discount on $400, due one year hence, if money bear interest at 5 per cent. per annum. Calculate the interest on this discount for the same time, and show that it is equal to the difference between the interest and the discount of $400.

(3) If £10 be the interest on £110 for a given time, what should be the discount of £110 for the same time?

(4) What must be the rate of interest in order that the discount on \$10292 payable at the end of 1 year 73 days may be \$372?

(5) A tradesman who is ready to allow 5 per cent. per annum Compound Interest for ready money, is asked to give credit for two years. If he charge \$110.25 in his bill, what ought the ready money price to have been?

III.

(1) A speculator borrowed \$5000, which he immediately invested in land. Six months afterwards he sold the land for \$7500, on a credit of 12 months, with interest. Money being at 6 per cent., what is the speculator's profit at the end of the 12 months' credit, at which time he returns the \$5000?

(2) A merchant bought 43 cwt. 3 qr. of sugar at \$5.25 per cwt., which he immediately sold at \$7 per cwt. on a credit of 90 days, and then had the purchaser's note for the amount discounted in the bank at 6 per cent. What profit did the merchant make?

(3) Find the Present Worth of \$1000 due $2\frac{1}{2}$ years hence at 5 per cent. per annum; and show that the Discount of the given sum is equal to the interest of the Present Worth for the same time and at the same rate of interest?

(4) A man having lent \$10000 at 5 per cent. interest, payable half-yearly, wishes to receive his interest in equal portions monthly, and in advance; how much ought he to receive every month?

(5) Show that the interest on £266 13*s.* 4*d.* for three months at $4\frac{1}{2}$ per cent. per annum, is equal to the discount of £83 for 15 mos. at 3 per cent. per annum.

IV.

(1) How much may be gained by hiring money at 5 % to pay a debt of \$6400, due in 8 months, allowing the present worth of this debt to be reckoned by deducting 5 % per annum discount?

(2) The difference between the simple and compound interests of a sum of money for 3 years at 8 per cent. is \$985.60. What is the sum?

(3) The interest on a certain sum of money for two years is £71 16*s.* $7\frac{1}{2}$*d.*, and the discount on the same sum for the same time is £63 17*s.*, simple interest being reckoned in both cases. Find the rate per cent. per annum, and the sum.

(4) *A* offers \$8000 for a farm; *B* offers \$9500, to be paid at the end of 4 years. Which is now the better offer, and by how much, allowing 5 per cent. compound interest?

(5) A person borrows money at 6 per cent. per annum, and pays the interest at the end of the year; he lends it out at 8 per cent. per annum, payable quarterly, and receives the interest at the end of the year; by this means he gains \$269·18592 a year. How much did he borrow?

XXIV. Equation of Payments.

184. When several sums of money are due from *A* to *B*, payable at different times, it is often required to find the time, called the EQUATED TIME, at which all may be paid together, without injustice to *A* or *B*.

When great exactness is demanded, interest must be added to the sums paid after they are due, and discount subtracted from the sums paid before they are due. But in practice the following rule is sufficiently accurate:

Multiply each debt by the number of days [or months] after which it is due: add the results together: divide this sum by the sum of the debts: the quotient will be the number of days [or months] in the equated time.

Take the following examples:

Ex. (1). If \$300 be due from *A* to *B* at the end of 5 months, and \$700 at the end of 9 months, when may both sums be paid in a single payment without unfairness to *A* or to *B*?

$$\text{Number of months in equated time} = \frac{300\times5+700\times9}{300+700} = \frac{7800}{1000} = \frac{78}{10} = 7\tfrac{4}{5}$$

∴ the whole amount of the debt should be paid at the end of $7\frac{4}{5}$ months.

The principle on which this solution depends is, that the interest of the money, the payment of which is delayed beyond the time at which it is due, is equal to the interest of that which is to be paid before it becomes due.

In the above example \$300 is kept $2\frac{4}{5}$ months after it is due, and the interest on it for that time is the same as the interest on \$840, $\$(300 \times 2\frac{4}{5})$, for one month.

But \$700 is paid $1\frac{1}{5}$ months before it is due, and the interest on it for that time is the same as the interest on \$840, $\$(700 \times 1\frac{1}{5})$ for one month.

Ex. (2). *A* is indebted to *B* in the following amounts: \$500 due in 6 months; \$600 due in 7 months; and \$800 due in 10 months. Find the time when all these payments should be made together.

$$\begin{array}{rcrcr}
500 & \times & 6 & = & 3000\\
600 & \times & 7 & = & 4200\\
800 & \times & 10 & = & 8000\\
\hline
1900 & & 1900 &) & 15200\\
\hline
 & & & & 8
\end{array}$$

$\therefore$ the equated time is 8 months.

Note.—This method is but a rough approximation, and can only be taken as equitable when the various times of payment are not widely apart. It will, in short, be applicable only to cases which occur in the ordinary course of trade, and is therefore all that we require in the present work.

It is also to be observed that the error involved in this method is *slightly in favor of the payer*, because interest is calculated on the payments made before they are due, instead of discount, in the algebraical process from which the method is derived. See Appendix.

Examples. (ci.)

What is the equated time of

(1) \$250 due 4 months hence, and \$350 due 10 months hence?

Find the equated time of

(2) \$300 due 3 months hence, \$400 due 4 months hence, and \$500 due 6 months hence.

(3) Of a debt of \$1400, \$100 is due immediately, \$600 at the end of 1 month, \$400 at the end of 7 months, and the

remainder at the end of a year. At what time might the whole debt fairly be paid in one sum?

(4) A grocer ought to receive from a customer \$50 at the end of 2 months, \$30 at the end of 4 months, and \$20 at the end of $6\frac{1}{2}$ months. What would be the proper time for receiving the whole sum together?

(5) A debt is to be paid as follows: One-sixth now, and one-sixth every three months until the whole is paid. When might the whole debt be paid at once?

(6) If \$450 be due in 16 months, and \$250 be due in $13\frac{1}{2}$ months; find the sum which, if paid now, would be equivalent to the whole debt at the equated time, interest at 4 per cent.

(7) There is due to a merchant \$800, one-sixth of which is to be paid in 2 months, one-third in 3 months, and the remainder in 6 months; but the debtor agrees to pay one-half *down.* How long may he retain the other half so that neither party may sustain loss?

(8) *A* sold goods to *B* at sundry times, and on different terms of credit, as follows: Sept. 30, 1868, \$80.75, on 4 months' credit; Nov. 3, 1868, \$150, on 5 months' credit; Jan. 1, 1869, \$30.80, on 6 months' credit; March 10, 1869, \$40.50, on 5 months' credit; April 25, 1869, \$60.30, on 4 months' credit. How much will balance the account June 2, 1869?

(9) *A* owes *B* on the 1st of March the following sums: £140 due on 20th of April, £120 due on 14th of May, £380 due on 15th of June. On what day may *B* pay these debts together?

(10) *M* buys goods of *N*, and has 6 months' credit from the date of invoice. The goods are delivered on 6 different days, to the following amount: £101 14*s.* 10*d.* on Aug. 8, £144 2*s.* 10*d.* on Sept. 5, £303 18*s.* 10*d.* on Sept. 18, £757 0*s.* 8*d.* on Nov. 13, £123 11*s.* 6*d.* on Nov. 28, £123 11*s.* 6*d.* on Dec. 5. On the 13th January, *N*, who desires to receive all the debts in one payment, reckons that this payment should be made in 100 days. Show that this is approximately correct.

EQUATION OF ACCOUNTS.

185. Equation of Accounts (also called "Averaging of Accounts" and "Compound Equation of Payments") is the process of finding at what time the *balance of an account* can be paid without gain or loss to either party.

The BALANCE OF AN ACCOUNT is the difference between the two sides of it, and is what one owes the other.

Ex.

Dr. A in Account with B. Cr.

1877.			1877.		
Jan. 1	To Mdse	$500.00	Feb. 10	By Cash	$1000.00
Feb. 4	" "	600.00	Mar. 4	" "	600.00
Mar. 10	" "	800.00			

Jan. 1, 500 × 0 = 0	
Feb. 4, 600 × 34 = 20400	Feb. 10, 1000 × 00 = 0
Mar. 10, 800 × 68 = 54400	Mar. 4, 600 × 22 = 13200
1900) 74800($39\frac{7}{19}$	1600) 13200($8\frac{1}{4}$
5700	12800
17800	400
17100	
700	
39 days from Jan. 1 is Feb. 9.	8 days from Feb. 10 is Feb 18.
Due Feb 9............$1900	Due Feb. 18 ....$1600

If the account be settled on Feb. 9 it is evident the credits would have been paid 9 days, or the time from Feb. 9 to Feb. 18, before they are due. This would have been a loss of interest to the credit side and a corresponding gain to the debit side. Now, as the settlement should be one of equity, we find how long it will take the balance, $300, to gain the same interest that $1900 would in 9 days.

If $1900 gain a certain interest in 9 days,
$1 will gain the same interest in 1900 × 9 days.
and $300 will gain the same interest in $\frac{1900\times9}{300}$, or 57 days.

Hence the balance became due 57 days before Feb. 19, or on Dec. 24.

NOTE.—Fractional parts of a day are not counted unless the fraction amounts to half a day or upwards; it then counts another day.

Hence we have the following rule:

First find the equated time for each side of the account separately. Then multiply the amount due on that side which falls due FIRST *by the number of days between the dates of the equated times, and divide the product by the balance of the account. The quotient will be the number of days to be counted* FORWARD *from the* LATEST DATE *when the smaller side of the account falls due* FIRST; *and* BACKWARD *when the larger side falls due* FIRST.

Examples. (cii.)

(1) Average the following account:

Dr. J. Hughes in account with S. Adams. Cr.

1875.			1875.		
July 4	To Balance	$375.90	Aug. 10	By Cash.	$316.00
Aug. 20	" Mdse.	815.58	Sept. 1	" "	675.00
Aug. 29	" "	178.25	Sept. 25	" Mdse.	512.25
Sept. 25	" "	387.20	Nov. 20	" Cash.	161.75
Dec. 5	" "	418.70	Dec. 1	" "	100.00

(2) When is the balance of the following account due?

Dr. A. B. Conron. Cr.

1877.			1877.		
Sep. 12	To Mdse. at 30 days	$927.30	Oct. 10	By Cash	$500.00
Oct. 15	" " 30 "	342.75	Nov 20	" "	300.00
Nov 18	" " 60 "	212.13	Nov 30	" "	250.00
Dec. 1	" " 30 "	175.50			

(3) When did the balance of the following accounts become due, the merchandise items being on 6 months?

Dr. J. Green in account with Adam Miller & Co. Cr.

1876.			1876.		
March 1	To Mdse.	$720.75	April 1	By Cash	$700.00
" 20	" "	815.30	May 30	" Mdse.	569.89
April 11	" "	587.80	July 20	" Cash	500.00
" 30	" "	300.00	Sept. 25	" "	100.00
June 15	" "	625.25	" 30	" Mdse.	750.20
July 18	" "	560.00	Oct. 30	" "	329.96
Aug. 30	" "	684.90	Nov. 20	" "	500.00
Sept. 25	" "	365.30	....	... .	......

XXV. Averages and Percentages.

186. The average of two or more groups of numbers is found by adding the numbers together and dividing the sum by the number of groups.

Thus to find the average of 13, 15, 74, 23, 6, and 31, we find the sum of the numbers to be 162, and as the number of groups is 6, the average will be $162 \div 6$, or 27.

Note.—Express any remainder, which may occur, *decimally.*

Examples. (ciii.)

(1) Find the average of 14, 26, 9, 18, 13, 24, 27, 39.

(2) Find the average of 1600, 276, 974, 0, 236, 845, 1239.

(3) Find the average population of three towns, consisting respectively of 34729, 46238, and 87296 inhabitants.

(4) Find the average of $15\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{5}{8}$, 0, $10\frac{3}{8}$, $74\frac{1}{5}$, $28\frac{1}{4}$, and 33.

(5) Find the average of $12\frac{12}{25}$, 21, $7\frac{3}{4}$, ·034, $3\frac{1}{8}$, 0, $24\frac{1}{2}$, and $12\frac{7}{20}$.

PERCENTAGES.

187. Business men regulate their affairs and calculate their profits and losses with reference to 100 as a standard, hence there are other applications of the term Per Cent. besides those already given.

When we speak of an agent getting 3 per cent. as a commission on the management of an estate, we mean that from every \$100 collected he deducts \$3 to remunerate himself for the trouble of collection.

When we read that the population of a town has increased 15 *per cent.* since the last census, we mean that if the number of inhabitants *then* had been divided into groups of 100, and the number of inhabitants *now* into groups of 115, the number of groups would be the same in both cases.

Ex. (1). How much is 3 per cent. on \$1479?

Since \$100 yields \$3,
\$1 yields $\$\frac{3}{100}$,
\$1479 yields $\$\frac{1479\times3}{100}$, or \$44.37.

Ex (2). The number of boys in a school increases in a certain period from 125 to 180; what is the increase per cent.?

On 125 the increase is 55.

On 1 the increase is $\frac{55}{125}$.

On 100 the increase is $\frac{100\times55}{125}$, or $\frac{220}{5}$, or 44;

∴ the increase is 44 per cent.

Examples. (civ.)

(1) Find 5 per cent. of $2400; 8 per cent. of 3475 horses.

(2) How much per cent. is 25 parts out of 75; 178 out of 8900; $\frac{1}{5}$ out of $\frac{1}{2}$?

(3) The population of London proper decreased 33·11 per cent. between 1861 and 1871. In 1861 it was 113,387; find what it was in 1871.

(4) How much per cent. is 9*d.* in the pound; $12\frac{1}{2}$ cents in the dollar; \$3 in every \$20?

(5) Find the number of which 21 is 7 per cent.; 750 is $3\frac{1}{5}$ per cent.; 215 is ·005 per cent.

COMMISSION AND BROKERAGE.

188. COMMISSION is the charge made by an agent for buying or selling goods, and is generally a percentage on the *money engaged in the transaction.*

BROKERAGE is the charge made by a broker for buying or selling stocks, bills of exchange, &c.

In computing Commission, care must be taken to calculate it on the money actually employed in the business.

Ex. (1). My agent has purchased wheat, on my account, to the amount of $18768. What is his commission at $1\frac{3}{4}$ per cent.?

$$\text{The Commission on } \$100 \text{ is } \$1.75;$$

$$\therefore \quad \text{“} \quad \$1 \text{ is } \$\frac{1\cdot75}{100};$$

$$\therefore \quad \text{“} \quad \$18768 \text{ is } \$\frac{18768\times1\cdot75}{100}$$

$$= \$328.44 \text{ Commission required.}$$

Hence the following rule may be used:

Multiply the given sum by the rate per cent. and divide the product by 100, and the result is the Commission or Brokerage.

Ex. (2). I send my agent $1827 with instructions to deduct his commission at $1\frac{1}{2}$ per cent., and invest the balance in silk. How much did he invest

Since the Commission on $100 is $1.50
out of $101.50 he can invest $100 ;
" $1 " " $$\frac{100}{101\cdot50}$;
" $1827 " " $$\frac{1827\times100}{101\cdot50}$
= $1800, sum required.

If in the above question the Commission is required, we reason as follows :

On $101.50 the Commission is $1.50 ;
" $1 " " $$\frac{1\cdot50}{101\cdot50}$;
" $1827 " " $$\frac{1827\times1\cdot50}{101\cdot50}$
= $27.
∴ the Commission required is $27.

Examples. (cv.)

Find the Commission on

(1) $7600 at $1\frac{2}{5}$ per cent. (2) $5600 at $12\frac{1}{2}$ per cent.

Find the brokerage on

(3) $2364 at $\frac{1}{4}$ per cent. (4) $375 at ·5 per cent.

(5) An agent collected rents to the amount of $578, and his Brokerage amounted to $26.01. What was the rate?

(6) Sent $3377 to my agent to invest after deducting his Commission at $2\frac{1}{3}$ per cent. What was his Commission?

(7) What is the ready money payment of an account amounting to $7680, allowing a discount of $2\frac{1}{2}$ per cent.?

(8) *A* receives a consignment of wheat from *B*. He is to sell it on a commission of 2 per cent., and invest the proceeds in silk, after deducting his commission on this new transaction at 4 per cent. *A*'s total commission was $600. What sum did he invest?

(9) What amount of money was invested when the broker's charges at $1\frac{1}{2}$ per cent. amounted to $576?

(10) Gave $20050 to a broker to invest with instruction, after deducting his brokerage at $\frac{1}{4}$ per cent., to invest the balance in Government bonds. What will be the sum invested, and how much will be the brokerage?

INSURANCE.

189. **Insurance** is security guaranteed by one party, on being paid a certain sum, to another against any loss.

The **Premium** is the sum paid for Insurance. It is always a certain per cent. of the sum insured.

The **Policy** is the written contract of Insurance.

Note.—As the Premium is always so much per cent. of the sum insured, it is found by the same rule as Commission.

Ex. What sum should be insured at 4 per cent., on goods worth $2940, that the owner may receive, in case of loss, the value of both goods and premium?

Since the premium on $100 at 4 per cent. is $4,

$96 worth of goods would be covered by $100;

∴ $1 " " " $\$\frac{100}{96}$;

∴ $2940 " " " $\$\frac{2940\times100}{96}$

= $3062.50, sum required.

Examples. (cvi.)

(1) What will be the premium of insurance on the furniture of a house valued at $2500 at ⅛ per cent.?

(2) What is the premium for insuring a cargo, valued at $21350, at 3½ per cent.?

(3) For what sum should goods worth £4384 0*s.* 3*d.* be insured at 2⅓ per cent. that the owner may recover, in case of loss, the value of both goods and premium?

(4) A person at the age of 40 insures his life in each of two offices for $5500, the premiums being at the rate of 3¾ and 3⅘ per cent. respectively. Find his annual payment.

(5) What sum must be paid to insure a cargo worth $26400, the premium being 1½ per cent., policy duty ⅙ per cent., and brokerage ⅛ per cent.?

(6) A trader gets 500 barrels of flour insured for 75 per cent. of its cost at 2⅕ per cent., paying $80.85 premium. At what price per barrel did he purchase the flour?

(7) A company took a risk at 2½ per cent., and re-insured ⅘ of it in another company at 3 per cent. The premium received exceeded that paid by $10. What was the amount of the risk?

(8) A shipment of apples was insured at 2⅜ per cent. to cover ⅝ of its value. The premium was $71.25. What were the apples worth?

TAXES.

190. A TAX is a sum of money assessed on a person in proportion to the value of his property, amount of income, etc., for public purposes.

In order to levy a tax persons, called assessors, are first employed to ascertain or appraise the value of all the property taxed. When this has been done the sum to be levied is apportioned amongst the property-owners according to the value of the property of each.

Ex. A certain town has property valued at $1,560,000 and levies a tax of $23,400; what should *B* pay whose property is valued at $7500?

$$\begin{aligned}\text{Since } \$1560000 &\text{ pays } \$23400;\\ \therefore \$1 &\text{ pays } \$\tfrac{23400}{1560000};\\ \therefore \$7500 &\text{ pays } \$\frac{7500\times 23400}{1560000}\\ &= \$112.50,\text{ tax required.}\end{aligned}$$

Examples. (cvii.)

(1) In a school section containing property valued at $100000 a tax has to be levied to pay the teacher's salary of $800, and $250 which had been expended in purchasing maps, etc. Find *A*'s tax, who owns property, real and personal, worth $5400.

(2) A man who owns $8500 worth of property pays a tax of $144.50. Find the rate on the dollar.

(3) If the property of Toronto be valued at $75000000, and *B*, who pays tax on $80000 worth of property, pays $1400, find the total tax levied in Toronto.

(4) In a certain section a school-house is to be built at an expense of $8400, to be defrayed by a tax upon property valued at $700000. What is the rate of taxation to cover both the cost of the school-house and the collector's commission at 4 per cent.?

DUTIES OR CUSTOMS.

191. DUTIES or CUSTOMS are sums of money required by Government to be paid on nearly all imported goods.

The law requires that all goods entering Canada shall be landed at certain places where CUSTOM HOUSES are established. These places are called PORTS OF ENTRY.

Duties are of two kinds, *ad valorem* and *specific.*

An Ad Valorem duty is a certain percentage on the cost of the goods in the country from which they are imported.

A Specific duty is a sum computed on the ton, yard, gallon, etc., without regard to the value of the goods.

Note.—As ad valorem duties are percentages, they are computed in the same manner as Commission, etc.

Ex. Find the Specific Duty on 760 lbs. of Sulphuric Acid at $\frac{1}{2}$ cent per lb.

Duty on 1 lb. is $\frac{1}{2}$ cent.
" 760 lbs. is $\frac{760}{2}$ cents = $3.80, duty required.

Examples. (cviii.)

(1) What is the Duty on 7635 lbs. of tea, valued at $3500, at 6 cents per lb.?

(2) Find the Ad Valorem Duty on an invoice of books which cost $1760 at 5 per cent.

(3) Find the Specific Duty on 750 gallons of wine worth $2150 at 60 cents per gallon.

(4) Find the Duty on 8400 lbs. of sugar worth $7\frac{1}{2}$ cents per lb., the specific duty being $\frac{1}{2}$ cent per lb. and the ad valorem duty 25 per cent.

(5) Paid $1662.50 duty on an invoice of cotton at the rate of $17\frac{1}{2}$ per cent. What was the value of the cotton?

STORAGE.

192. Storage is a charge made by a person who stores movable property or goods for another. It is usually reckoned by the month of 30 days at a certain price per bushel, cask, box, bale, etc.

The owners of the goods pay for putting the goods in store, stowing away, and the expenses of delivery.

When goods are received and delivered at the pleasure of the consignor, the dues for storage are usually determined by an average.

Ex. What is the cost of storage, at 1c. per bushel per month, of wheat received and delivered as per following:

Account closed October 2nd, 1877.

ACCOUNT OF STORAGE OF WHEAT, RECEIVED AND DELIVERED FOR ACCOUNT OF JOHN JONES, TORONTO.

DATE.		Received.	Delivered.	Balance.	Days	Products.
1877.						
July	2	200	......	200	9	1800
"	11	......	150	50	5	250
"	16	350	......	400	5	2000
"	21	......	300	100	20	2000
August	10	400	......	500	5	2500
"	15	......	450	50	5	250
"	20	......	50	0	0	000
September	5	200	......	200	5	1000
"	10	100	......	300	5	1500
"	15	......	200	100	17	1700
		1250	1150			30)13000
Bal. on hand Oct. 2		......	100			
		1250	1250			$433\frac{1}{3}$

$433\frac{1}{3} \times 1$ cent $= \$4.33\frac{1}{3}$.

The storage of 200 bu. for 9 days + 50 bu. for 5 days + 400 bu. for 5 days + 100 bu. for 20 days + 500 bu. for 5 days + 50 bu. for 5 days + 200 bu. for 5 days + 300 bu. for 5 days + 100 bu. for 17 days is the same as the storage of 13000 bu. for 1 day, or of $433\frac{1}{3}$ bu. for a month of 30 days. The storage of $433\frac{1}{3}$ bu. at 1 cent per bushel is $\$4.33\frac{1}{3}$.

Examination Papers.

I.

(1) If a grocer's pound weight is 10 drams too light, find his gain per cent. from this source alone.

(2) If a debt, after a deduction of 5 per cent., becomes $228, what should it have become if a deduction of $6\frac{1}{4}$ per cent. had been made?

(3) Find the value of the goods imported when an *ad valorem* duty of $17\frac{1}{2}$ per cent. produces $637.

(4) The population of a city has increased by 5975 persons between 1860 and 1870; this increase is $12\frac{1}{2}$ per cent. of the population of 1870. What was the population in 1860?

(5) In 1850 the population of a town was $7600; in 1870

it was found to be 9196. If the increase per cent. during the first decade was the same as during the last, what was this per cent.?

II.

(1) *A*, after paying an income tax of $1\frac{1}{2}$ per cent. on all his salary over $400, has $1739.60 left. Find his salary.

(2) A town has levied a tax of $7340, which sum includes the amount voted for building a bridge and the collector's fees at 3 per cent. What was expended on the bridge?

(3) The average of ten results was 17·5; that of the first three was 16·25, and of the next four 16·5; the eighth was 3 less than the ninth, and 4 less than the tenth. What was the tenth?

(4) The gross receipts of a railway company in a certain year are apportioned thus: 40 per cent. to pay the working expenses, 54 per cent. to give the shareholders a dividend at the rate of $3\frac{1}{2}$ per cent. on their shares; and the remainder, $42525, is reserved. What was the paid-up capital of the company?

(5) *A* can do 5 per cent. of a piece of work in 3 days of 10 hours each; *B* can do $7\frac{1}{2}$ per cent. of it in 5 days of 8 hours each. If both men work together and the whole work be worth $85, what does each get?

III.

(1) A cargo is valued at $7905.45; the premium of insurance is at the rate of $5\frac{1}{4}$ per cent., policy duty at $\frac{1}{5}$ per cent., and commission at $\frac{7}{16}$ per cent.; what sum must be insured to cover the cargo and the expenses of insurance?

(2) Received and delivered, on account of James Smith, sundry bales of cotton, as follows: Received Jan. 1, 1877, 2310 bales; Jan. 16, 120 bales; Feb. 1, 300 bales. Delivered Feb. 22, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales. Required the number of bales remaining in store May 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month.

(3) If the increase in the number of male and female criminals is $2\frac{1}{2}$ per cent., while the decrease in the number of males alone is $7\frac{1}{2}$ per cent., and the increase in the number of females is $10\frac{1}{4}$ per cent., compare the antecedent numbers of male and female prisoners.

(4) A person takes a railway return-ticket for a month, paying 25 per cent. more for it than he would have done for a single ticket. At the end of a month he obtains an ex-

tension of time for a week by paying 5 per cent. on the monthly ticket. The whole sum paid is $10.50; find the price of the single ticket.

(5) The paper duty was $1\frac{1}{2}$*d.* per lb., and the weight of a certain book $1\frac{1}{2}$ lbs. The paper manufacturer realized 10 per cent. on his sale, and the publisher 20 per cent. on his outlay. What reduction might be made in the price of the book on the abolition of the paper duty, allowing to each tradesman the same rate of profit as before?

IV.

(1) A merchant bought 37 yds. 2 qrs. of cloth at \$4.87$\frac{1}{2}$ per yard, and 49 yds. $2\frac{1}{2}$ qrs. of silk at $93\frac{3}{4}$ cents per yard. For what sum must the whole be sold to make a profit of $33\frac{1}{3}$ per cent.?

(2) A commission merchant is to sell 12000 lbs. of cotton and invest the proceeds in sugar, retaining $1\frac{3}{4}$ per cent. on the sale and the same on the purchase. Cotton selling at 7 cents, and sugar at 5 cents per pound, what quantity of sugar can the merchant buy?

(3) In an examination of 750 candidates, ·22 on the whole do well, ·34 barely pass, and the rest fail. How many do well, barely pass, and fail, respectively?

(4) Sold grain on commission at 5 per cent.; invested net proceeds in groceries at 2 per cent. commission. My whole commission was \$70. What was the value of the grain and groceries?

(5) A commission merchant receives 125 bbls. of flour from *A*, 150 bbls. from *B*, 225 bbls. from *C*; he finds on inspection that *A*'s is 10 per cent. better than *B*'s, and *C*'s $5\frac{5}{11}$ per cent. better than *A*'s; he sells the whole lot at \$7 per barrel, and charges 4 per cent. commission. How much does he remit to each?

V.

(1) A broker charges me $1\frac{1}{3}$ per cent. commission for purchasing some uncurrent bank bills at 25 per cent. discount; of these bills three of \$10 each and one of \$50 became worthless; I dispose of the remainder at par, and thus make \$520. What was the amount of bills purchased?

(2) A wholesale merchant sent a quantity of goods into the country to be sold by auction, on a commission of $4\frac{1}{2}$ per cent What amount of goods must be sold that his agent may buy produce with the avails to the amount of \$1910, after retaining a commission of 2 per cent.?

(3) A factor receives \$30056, and is directed to purchase cotton at \$289 per bale; he is to receive 4 per cent. commission. How many bales does he buy?

(4) Sold goods to a certain amount on a commission of 5 per cent., and having remitted the net proceeds to the owner, received for prompt payment $\frac{1}{3}$ per cent., which amounted to \$16.15. What was the amount of commission?

(5) A man obtained an insurance for life at the age of 37, and died when 51 years old. The policy required annual payments during life at \$2·8674 per \$100, and secured to the heirs \$1709.69 more than the amount of all the premiums paid. What was the face value of the policy?

XXVI. Profit and Loss.

193. If I sell for \$105 that for which I gave \$100, I gain \$5 on an outlay of \$100.

If I sell for \$95 that for which I gave \$100, I lose \$5 on an outlay of \$100.

The following Examples will show the method of solving questions relating to Profit and Loss, the principles laid down in Section xx being followed.

Ex. (1). I sell for \$6 that for which I gave \$5. What is my gain per cent.?

On an outlay of \$5 my gain is \$1;
On an outlay of \$1 my gain is \$$\frac{1}{5}$;
On an outlay of \$100 my gain is \$$\frac{100}{5}$, or \$20;
∴ I gain 20 per cent.

Ex. (2). I bought some goods for \$17. How must I sell them in order to gain $17\frac{11}{17}$ per cent.?

That for which I gave \$100 I must sell for \$$117\frac{11}{17}$;
That for which I gave \$1 I must sell for \$$\frac{2000}{100\times17}$;
That for which I gave \$17 I must sell for \$$\frac{17\times2000}{100\times17}$, or \$20.

Ex. (3). By selling goods for \$7.20 I made a profit of 20 per cent. What did I give for them?

That which I sold for \$120 I bought for \$100;
That which I sold for \$1 I bought for \$$\frac{100}{120}$;
That which I sold for \$7.20 I bought for \$$\frac{7.20\times100}{120}$, or \$6.

Ex. (4). If by selling coffee at 1*s.* 7*d.* per lb. I lose 5 per cent., what must I sell it at to gain 5 per cent.?

That which I sell at 95*d.* I bought for 100*d.*;
that which I sell at 1*d.* I bought for $\frac{100}{95}$*d.*;
that which I sell at 19*d.* I bought for $\frac{19 \times 100}{95}$*d.*, or 20*d.*

Having thus found the *cost* price, we proced thus:

To gain 5 per cent.,
that for which I gave 100*d.* I must sell for 105*d.*;
that for which I gave 1*d.* I must sell for $\frac{105}{100}$*d.*;
that for which I gave 20*d.* I must sell for $\frac{20 \times 105}{100}$*d.*, or 1*s.* 9*d.*

Or thus:

In the first case,
that which costs 100*d.* sells for 95*d.*

In the second case,
that which costs 100*d.* sells for 105*d.*;
∴ that which sells for 95*d.* must bring 105*d.*;
“ “ 1*d.* must bring $\frac{105}{95}$*d.*;
“ “ 19*d.* must bring $\frac{19 \times 105}{95}$*d.*,
or 1*s.* 9*d.*, as before.

Ex. (5). A quantity of tea is sold for $83\frac{1}{3}$ cents per pound; the gain is 10 per cent., and the total gain is \$48. What is the quantity of the tea?

That which sells for \$110 costs \$100;
“ “ \$1 “ $\$\frac{100}{110}$;
“ “ $83\frac{1}{3}$ cts. “ $\$\frac{0·83\frac{1}{3} \times 100}{110}$;

∴ the cost price per lb. = $\frac{10}{11}$ of \$0·83$\frac{1}{3}$;
∴ the gain on 1 lb. = $\frac{1}{11}$ of \$0.83$\frac{1}{3}$;
but the gain per lb. × No. of lbs. sold = total gain,
or $\frac{1}{11}$ of \$0·83$\frac{1}{3}$ × No. of lbs. sold = \$48;

$$\text{No. of lbs. sold} = \$\frac{48}{\frac{1}{11} \text{ of } \$0·83\frac{1}{3}}$$
$$= 633\tfrac{3}{5} \text{ lbs.}$$

∴ $633\frac{3}{5}$ lbs. is the quantity sold.

194. When tea, spirits, wine, and such commodities are *mixed* it must be observed that

quantity of ingredients = quantity of mixture,
cost of ingredients = cost of mixture.

Thus, if a mixture is made of 1 gallon of ale at 8 cts. a gallon, 3 at 15 cts., 4 at 20 cts., and 12 at 7 cts.

quantity of ingredients $= (1 + 3 + 4 + 12)$ galls., or 20 galls.
cost of ingredients $= (8 + 45 + 80 + 84)$ cts., or $2.17.

If I want to know what gain per cent. I shall make by selling this mixture at 26 cts. a gallon, I reason thus:

20 gall. at 26 cts. will sell for $5.20;
∴ that for which I gave $2.17, I sell for $5.20;
∴ $2.17 gains ($5.20 − $2.17) = $3.03;
∴ $1 gains $\$\frac{3\cdot 03}{2\cdot 17}$;
∴ $100 gains $\$\frac{100 \times 3.03}{2\cdot 17}$, or $139.63.
∴ I gain 139·63 per cent.

195. In solving questions on Profit and Loss the student must be very careful to notice whether the gain is calculated on the selling price or cost price. Thus, it is sometimes said that a retailer's profit is 25 per cent., meaning that he gave 75 cents for an article which he sells for $1. His profit in this case is $33\frac{1}{3}$ per cent. on his outlay. Care must therefore be taken to express distinctly what is meant. The profit on a single transaction or set of transactions by no means represents a net profit, as it is not charged with a variety of expenses which belong to the business in general rather than to the set of transactions in question.

Ex. If 100 articles of a given kind can be made in a week out of $40 worth of raw materials, cost of labor, etc., being $10, fixed charges for rent, etc., being $250 a year, find (1) the cost price of each article, (2) the invoice price in order that a profit of 30 per cent. on the cost price may be realized, the following allowances being necessary, viz.: 10 per cent. commission to agents on money received for sales, and 12 per cent. for bad debts, and (3) the amount of profit in a year.

(1) The fixed charges must be referred to the same unit of time as the rest of the estimate, viz.: 1 week $= \$\frac{250}{52} = \$\frac{125}{26}$.

$$\text{Cost of 100 articles} = \$50 + \$\tfrac{125}{26} = \$54\cdot8077\,;$$

$$\therefore \text{ cost of 1 article} = \$0\cdot548077.$$

(2) The profit on capital may be regarded as part of the cost of production. It would be so, in fact, if the money were borrowed at 30 per cent. interest. 30 per cent. added to $0·548077 gives $\$\frac{130\times\cdot548077}{100}$.

Again, the commission is paid on the money actually received; to provide for it we must take the $\frac{10}{9}$ of

$$\$\frac{130\times\cdot548077}{100}, \text{ or } \$\frac{10\times130\times\cdot548077}{9\times100}.$$

Next: 12 per cent. on bad debts means that 12 do not pay for 88 who do. To provide for it we take $\frac{100}{88}$ of the selling price. The invoice price will therefore be

$$\$\frac{100\times10\times130\times\cdot548077}{88\times9\times100}, \text{ or } \$\cdot899.$$

(3) To find the profit we must take $\frac{30}{100}$ of the cost price and multiply by 100 × 52.

$$\text{Annual profit} = \$\frac{30\times100\times52\times\cdot548077}{100} = \$855.$$

Examples. (cix.)

(1) If I buy an article for $3.20 and sell it for $4, what is my gain per cent.?

(2) If I sell goods for $2240 and gain 12 per cent., what was the cost price?

(3) If 375 yards of silk be sold for $1960, and 20 per cent. profit be made, what did it cost per yard?

(4) If, by selling wine at 17*s*. 5*d*. a gallon, I lose 5 per cent., at what price must I sell it to gain 15 per cent.?

(5) If, by selling goods for $544, I lose 16 per cent., how much per cent. should I have lost or gained if I had sold them for $672?

(6) The manufacturer will supply a certain article at 1½*d*. If a tradesman charges 2*d*., what profit per cent. will he make?

(7) A tradesman's prices are 20 per cent. above cost price. If he allows a customer 10 per cent. on his bill, what profit does he make?

(8) A tradesman's prices are 25 per cent. above cost price. If he allow a customer 12 per cent. on his bill, what profit does he make?

(9) A man buys goods at £23 5*s.* 5*d.* and sells them at £22 2*s.* 1¾*d.* How much does he lose per cent.?

(10) A man buys goods at £15 6*s.* 3*d.* and sells them again at £11 15*s.* 9¾*d.* How much does he lose per cent.?

(11) A man buys goods at the rate of $96 per cwt., and sells 2 tons, 14 cwt. 3 qr. 12 lbs. for $6000. How much has he gained or lost per cent. on his outlay?

(12) If 8 per cent. be gained by selling a piece of ground for $4125.60, what would be gained per cent. by selling it for $4202?

(13) If 3 per cent. more be gained by selling a horse for $333 than by selling him for $324, what must his original price have been?

(14) A grocer mixes 12 lb. of tea at 2*s.* 6½*d.* per lb. with 4 lbs. at 3*s.* 2¼*d.* At what price must he sell the mixture so as to gain 33⅓ per cent. upon his outlay?

(15) How many pounds of tobacco at $1.05 a pound must a tobacconist mix with 4 lb. at $1.30, that he may sell the mixture at $1.56⅔ per pound, and gain 33⅓ per cent. upon his outlay?

(16) A spirit merchant buys 80 gallons of whiskey at $3.60 per gallon, and 180 gallons more at $3.00 per gallon, and mixes them. At what price must he sell the mixture to gain 8⅓ per cent. upon his outlay?

(17) I mix 80 gallons of gin at $3.10 per gallon with 96 gallons at $3.41⅔, and sell the mixture so as to gain 10 per cent. At what price per gallon do I sell it?

(18) A grocer buys two sorts of tea at 55 cents and 61⅔ cents per lb. respectively. He mixes them so as to have 3 lb. of the dearer for every 1 lb. of the cheaper sort, and sells the mixture at 80 cents per lb. What does he gain per cent.?

XXVII. Stocks and Shares.

196. The Government of a country, the authorities of a city, etc., often find it necessary to borrow money to carry on public works, etc. A loan is then contracted and the borrower pledges the credit of the country, city, etc., to pay a fixed rate of interest on the sum borrowed until the debt is paid off.

The term STOCK is applied to any such government loan. It also denotes the capital of a joint-stock company.

Banks, Railway Companies, and others have their capital divided into *shares* of so many dollars each, usually $50 or $100.

The price of stock is always quoted at so many dollars for $100 stock. Thus, when we read that the stock of the Toronto Bank is at 155 it means that $155 of money will purchase $100 stock in that bank.

The price of stock is always fluctuating, owing to a change in the value of money, *i.e.*, at times money is scarce and consequently in large demand, and hence the rate of interest will be high; at other times it is plentiful and therefore cheap. Thus if *A* has money to loan and can get 8 per cent. for it, he will not invest it in the Dominion stock, which pays 6 per cent., unless the latter is so cheap that he can make 8 per cent., *i.e.*, unless he can buy at 75. Hence if *B* wished to sell Dominion 6 per cent. stock he would have to sell it at a *discount*.

Again, if money could only be loaned at 5 per cent., *B* would be able to sell $100 of such stock for more than $100 money, in this case he would sell at a *Premium*. Among the other causes which determine the value of stock, we may mention its desirability of a safe investment, commercial and political changes at home and abroad, etc.

197. Stock is at PAR when it sells for its *nominal* value, as when $100 stock sells for $100 money.

It is at a PREMIUM when it sells for more than its *nominal* value. Thus, when $100 stock sells for $109 money it is at a *Premium* of 9 per cent.

It is at a DISCOUNT when it sells at less than its nominal value. Thus, when $100 stock sells for $85 money, it is at a *discount* of 15 per cent.

The purchase and sale of stocks are usually effected by means of a stock-broker, who is paid a certain percentage on all *stock* that passes through his hands. Thus, if stock is at 92½ and the broker charges ½ per cent., the buyer will have to pay \$93 (\$92$\frac{1}{2}+\frac{1}{2}$) for \$100 stock, and the seller would receive \$92 (\$92$\frac{1}{2}-\frac{1}{2}$) for it.

198. Stock is often named from the interest which is paid to the owners of the stock. Thus, the Dominion Government stock, paying interest at the rate of 6 per cent., is spoken of as the Dominion 6 per cents., or Dominion 6's.

Consols are a part of the National Debt of Great Britain, so called from the Consolidation of the stock of various annuities into a joint 3 per cent. stock.

The National Debt of Great Britain, which now amounts to about 773 millions, has been incurred by loans made to the State by individuals. Interest is paid upon the main part of this debt at the rate of 3 per cent. The names of the persons who have a claim on the nation for such interest, are registered in books kept by the Bank of England on behalf of the Government. Such persons are called *Fundholders;* the debt itself is often called *The Funds;* and the interest, which is payable half-yearly, is called *Dividends.*

Suppose *A* to be a Fundholder in that particular part of the National Debt called *The Three Per Cent. Consols,* and suppose the amount of the debt, which he is acknowledged by the Register to hold, be £5000, he is then said to hold £5000 stock. *A* cannot demand the payment of 5000 sovereigns, or any smaller sum, from the Government, as a redemption of the debt, but the Government undertakes to pay him (or any one to whom he may assign his claim) 75 sovereigns, every half-year, that being the amount of interest on £5000 for half a year at 3 per cent.

Now suppose *A* to be desirous of selling his claim to *B.* The value of the claim does not vary much from time to time in the case before us, for England is known to be willing and is acknowledged to be able to pay the interest on her debt, and the security of the claim makes the Fundholder satisfied with a low rate of interest, punctually paid and easily obtained. The value of £100 Stock in Consols is at the present time (July 12, 1877) $92\frac{3}{8}$, that is, *A* can obtain £$92\frac{3}{8}$ for each £100 Stock that he holds, and *B*, on the payment of 50 × £$92\frac{3}{8}$, or £4618 15*s*., can have the £5000 Stock, now held by *A*, transferred to him.

A's name is then removed from the Register, and *B*'s name is inserted in it, and the process is called a *Transfer*. *A* is said to *sell out* of the Funds, and *B* is said to *invest* in them.

199. United States securities are of two kinds: Notes and Bonds.

United States 6's, 5-20 are bonds bearing interest at 6 per cent., and payable in 20 years, but may be paid in 5 years if the Government choose. When it is necessary to distinguish different issues of bonds bearing the same rate of interest, the year at which they become due is mentioned; thus U. S. 6's, 5-20 of '84; U. S. 6's, 5-20 of '85.

Notes are of two kinds:

First, those payable on demand, without interest, known as United States Legal-tender Notes, or "Green Backs."

Second, Treasury Notes payable at a specified time, with interest. Of this kind are notes bearing interest at $7\frac{3}{10}$ per cent., and known as 7-30's. These have all been redeemed.

200. CURRENCY is a term used in commercial language,

First, to denote the aggregate of Specie, Bills of Exchange, Bank Bills, Treasury Notes, and other substitutes for money employed in buying, selling, and carrying on exchange of commodities between various countries.

Second, to denote whatever circulating medium is used in any country as a substitute for the Government standard. It sometimes happens that the paper currency of a country becomes depreciated in value. Thus, when we read in Stock quotations of buying at $94\frac{3}{4}$ and selling at $95\frac{1}{2}$, it is meant that a broker would give \$$94\frac{3}{4}$ gold for \$100 of paper currency, and that he would sell \$100 of paper currency for \$$95\frac{1}{2}$ gold. Also, when we read that gold is $105\frac{1}{4}$, it is meant that the paper currency is taken as the standard for the time being, and \$$105\frac{1}{4}$ of such currency would be given for \$100 gold.

201. In Canada the liability on all Bank Stocks is limited to double the amount of the subscribed capital. On all other stocks the liability of shareholders is strictly limited to the amount of the subscribed capital.

When all the Capital of a company has been paid up, it is often changed from Shares to Stock, because in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those Shares cannot be transferred.

Three points must now be clearly marked:

(1) We shall know the amount of money received by *A* for any given amount of stock, if we know the price of the stock at the time of sale.

(2) We shall know how much stock can be bought by *B* for any given amount of money, if we know the price of the stock at the time of sale.

(3) We shall know the amount of income received by *A* (and subsequently by *B*) on any given amount of stock, if we know the rate of interest payable on the stock; the income depending in no way on the price of the stock.

These three cases we now proceed to illustrate:

Ex. (1). What is the value of \$2500 stock in the Dominion 5's at $98\frac{1}{4}$?

The value of \$100 stock is \$98.25;

∴ " \$1 stock is $\$\frac{98.25}{100}$;

∴ " \$2500 stock is $\$\frac{2500 \times 98.25}{100}$;

$= \$2456.25.$

Ex. (2). How much stock can be purchased at $92\frac{1}{2}$ for \$740?

For \$92.50 I can purchase \$100 stock;

for \$1 " $\$\frac{100}{92.50}$;

for \$740 " $\$\frac{740 \times 100}{92.50}$, or \$800 stock.

Ex. (3). What annual income is derived from investing \$3920 in the 6 per cents. at 98?

Here, the owner of \$100 stock has an income of \$6, and to purchase this stock he must pay \$98;

∴ \$98 gives an income of \$6;

∴ \$1 " $\$\frac{6}{98}$;

∴ \$3920 " $\$\frac{3920 \times 6}{98}$, or \$240.

Ex. (4). What sum must be invested in the Dominion 6's at 95 so that I may have an annual income of \$1200?

Since \$6 is got from investing \$95,

∴ \$1 " $\$\frac{95}{6}$;

∴ \$1200 " $\$\frac{1200 \times 95}{6}$, or \$19000.

Ex. (5). What annual income is derived from \$3550 stock in the U.S. 5's, 10-40?

Income on \$100 stock is \$5;

" \$1 " $\$\frac{5}{100}$;

" \$3550 " $\$\frac{3550 \times 5}{100}$, or \$177.50.

This is merely a case of finding the Interest, where the stock is the Principal.

Ex. (6). Bought stock in the Bank of Commerce at 120. The last dividend was at 8 per cent.; what per cent. did I make on the investment?

\$120 gives an income of \$8;

∴ \$1 " $\$\frac{8}{120}$;

∴ \$100 " $\$\frac{100 \times 8}{120}$, or $\$6\frac{2}{3}$;

∴ the per cent. required is $6\frac{2}{3}$.

Ex. (7). When stock is at 84, how much stock must be sold to raise \$462?

Since \$84 is got from selling \$100 stock;
∴ \$1 " $\$\frac{100}{84}$ stock;
∴ \$462 " $\$\frac{462 \times 100}{84}$ stock;
or \$550 stock.

Ex. (8). What is the price of Ontario Bank stock when \$6000 stock produces \$5880?

Since \$6000 stock is worth \$5880,
∴ \$1 " $\$\frac{5880}{6000}$;
∴ \$100 " $\$\frac{100 \times 5880}{6000}$, or \$98;
∴ the stock was selling at 98.

Ex. (9). By investing in the Dominion 6's I make $6\frac{1}{2}$ per cent. What was the selling price of this stock?

Since \$6.50 is got from investing \$100,
∴ \$1 " $\$\frac{100}{6 \cdot 50}$;
∴ \$6 " $\$\frac{6 \times 100}{6 \cdot 50}$, or $\$92\frac{4}{13}$;
∴ the selling price was $92\frac{4}{13}$.

Ex. (10). Which is the more advantageous stock to invest in, 6 per cents. at 95, or 5 per cents. at $87\frac{1}{2}$, and how much per cent. is it better

Income for \$95 in the 6 per cents. is \$6;
∴ Income for \$1 in the 6 per cents. is $\$\frac{6}{95}$.

Income for \$1 in the 5 per cents. is $\$\frac{5}{87\frac{1}{2}}$, or $\$\frac{10}{175}$.

We have now to compare the fractions $\frac{6}{95}$ and $\frac{10}{175}$.

Reduced to a common denominator these become $\frac{222}{3515}$ and $\frac{190}{3515}$;

∴ Income for \$1 in the 6 per cents. is $(\frac{222}{3515} - \frac{190}{3515})$ of \$1 better than in the 5 per cents.

∴ Income for \$100 in the 6 per cents. is $100 \times (\frac{222}{3515} - \frac{190}{3515})$ of \$1 better than in the 5 per cents.;

Now $100 \times (\frac{222}{3515} - \frac{190}{3515}) = \cdot 91$....per cent. required.

Ex. (11). A person transfers £5000 stock from a 3 per cent. stock at 72, and invests the proceeds in a 4 per cent. stock at 90. Find the difference in his income

First, he sells £5000 stock at 72, and gets £(72×50) or £3600.

Then he invests £3600 in the 4 per cent. stock at 90, and buys £$\frac{3600\times100}{90}$ stock, or £4000 stock.

Now his *first* income on the £5000 stock was £$\frac{5000\times3}{100}$, or £150.

And his *second* income on the £4000 stock is £$\frac{4000\times4}{100}$, or £160;

∴ he increases his income by £10.

Ex. (12). A person invests £1075 10*s.* in Consols when they are at $89\frac{1}{2}$, and sells out when they are at $93\frac{3}{8}$. What is his gain, brokerage at $\frac{1}{8}$ per cent. on each transaction?

Here an annuity which costs £$(89\frac{1}{2}+\frac{1}{8})$ is sold for £$(93\frac{3}{8}-\frac{1}{8})$.

∴ on £$89\frac{5}{8}$ the gain is £$3\frac{5}{8}$;

∴ on £1 the gain is £$\frac{3\frac{5}{8}}{89\frac{5}{8}}$, or £$\frac{29}{717}$;

∴ on £1075 10*s.* the gain is £$1075{\cdot}5\times\frac{29}{717}$, or £43 10*s.*

Ex. (13). A person invested in Bank stock at $89\frac{3}{4}$ and sold out at $103\frac{1}{2}$, and cleared \$397.50. How much did he invest, brokerage being $\frac{1}{4}$ per cent. on each transaction?

Here what cost \$90 is sold for $\$103\frac{1}{4}$;

∴ he gained \$13.25 by investing \$90;

∴ he gained \$1 by investing $\$\frac{90}{13\cdot25}$;

∴ he gained \$397.50 by investing $\$\frac{397\cdot50\times90}{13\cdot25}$, or \$2700.

Ex. (14). A person having to pay $\$3606\frac{2}{33}$ two years hence, invested a certain sum in the Toronto 6 per cent. city bonds, to accumulate interest until the debt be paid, and also an equal sum next year; supposing the investments to be made when the stock was at 99, and the first year's interest also invested in stock, and the price to remain the same, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

Every \$99 invested will give \$6 interest;

∴ every \$1 invested will give $\$\frac{6}{99}$ interest;

∴ \$ sum invested will give \$ sum × $\frac{6}{99}$ interest.

Now \$ sum × $\frac{6}{99}$ invested will give \$ sum × $\frac{6}{99} \times \frac{6}{99}$ interest.

Hence at the end of the second year there were on hand the two sums invested.

Two years' interest on the first investment = 2 × sum × $\frac{6}{99}$ ·

One year's interest on the second investment = sum × $\frac{6}{99}$:

And the interest on the first year's interest = sum × $\frac{6}{99} \times \frac{6}{99}$.

Or 2 sums + 3 × sum × $\frac{6}{99}$ + sum × $\frac{6}{99} \times \frac{6}{99}$ to meet $\$3606\frac{2}{33}$;

$$\therefore \left(2 + \frac{18}{99} + \frac{36}{9801}\right) \text{sum} = \$3606\tfrac{2}{33};$$

$$\therefore \text{sum} = \$\frac{3606\frac{2}{33}}{\frac{21420}{9801}} = \$1650.$$

Examples. (cx.)

Find the value of

(1) \$7645 stock in the 6 per cents. at 95.

(2) \$9800 stock in the 5 per cents. at 80.

(3) \$7650 stock in the 7 per cents. at 118½.

(4) £3850 stock in the 3 per cents. at 92.

(5) £572 10s. stock in the 3 per cents. at 91½.

How much stock will

(6) \$8400 buy in the 4 per cents. at 75?

(7) \$3757.50 buy in the 8 per cents. at 125¼?

(8) \$994.50 buy in the 7 per cents. at 117?

(9) £2199 buy in the 3 per cents. at 91⅝?

(10) £5527 10s. buy in the 3 per cents. at 92⅛?

What income is got from investing

(11) \$934.25 in the 6 per cents. at 101?

(12) \$4147 in 4 per cent. stock at 72½?

(13) \$6720 in 5½ per cent. stock at 96?

(14) \$3725 in 3 per cent. stock at 74½?

(15) £8475 10s. in 3 per cent. stock at 92⅛?

What amount of stock must be sold

(16) In the 8 per cents. at 125 to produce $750?

(17) In the Dominion 5's at 92½ to produce $629?

(18) In the 6 per cents. at 101 to produce $959.50?

(19) In the 7½ per cents. at 128 to produce $409.60?

What per cent. is made by investing in the

(20) 8 per cents. at 120?

(21) 5 per cents. at 95?

(22) 6 per cents. at 104?

(23) 3½ per cents. at 75?

When Greenbacks are at

(24) 90, what is the price of gold?

(25) 92¼, what is the price of gold?

(26) 84, what is the price of gold?

When gold is at a premium of

(27) 10 per cent., what are "Greenbacks" quoted at?

(28) 25 per cent., what are "Greenbacks" quoted at?

(29) 14 per cent., what is $5700 of American currency worth?

What sum must be invested in the

(30) 8 per cents. at 120 so as to produce an income of $640?

(31) 5 per cents. at 90 so as to produce an income of $3750?

(32) 4½ per cents. at 67 so as to produce an income of $2790?

What is the selling price of stock when

(33) $550 stock in the 6 per cents. produce $558.25?

(34) $7840 stock in the 4 per cents. produce $6664?

(35) £840 stock in the 3 per cents. produce £773 17*s*.

(36) What must I pay for U.S. 10–40's (Interest at 5 %) that my investment may yield 6 per cent.?

(37) Which is the better investment, the buying of 9 per cent. stocks at 25 per cent. advance, or 6 per cent. stocks at 25 per cent. discount, and how much per cent. better?

(38) The difference between the incomes derived from investing a certain sum in 6 per cent. stock at 126, and in 9

per cent. stock at 210, is £22 10*s.* What is the amount invested?

(39) I sell out of the 3 per cents. at 96, and invest the proceeds in Railway 5 per cent. stock at par. Find by how much per cent. my income is increased.

(40) If a $3\frac{1}{2}$ per cent. stock be at 91, how much must I invest in it so as to have a yearly income of £932, after paying 7*d.* in the pound income-tax?

(41) By selling out £4500 in the India 5 per cent. stock at $112\frac{1}{2}$, and investing the proceeds in Egyptian 7 per cent. stock, a person finds his income increased by £168 15*s.* What is the price of the latter stock?

(42) Find the alteration in income occasioned by shifting £3200 stock from the 3 per cents. at $86\frac{3}{8}$ to 4 per cent. stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent.

(43) *A* owns a farm which rents for $411.45 per annum. If he sells the farm for $8229, and invest the proceeds in U. S. 6's, 5–20's of '84, at 105, paying $\frac{1}{2}$ per cent. brokerage, will his yearly income be increased or diminished, and how much?

(44) Through a broker I invested a certain sum of money in U.S. 6's, 5–20 at $107\frac{1}{2}$, and twice as much in U.S. 5's, 10–40 at $98\frac{1}{2}$, brokerage in each case $\frac{1}{2}$ per cent. My income from both investments was $1674. How much did I invest in each kind of stock?

(45) *A* purchased goods for which he was to pay $7000 in currency, or $5500 in gold. Will he gain or lose by accepting the latter proposal, gold being at 125, and how much?

(46) I invest in the 3 per cents. at 92. They fall to 85, and I sell out and obtain a safe investment paying 5 per cent., but not subject to fluctuation of value. How long must I hold it before I shall make a profit by the change, in case 3 per cents. rose to their former value?

(47) I own $4000 Montreal Bank stock paying an annual dividend of 14 per cent. I sell at 180 and invest in Toronto Gas Company stock at 125, and receive an annual dividend of 9 per cent. What change is made in my income, brokerage being $\frac{2}{5}$ % and $\frac{3}{5}$ % on the respective transactions?

(48) A person bought stock at $95\frac{1}{4}$, and after receiving the half-yearly dividend at the rate of 7 per cent. per annum. sold out at $92\frac{3}{8}$ and made a profit of $37.50. How much stock did he buy?

(49) Whether is it better to invest in the 6 per cents. at $98\frac{1}{2}$, or in the 5 per cents. at 85, brokerage being $\frac{1}{2}$ per cent.?

(50) What sum must a man invest in the Dominion 6's at 101 in order to have a clear income of $1775.50, after paying an income tax of 1¾ cents on the dollar on all over $400?

(51) A gentleman has been receiving 12 per cent. on his capital in Canada. He goes to England to reside, and invests it in the 3 per cents. at 94⅜, and his income in England is £2400. What was his income in Canada, £ being equal to $4.86⅔?

(52) By selling out £4500 in the India five per cent. stock at 112¾, and investing the proceeds in Egyptian seven per cent. stock, *A* finds his income increased by £168 15*s*. What was the price of the latter stock, brokerage on each transaction being ¼ per cent.?

(53) The 6 per cents. are at 91½ and the 7 per cents. at 102. A person has a sum of money to invest which will give him $3500 more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

(54) One company guarantees to pay 5 per cent. on shares of $100 each; another guarantees at the rate of 4⅝ per cent. on shares of $30 each; the price of the former is $124½, and of the latter $34 each. Compare the rates of interest which the shares return to the purchasers.

(55) The present income of a railway company would justify a dividend of 3¾ per cent. if there were no preference shares; but as $1200000 of the stock consists of such shares, which are guaranteed 5 per cent. per annum, the ordinary shareholders receive only 3 per cent. What is the whole amount of stock?

(56) Received from my correspondent in New York $6150 U.S. currency, with instructions to deduct my commission at 2½ per cent., and invest the remainder in Canadian tweeds worth $1.03½ per yard. How many yards should I send him, gold being quoted at 115?

Examination Papers.

I.

(1) In a sale of goods for $728 there is a loss of 9 per cent.; for what must 3 times the quantity be sold in order to gain 7 per cent.?

(2) If 20 per cent. be gained by selling an article for $2.10, what is the gain or loss per cent. when it is sold for $1.60?

(3) A grocer had 150 lbs. of tea, of which he sold 50 lbs. at $1.80 per pound, and found he was gaining only $7\frac{1}{2}$ per cent., but he wished to gain 10 per cent. on the whole. At what rate must the remaining 100 lbs. be sold that he may attain his wishes?

(4) A tradesman adds 35 per cent. to the cost price of his goods, and gives his customers a reduction of 10 per cent. on their bills. What profit does he make?

(5) A bill of $2520 due a year hence can be taken up now at 5 per cent. discount. Supposing a tradesman can employ his capital so as to obtain interest at the end of every quarter at the rate of $4\frac{1}{2}$ per cent. per annum, had he better so employ it or take up the bill? and what will be the difference to him?

II.

(1) A tradesman marks his goods with two prices, one for ready money and the other for one year's credit, allowing discount at 5 per cent. If the credit price be marked $2.45, what ought the cash price to be?

(2) If goods be sold on condition to allow 10 per cent. discount, if payment be made at the end of six months what discount ought to be allowed if payment be actually made (1) three months *before*, and (2) three months *after* the stated time, if money bear interest at 5 per cent. per annum?

(3) A person purchases goods at $1.20 per pound Troy weight and sells them again by Avoirdupois weight. At what rate per ounce must he sell so as exactly to reimburse his outlay?

(4) What is meant when it is said that Consols are at $88\frac{1}{4}$? What are they at when £9000 is paid for £10000 Consols?

(5) A person sells $1200 stock in the 3 per cents. at 86, in order to invest in Bank stock paying 8 per cent. What price must he pay for it to be neither a gainer nor loser?

III.

(1) I send $3060 to my agent in Montreal to invest in tea at 75c. per lb. He deducts his commission of 2 per cent. and purchases the tea. How many pounds do I receive, and at what must I sell per lb. so as to make a profit of 40 per cent. after paying freightage $30 and insurance at the rate of $\frac{1}{3}$ per cent.?

(2) Bought land at $50 an acre; how much must I ask an acre that I may take off 25 per cent. from my asking

price and still make 20 per cent. profit on the purchase money?

(3) *A* buys silks at $2.25 per yard on a credit of 6 months. *B* buys the same quality of silks for $2.15 per yard cash. Which makes the best purchase, money being worth 10 per cent., and what must the goods be marked at to insure a gain of 25 per cent.? Or, if the silks be sold at $3 per yard, what profit per cent. does each make?

(4) A person buys an article and sells it so as to gain 5 per cent. If he had bought it at 5 per cent. less and sold it for 5 cents less he would have gained 10 per cent. Find the cost price.

(5) A person buys 6 per cent. City of Toronto bonds, the interest on which is paid yearly, and which are to be paid off at par 3 years after the time of purchase. If money be worth 5 per cent., what price should he give for the bonds?

IV.

(1) Bought cloth at $3 in gold and sold at $4 in currency. Did I gain or lose by the transaction, and how much per cent. in currency, gold being at 118?

(2) A merchant sold 24 cheese at $30 each. On one-half he gained 30 per cent., and on the remainder he lost 30 per cent. Did he gain or lose on the whole, and how much?

(3) A man wishing to sell his farm asked 36 per cent. more than it cost him, but he finally sold it for 20 per cent. less than his asking price. He gained $528 by the transaction. How much did the farm cost, what was his asking price, and for how much did he sell it?

(4) A person having to pay $1085 at the end of 2 years, invested a certain sum in 3 per cent. stock, allowing the dividends to accumulate until the payment of the debt, and also an equal sum next year, and also the previous year's interest. If the investment is made and the debt paid when stock was at 73, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

(5) A merchant's stock-in-trade is valued on Jan. 1, 1875, at $40000; he has $1750 in cash and owes $9350; during the year his personal expenses, $1500, are paid out of the proceeds of the business, and on Jan. 1, 1876, his stock is valued at $39750, he has $2850 in cash and owes $7550. What is the whole profit of the year's transactions after deducting 5 per cent. interest on the capital with which he began the year?

V.

(1) I received an 8 per cent. dividend on railway stock, and invested the money in the same stock at 80. My stock having increased to $13750, what was the amount of my dividend?

(2) How many shares of $50 each must be bought at 25 per cent. discount, brokerage $1\frac{2}{3}$ per cent., and sold at 16 per cent. discount, brokerage $1\frac{1}{4}$ per cent., to gain $\$121.66\frac{2}{3}$?

(3) What sum must be invested in United States 10-40's bearing interest at 5 per cent., payable in gold purchased at par, to produce a semi-annual income of $400 U.S. currency, when gold is quoted at 175 per cent.?

(4) The charter of a new railroad company limits the stock to $1500000, of which 3 instalments of 10 per cent., 20 per cent., and 40 per cent. respectively having been paid in; the cost of construction has reached $850000, and the estimated cost of completion is $850000. If the company call in the final instalment of its stock, and assess the stockholders for the remaining outlay, what will be the rate per cent.?

(5) A person invests $16380 in the 3 per cents. at 91; he sells out $12000 stock when they have risen to $93\frac{1}{2}$, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction? If he invests the produce in $4\frac{1}{2}$ per cent. stock at 102, what is the difference in his income?

XXVIII. Division into Proportional Parts.

202. Suppose 3 persons, *A*, *B*, and *C*, to be in partnership, and an arrangement made that the profits of the business in which they are engaged are to be divided into 6 *equal* parts, of which *A* is to take 3 parts, *B* 2 parts, and *C* 1 part. The shares of *A*, *B*, and *C* are then said to be in the proportion of 3, 2, and 1.

Ex. (1). Divide $1275 among 3 persons, whose shares are to be in the proportion of 3, 5, and 7.

This may be regarded as a case in which one holds 3 shares, one 5, and one 7, and they hold 15 shares in all.

Hence, if we divide $1275 by 15, and we get the amount of *one* share, that is, amount of one share $= \$\frac{1275}{15} = \$85$.

Then one of the persons receives $3 \times \$85$, or $255;
the second receives $5 \times \$85$, or $425;
the third receives $7 \times \$85$, or $595.

Ex. (2). Divide $837 among three partners, whose shares are to be in proportion of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$.

The common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ is 30;

∴ the shares are to be in the proportion of $\frac{15}{30}$, $\frac{10}{30}$, and $\frac{6}{30}$; that is the proportion of 15, 10, and 6.

Now $15 + 10 + 6 = 31$;

∴ amount of *one* share out of 31 shares $= \$\frac{837}{31} = \$27$.

Then one of the partners receives 15 × $27, or $405;
the second receives 10 × $27, or $270;
the third receives 6 × $27, or $162.

Ex. (3). A rate of $4212 is to be paid by three townships, and the property on which it is levied is $24700 in the first, $37250 in the second, and $43350 in the third. What sum is paid by each?

Amount of property on which the rate is levied is $105300.
Then $105300 has to pay a rate of $4212;
∴ $1 " $\$\frac{4212}{105300}$;
∴ $24700 " $\$\frac{24700 \times 4212}{105300}$, or $988;
$37250 " $\$\frac{37250 \times 4212}{105300}$, or $1490;
$43350 " $\$\frac{43350 \times 4212}{105300}$, or $1734.

Ex. (4). Divide $1000 among *A*, *B*, and *C*, so that *A* may have half as much again as *B*, and *B* a third as much again as *C*.

Representing *C*'s part by 1,
B's part will be $1\frac{1}{3}$,
and *A*'s part will be $1\frac{1}{3} + \frac{1}{2}$ of $1\frac{1}{3} = 2$;
and, therefore, the parts are to be as the numbers 2, $1\frac{1}{3}$, 1;
∴ all the shares $= 2 + 1\frac{1}{3} + 1 = 4\frac{1}{3}$ times *C*'s share.

$4\frac{1}{3}$ times *C*'s share = $1000;

$$C\text{'s share} = \frac{\$1000}{4\frac{1}{3}} = \$230{\cdot}769\,;$$

B's share $= \frac{4}{3}$ of *C*'s = $307·692.
A's share = 2 times *C*'s = $461·538.

Ex. (5). Divide the number 237 into three parts, such that 3 times the first may be equal to 5 times the second and to 8 times the third.

Take the first part as the *unit;* then by the question the second part will be $\frac{3}{5}$ of the first, and the third will be $\frac{3}{8}$ of the first.

$$\begin{aligned}
&\text{Sum of the parts} = 1 + \tfrac{3}{5} + \tfrac{3}{8} = \tfrac{79}{40} \text{ times the first.}\\
&\text{Hence } \tfrac{79}{40} \text{ times the 1st} = 237.\\
&\qquad\text{The 1st} = 237 \div \tfrac{79}{40} = 120.\\
&\qquad\text{The 2nd} = \tfrac{3}{5} \text{ of 1st} = \tfrac{3}{5} \text{ of } 120 = 72.\\
&\qquad\text{The 3rd} = \tfrac{3}{8} \text{ of 1st} = \tfrac{3}{8} \text{ of } 120 = 45.
\end{aligned}$$

Ex. (6). Divide \$3400 among *A*, *B*, and *C*, so that *A* may have \$800 more than $\frac{2}{3}$ of *B*'s share, and *B* \$600 less than $\frac{3}{4}$ of *C*'s share.

Representing *C*'s share by 1, then

$$\begin{aligned}
B\text{'s share} &= \tfrac{3}{4} \text{ of } C\text{'s share} - \$600\,;\\
A\text{'s share} &= \tfrac{2}{3} \text{ of } B\text{'s share} + \$800\\
&= \tfrac{2}{3}\left(\tfrac{3}{4} \text{ of } C\text{'s} - \$600\right) + \$800\\
&= \tfrac{1}{2} \text{ of } C\text{'s} + \$400.\\
\text{Sum of all the shares} &= C\text{'s} + \tfrac{3}{4}\,C\text{'s} - \$600 + \tfrac{1}{2}\,C\text{'s} + \$400\\
&= \tfrac{9}{4}\,C\text{'s} - \$200\,;\\
\therefore \tfrac{9}{4}\,C\text{'s} - \$200 &= \$3400.\\
\tfrac{9}{4}\,C\text{'s} &= \$3400 + \$200\\
&= \$3600.\\
C\text{'s} &= \$1600.\\
B\text{'s} &= \tfrac{3}{4} \text{ of } \$1600 - \$600\\
&= \$600.\\
A\text{'s} &= \tfrac{1}{2} \text{ of } \$1600 + \$400\\
&= \$1200.
\end{aligned}$$

Examples. (cxi.)

(1) Divide \$60 into two parts proportional to 11 and 9.

(2) Divide \$2500 into parts proportional to 2, 3, 7, 8.

(3) Divide \$8470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

(4) Gunpowder is made of saltpetre, sulphur, and charcoal, in parts proportional to 75, 10, and 15; how many pounds of each are contained in 12 cwt. of gunpowder?

(5) The sides of a triangle are as 3, 4, 5, and the sum of the lengths of the sides is 480 yards; find the sides.

(6) Divide \$640 among *A*, *B*, and *C*, so that *A* may have three times as much as *B*, and *C* as much as *A* and *B* together.

(7) Divide 100 apples among three boys, so that the first may receive 7 as often as the second receives 8, and the third may receive 5 as often as the second receives 4.

(8) A bankrupt owes £272 10*s.* to *A*, £354 5*s.* to *B*, and £490 10*s.* to *C*; his assets are £418 19*s.* 4½*d.* What will each of the creditors receive?

(9) A force of police 1921 strong is to be distributed among 4 towns in proportion to the number of inhabitants in each, the population being 4150, 12450, 24900, and 29050 respectively. Determine the number of men sent to each.

(10) Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences, and fourpences.

(11) A piece of land of 200 acres is to be divided among four person, in proportion to their rentals from surrounding property; supposing these rents to be £500, £350, £800, and £90, how many acres must be allotted to each?

(12) Divide £2 5*s.* among *A*, *B*, and *C*, so that for each threepenny piece received by *A*, *B* may receive a fourpenny piece, and that there are as many shillings in the sum received by *C* as there are sixpences in the sum received by *B*.

(13) Divide $10.40 among 5 men, 7 women, and 14 boys, so that each woman may have $\frac{3}{7}$ of each man's share, and each boy $\frac{2}{5}$ of each woman's share.

(14) A number of men, women, and children are in the proportions 2, 3, 5; divide $517.65 among them, so that the shares of a man, a woman, and a child may be proportional to 3, 2, 1, there being 9 women.

(15) A man left his property to be divided among his 3 sons in proportion to their ages, which are 20, 18, and 12 years. The share of the youngest is $1440. What was the value of the property?

(16) Divide $5000 among *A*, *B*, and *C*, so that *A* may get $300 less than $\frac{5}{9}$ of *C*'s share, and *C* $800 more than $\frac{2}{5}$ of *B*'s share. What are the shares of each?

(17) Divide $5000 among *A*, *B*, *C*, and *D*, so that *A* may get $\frac{2}{3}$ of *B*'s share, and $250; *B*, $200 more than $\frac{4}{5}$ of *C*'s share; *C*, $100 less than $\frac{9}{10}$ of *D*'s share. What are the shares of each?

(18) The sum of three fractions is $\frac{183}{242}$; and 22 times the first, 23 times the second, and 24 times the third give equal products. Find the fraction.

(19) Divide the simple interest on $1171 for 13 years at 6 per cent. in parts which shall have the same relation as $\frac{5}{8}$, $\frac{7}{9}$, $\frac{9}{10}$, $\frac{5}{12}$, $\frac{8}{15}$.

(20) Of the boys in a school one-third are over 15 years of age, one-third between 10 and 15. A legacy of $400 can be exactly divided amongst them by giving $\$\frac{1}{2}$ to each boy over 15, $\$\frac{1}{3}$ to each between 10 and 15, and $\$\frac{1}{6}$ to each of the rest. How many boys are there in the school?

PARTNERSHIP.

203. When persons unite to carry on any particular branch of business the connection so formed is called a PARTNERSHIP. The method of working questions in Partnership is the same as that explained in the preceding article.

Ex. (1). *A*, *B*, and *C* entered into partnership to carry on a mercantile business for two years. *A* puts in \$9000, *B* \$6000, and *C* \$3000. They gained \$4500. What is each one's share of the gain?

The whole capital invested is \$18000.

Then \$18000 gains \$4500;

$\therefore$ \$1 gains $\$\frac{4500}{18000}$, or $\$\frac{1}{4}$.

\$9000 gains $\$\frac{9000}{4} = \$2250$.

\$6000 gains $\$\frac{6000}{4} = \$1500$.

\$3000 gains $\$\frac{3000}{4} = \$750$.

Hence *A*'s share of the gain is \$2250; *B*'s, \$1500; and *C*'s, \$750.

Ex. (2). *A*, *B*, and *C* entered into partnership for trading. *A* put in \$600 for 4 months, *B* \$400 for 5 months, and *C* \$200 for 6 months. They gained \$980. What was each man's share of the gain?

\$600 for 4 months = \$2400 for 1 month.
\$400 " 5 " = \$2000 " "
\$200 " 6 " = \$1200 " "

The whole capital is equivalent to \$5600 for 1 month.

Then \$5600 gains \$980;

$\therefore$ \$1 gains $\$\frac{980}{5600} = \$\frac{7}{40}$.

\$2400 gains $\$\frac{2400 \times 7}{40} = \$420$.

\$2000 gains $\$\frac{2000 \times 7}{40} = \$350$.

\$1200 gains $\$\frac{1200 \times 7}{40} = \$210$.

$\therefore$ *A*'s share is \$420, *B*'s \$350, and *C*'s \$210.

Examples. (cxii.)

(1) Two men jointly purchased a house for \$2592, the first contributing \$864 towards the purchase and the second

$1728. They afterwards rented the house for $132.75 annually. What share of the rent ought each to have?

(2) *A*, *B*, and *C* jointly rented a pasture for 3 months, agreeing to pay $22.50 for the use of the same. *A* put in 6 horses, *B* put in 18 cows, and *C* 90 sheep. Considering each horse as equivalent to 2 cows, and each cow as equal to 3 sheep, what part of the rent ought each to pay?

(3) *A*, *B*, and *C* entered into partnership for speculating in cotton, their joint capital being $25780, of which *A* furnished $\frac{2}{5}$, *B* contributed $\frac{3}{4}$ of the remainder, and *C* the balance. Their clear profit was 20 per cent. of the original investment. How should it be divided?

(4) *A* starts a business with a capital of $2400 on the 19th of March, and on the 17th of July admits a partner *B* with a capital of $1800. The profits amount to $943 by the 31st of December. What is each person's share?

(5) *D* and *E* enter into partnership; *D* puts in $40 for 3 months, and *E* $75 for 4 months. They gain $70. What is each man's share in the gain?

(6) *A*, *B*, and *C* are partners; *A* puts in $500 for 7 months, *B* $600 for 8 months, and *C* $900 for 9 months. The profit is $410. What is the share of each?

(7) Three graziers hire a pasture for their common use, for which they pay $106. One puts in 10 oxen for 3 months, another 12 oxen for 4 months, and the third 14 oxen for 2 months. How much of the rent should each pay?

(8) Two men complete in a fortnight a piece of work for which they are paid $29.55. One of them works alternately 9 hours and 8 hours a day. The other works 9½ hours for 5 days in the week, and does nothing on the remaining day. What part of the sum should each receive?

(9) *A* and *B* begin to trade in partnership. *A* puts in $400 at first, and $500 at the end of 2 months. *B* puts in $300 at first, and $600 at the end of three months. The profit at the end of the year is $470. How should this be divided?

(10) Johnston and Wilson formed a co-partnership in business for 2 years. Johnston at first contributed $3000 to joint capital, and at the end of 12 months put in $1500 more. Wilson at first put in $3500, but at the end of 15 months from the beginning withdrew $1000. At the end of the first year they admitted Miller into the firm, he contributing $2250. Their joint profits were $1248. How ought this to be apportioned?

(11) *A* and *B* rent a field for $88.20. *A* puts in 10 horses for $1\frac{1}{2}$ months, 30 oxen for 2 months, and 100 sheep for $3\frac{1}{4}$ months; *B*, 40 horses for $2\frac{1}{2}$ months, 50 oxen for $1\frac{1}{4}$ months, and 115 sheep for 3 months. If the food consumed in the same time by a horse, an ox, and a sheep be as the numbers 3, 2, 1, what proportion of the rent must each pay?

(12) A person in his will directed that $\frac{1}{2}$ his property should be given to *A*, $\frac{1}{3}$ to *B*, $\frac{1}{4}$ to *C*, and $\frac{1}{5}$ to *D*. Shew that this disposition cannot be fulfilled. If his property amount to $1886.50, dispose of it so that their shares may have to one another the relation he intended.

(13) *A*, *B*, and *C* had each a cask of rum containing respectively 36, 54, and 78 gallons. They blended their rum, and then refilled their casks from the mixture. How much of the rums of *A* and *B* are contained in *C*'s cask?

(14) *A* rents a house for $187.20; at the end of 4 months he takes in *B* as a co-tenant, and they admit *C* in like manner for the last $2\frac{1}{2}$ months. What portion of the rent must each of them pay?

PARTNERSHIP SETTLEMENTS.

204. When a partnership is dissolved, either by mutual consent or by limitation of contract, the adjustment of the proceeds between the members is called a Partnership Settlement. If the Resources are found to exceed the Liabilities, the difference is termed Net Capital; if the Liabilities exceed the Resources, the difference is Net Insolvency. The investment of the partners is the Net Capital at commencement. If the net capital at closing exceeds the net capital at commencement, the difference is the Net Gain; if the opposite, Net Loss. This net gain, or net loss, is then shared between the partners in accordance with the original agreement between them. This division is frequently not made in exact proportion to the amount invested; sometimes the skill of one partner is considered equal to the capital of another; sometimes a stated salary is allowed each partner according to his ability or reputation; and sometimes, where unequal amounts are invested, interest is allowed each partner on his investment; but whatever allowance is made *such allowance must be classed as a liability and go to reduce the gain.*

Ex. (1). *A* and *B* are partners. The following is a statement of their property and debts: they have Cash, $3240; Merchandise, $2575; Bills Receivable, $860; J. Brown owes on account, $375. They owe on Bills Payable, $1250, and J. Jones on account, $370. *A* invested at commencing $2500, and drew out, during business, $560. *B* invested $2500, and drew out, during business, $280. They agreed to share equally in gains and losses. What was the net gain? and what was the net capital of each at closing?

RESOURCES AND LIABILITIES.

Dr.	Cr.
$3240	$1250
2575	370
860	
375	$1620

7050 Resources at Closing.
1620 Liabilities "

5430 Present Worth of Firm.
4160 Credit excess of Ownership.

1270 Net Gain.

635 *A*'s share of net gain.
635 *B*'s " "

OWNERSHIP.

Dr.		Cr.
$560	*A* withdrew.	$2500
280	*B* "	2500
$840	Total investment	$5000
	" withdrawn	840
	Firm's net investment	4160

Hence *A*'s present net capital = $2500 − $560 + $635 = $2575, and *B*'s present net capital = $2500 − $280 + $635 = $2855.

Examples. (cxiii.)

(1) *A* and *B*, having conducted business 1 year as partners, close with the following resources and liabilities: They have Cash, $3456; Mdse., $2120; Bills Receivable, $1874; E. Corby owes $630. They owe on Bills Payable, $3250; W. Smith on account, $346. *A* invested $1500, and withdrew $175. *B* invested $1500, and drew out $315. What is the net gain, and net capital of each at closing?

(2) *A* and *B* close business as follows: They have Cash, $1424; Mdse., $1562; Fixtures, $383; Mortgages Receivable, $3485; Bills Receivable, $826. They owe on Bills Payable, $2450; on accounts, $1240. *A* invested $6000, and a debt for $1000 was assumed by the firm, and paid during business. He drew out $685; and is allowed interest on capital invested, $420. *B* invested $4000, and drew out $1860, and is allowed interest on capital, $280. *A* is to share $\frac{3}{5}$ and *B* $\frac{2}{5}$ of gains and losses. What is the net loss? What is the net capital of each?

(3) *A* and *B* close business, and wish to know the financial standing of each. They have cash, $2263, and Real

Estate worth $5000. They owe on Mortgages, $3846; on Notes, $4462; on Personal Accounts, $675. *A* invested $6000, and drew out $2860. *B* invested $4000, drew out $5560, and is allowed for extra services $250. *A* shares $\frac{3}{5}$ and *B* $\frac{2}{5}$ of the gains and losses. What is the net loss? What is the financial standing of each?

XXIX. Alligation.

205. Alligation is the process by which we find the mean or average price of a compound when we mix or unite two or more articles of different values.

Ex. (1). A merchant has brown sugar worth 8 cents per pound, New Orleans worth 9 cents, and refined sugar worth 14 cents. How many pounds of each kind must he use in order to form a mixture worth 12 cents per pound?

By selling the mixture at 12 cents per lb., we see that 8 cents (brown) gains 4 cents on 1 lb.; ∴ 1 cent is gained on $\frac{1}{4}$ lb.

9 cents (New Orleans) gains 3 cents on 1 lb.; ∴ 1 cent is gained on $\frac{1}{3}$ lb.

14 cents (refined) loses 2 cents on 1 lb.; ∴ 1 cent is lost on $\frac{1}{2}$ lb.

Now with every cent *gain* he must combine a cent *loss*, hence he must have

$$\left.\begin{array}{l}\frac{1}{4}\text{ lb. at } 8\text{ cts.}\\ \frac{1}{2}\text{ lb. `` } 14\text{ cts.}\\ \frac{1}{3}\text{ lb. `` } 9\text{ cts.}\\ \frac{1}{2}\text{ lb. `` } 14\text{ cts.}\end{array}\right\} = \left\{\begin{array}{l}3\text{ lbs. at } 8\text{ cts.}\\ 6\text{ lbs. `` } 14\text{ cts.}\\ 4\text{ lbs. `` } 9\text{ cts.}\\ 6\text{ lbs. `` } 14\text{ cts.}\end{array}\right.$$

He must, therefore, have 3 lbs. brown sugar, 4 lbs. New Orleans, and 12 lbs. refined.

We may show that these quantities will make the mixture required, as follows:

3 lbs. at 8 cts. per lb. = 24 cts.
4 lbs. " 9 cts. " = 36 cts.
12 lbs. " 14 cts. " = 168 cts
— —
19 lbs. = whole mixture. 228 cts. = value of mixture.

Hence, if 19 lbs. be worth 228 cents,

1 lb. is worth $\frac{228}{19} = 12$ cts.

Or we may reason thus: The 1 ct. *gained* on the $\frac{1}{4}$ lb. of brown exactly balances the 1 ct. *lost* on the $\frac{1}{2}$ lb. of the refined. Hence he must take $\frac{1}{4}$ lb. of the brown and $\frac{1}{2}$ lb. of the refined, or 2 lbs. of the one and 4 lbs. of the other.

Similarly, for every 2 lbs. of New Orleans, there must be 3 lbs. of refined. As 4 lbs. of refined were required to balance the brown, and 3 lbs. of the refined to balance the New Orleans, there must be 7 lbs. of the refined in the compound. Therefore the respective quantities are 2 lbs. brown, 2 lbs. New Orleans, and 7 lbs. refined.

From the above we see that in examples of this kind a variety of answers may frequently be obtained, and all of them may be correct. To ascertain their correctness we resort to the method of proof given in this example.

206. From the above analysis we derive an easy practical method of solving such questions.

Ex. (2). How much sugar at 10, 13, 15, 17, and 18 cents per pound must be taken to make a mixture worth 16 cents per lb.?

We proceed as follows:

Differences.	16	
	—	
6	10	1
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8
2	18	4, 3, 2, 1

Write down the prices in a vertical column, and place the *differences* between these prices and the *mean* in a second vertical column to the left. Now take 1 @ 10, 1 @ 13, and 1 @ 15, (the lowest that can be taken); this would represent a *loss* of 10 as compared with the *mean;* and this loss must be balanced by taking the necessary multiples of the differences 1 and 2, which represent *gain* as compared with the mean.

It is seen that this loss of 10 can be made up in *four* ways: by 2 @ 17, 4 @ 18, 4 @ 17, 3 @ 18, 6 @ 17, 2 @ 18, 8 @ 17, and 1 @ 18.

Other combinations may be made, as *e.g.:*

6	10	1
3	13	1
1	15	2
..	..	..
1	17	1, 3, 5, 7, 9
2	18	5, 4, 3, 2, 1

Here 1 @ 10, 1 @ 13, and 2 @ 15, give *loss* of 11, which can be made up by multiples of the differences 1 and 2 (opposite 17 and 18) in *five* ways, as indicated.

Also,

6	10	1
3	13	2
1	15	1
..	..	..
1	17	1, 3, 5, 7, 9, 11
2	18	6, 5, 4, 3, 2, 1

Where 1 @ 10, 2 @ 13, and 1 @ 15 give 13 loss, which may be made up in *six* different ways.

Again,

6	10	2
3	13	1
1	15	1
..	..	..
1	17	2, 4, 6, 8, 10, 12, 14
2	18	7, 6, 5, 4, 3, 2, 1

Where 2 @ 10, 1 @ 13, and 1 @ 15 give *loss* of 16, which may be made up in *seven* ways.

Also,

6	10	1
3	13	1
1	15	3
..	..	..
1	17	2, 4, 6, 8, 10
2	18	5, 4, 3, 2, 1

Where 1 @ 10, 1 @ 13, and 3 @ 15 give *loss* of 12, which may be made up in *five* ways; and thus an indefinate number of combinations may be formed.

It should be observed that if the differences opposite the prices *less* than the *mean* are together greater than the sum of the other differences (as in the example), we assign numbers (the *lowest* possible) to the prices less than the mean FIRST, and *vice versa; e.g.* of the latter case:—

How much coffee at 25, 24, 23, 22, 21, 19, 18, and 17 cents per pound must be taken to make a mixture worth 20 cents per pound?

Diff's.	20	
	—	
3	17	4, 3, 2, 1, 1, 2
2	18	1, 2, 3, 5, 4, 2, &c.
1	19	1, 2, 3, 2, 4, 3
..	..	..
1	21	1
2	22	1
3	23	1
4	24	1
5	25	1

Here the sum of the differences in excess of the mean is greater than that of the differences below the mean; we therefore assign *first* numbers to the prices which are greater than the mean, viz., 1 @ 21, 1 @ 22, 1 @ 23, 1 @ 24, and 1 @ 25; this gives a *gain* of 15, which may be balanced as above by 1 @ 19, 1 @ 18, and 4 @ 17; or by 2 @ 19, 2 @ 18, and 3 @ 17, &c., &c.

Ex. (3). A grocer has 12 lbs. of brown sugar, worth 10 cents per pound, which he wishes to mix with clarified sugar worth 16 cents per pound, so that the mixture may be worth 14 cents per pound. How many pounds of clarified sugar must he take?

Proceeding as in the previous examples, without reference to the *quantity* of the brown sugar, we find that there must be 1 lb. brown sugar to 2 lbs. clarified sugar. But as 12 lbs. of brown sugar are required, we must multiply each of these quantities by 12 in order that the gain and loss may be equal. We shall therefore have 12 × 2 = 24 lbs. of clarified sugar.

Ex. (4). A grocer wishes to mix 20 lbs. of sugar worth 9 cents per pound, and 10 lbs. worth 12 cents per pound, with clarified sugar worth 15 cents, so that the compound may sell for 13 cents. How much of the clarified must he take?

20 lbs. at 9 cents = $1.80
10 lbs. at 12 cents = $1.20

30 $3.00

Then, if 30 lbs. is worth $3,
1 lb. " $\$\frac{3}{30}$ = 10 cents.

The value of 1 lb. of the mixture is, therefore, worth 10 cents. The question may then be read as follows:

How many pounds of clarified sugar, worth 15 cents per pound, must be mixed with 30 lbs. of another kind of sugar, worth 10 cents per pound, so that the mixture may be sold for 13 cents per pound?

The question in this form has already been fully explained.

Ex. (5). A merchant has West India sugar worth 8 cents per pound, and New Orleans sugar worth 13 cents. He wishes to combine these so as to make a barrel containing 175 lbs., which he may sell at 11 cents per pound. How many pounds of each kind must he take?

Solving the question without reference to the 175 lbs., we find that 2 lbs. of West India sugar and 3 lbs. of New Orleans sugar will form a mixture worth 11 cents per pound. Adding these quantities we find that they form a mixture of 5 lbs. But the required mixture is to contain 175 lbs., or 35 times 5. We shall therefore have

35 × 2 lbs. = 70 lbs. West India sugar.
35 × 3 lbs. = 105 lbs. New Orleans sugar.

Examples. (cxiv.)

(1) What quantities of coffee, worth 23 and 35 cents respectively per pound, must be mixed together so that the compound may be sold for 30 cents a pound?

(2) What quantity of oats at 35 cents per bushel, rye at 60 cents per bushel, and barley at 80 cents, must be taken to form a mixture worth 55 cents per bushel?

(3) How much tea, worth respectively 55 cents and 75 cents per pound, must be mixed with 30 lbs. worth 90 cents per pound, in order that the compound may be sold for 70 cents per pound?

(4) How much water will it require to dilute 60 gallons of alcohol, worth $1.50 per gallon, so that the mixture may be worth only $1.20 per gallon?

(5) How many gallons of kerosene oil, worth 60 cents per gallon, must be mixed with 12 gallons of coal oil, worth 36 cents, and 8 gallons of Aurora oil, worth 56 cents, so that the compound may be sold for 50 cents per gallon?

(6) A farmer has 16 bushels of corn, worth 48 cents per bushel, and 12 bushels of oats at 34 cents per bushel, which he wishes to mix with rye at 60 cents and barley at 80 cents, in order to sell the compound at 56 cents per bushel. How many bushels of rye and barley will be required?

(7) A confectioner mixes three different qualities of candy worth respectively 14 cents, 18 cents, and 30 cents per pound, so as to make a box of 84 lbs.; how many pounds of each sort must he take so as to sell the compound at an average price of 24 cents per pound?

(8) A farmer has three different qualities of wool, worth respectively 33 cents, 37 cents, and 45 cents per pound. He wishes to make up a package amounting to 120 lbs., which he can afford to sell at 39 cents per pound. How many pounds of each kind must he take?

XXX. Exchange.

207. The term *Exchange* is here used for giving or receiving in the money of one country a sum equal in value to a sum of money of another country. For example, if an English merchant pays to a French merchant 100 sovereigns and receives in return 2500 francs, it is a case of Exchange.

In countries which carry on considerable trade with each other, the debts reciprocally due from the one to the other are generally nearly equal. In England there is always a large number of persons indebted to others in America, and likewise a large number in America owing money in England. Now if coin, or specie, as it is called, were sent from England to pay the debts in America, and from America to England, the specie would have to be transmitted twice, and would necessarily involve risk, loss of interest, and expense of transportation. To avoid this risk, &c., Bills of Exchange are used to liquidate debts reciprocally due between two places without any actual transmission of money.

208. A Bill of Exchange is a written order, addressed to a person in a distant place, directing him to pay a certain sum of money, at a specified time, to another, or to his order. The person who signs the bill is called the Drawer, or Maker. The person to whom it is addressed is the Drawee, and after the Drawee agrees to pay it, and writes "accepted" with his signature and the date across the face of it, he becomes the Acceptor. The person to whom the money is to be paid is the Payee; if he transfers payment to another he endorses it, *i.e.*, he writes his name across the back of it and becomes responsible for its payment in case the Drawee fails to make payment.

209. The *Par of Exchange* between two countries denotes the nominal value of a unit of coinage in one country, as estimated in terms of a unit of coinage in the other country.

As we supposed the exports from England and America to be equal, creditors in England will be as anxious to sell bills on America as debtors to buy them, and the exchange will deviate but slightly from the *par* of Exchange. But if the exports from America are in excess of those from England, or the *Balance of Trade* is in favor of America, the claims of America in England will exceed its liabilities, and the English will give more than the par value of such bills to avoid the cost of transmitting specie; and on the other hand, the exporters in America, not finding sufficient purchasers for all their bills on England, will sell them at less than their par value. Now the real rate of exchange, depending on the balance of trade, is called the COURSE OF EXCHANGE; and it is at a *premium* or *discount*, according as it is above or below the par of exchange. Of course no one would give a premium greater than the cost of transmitting specie. But if the balance of trade is against England as regards America, but in favor of England as against France, the English merchant may find it advantageous to remit to France, and then for France to remit to America, and this mode is adopted when the course of exchange by this circuitous route is less than the direct course of exchange. The finding the course of exchange between two places, by comparing the courses of exchange between them and one or more intervening places, is called ARBITRATION OF EXCHANGE. The arbitration is *Simple* when only *one* place intervenes, and *Compound* when more than one.

Bills of Exchange are usually drawn in sets, three bills constituting a set. These are distinguished from one another by being called the *first*, *second*, and *third* of exchange. These are forwarded by different routes so as to guard against delay or their being lost. The first that arrives is paid, and the other two become void.

210. By Act of Parliament the value of the pound sterling was fixed at $\$4\frac{4}{9}$. This was much below its intrinsic value, which is now fixed at $\$4.86\frac{2}{3}$. The rates of exchange which are quoted in commercial papers are still calculated at a certain per cent. *on the old par of exchange*. Exchange is at par between Great Britain and Canada when it is at a premium of $9\frac{1}{2}$ per cent., for $\$4\frac{4}{9}$ increased by $9\frac{1}{2}$ per cent. equals $\$4.86\frac{2}{3}$.

FORM OF DRAFT OR INLAND BILL OF EXCHANGE.

$1000. Toronto, July 12, 1889.

At ten days' sight, pay to the order of Adam Miller & Co., One Thousand Dollars, value received, and charge to account of

Stamp.

W. E. Jones.

To J. Smith & Co.,
Montreal.

FORM OF A FOREIGN BILL OF EXCHANGE.

Exchange for £200. Toronto, July 12, 1889.

Three days after sight of this first of exchange (second and third of same date and tenor unpaid), pay to Adam Miller & Co., or order, Two Hundred Pounds Sterling, value received, and charge the same to the account of

Stamp.

W. B. Taylor.

To Geo. H. Simpson,
Banker, London.

FOREIGN MONEYS OF ACCOUNT.

With the par value of the unit, as fixed by commercial usage, expressed in dollars and cents.

Austria.—60 kreutzers = 1 florin (silver) = ...	$·485
Belgium.—100 cents = 1 guilder or florin; 1 guilder (silver) = ...	·40
Brazil.—1000 rees = 1 milree = ...	·828
British India.—12 pice = 1 anna; 16 annas = 1 Company's rupee = ...	·445
Buenos Ayres.—8 rials = 1 dollar currency, mean value = ...	·93
Canton.—10 cash = 1 candarines; 10 cand. = 1 mace; 10 mace = 1 tael = ...	1·48
Cuba, Columbia, and Chili.—8 rials = 1 dollar = ...	1·00
Denmark.—12 pfenning = 1 skilling; 16 skilling = 1 marc; 6 marcs = 1 rix-dollar = ...	·52
France.—10 centimes = 1 decime; 10 decime = 1 franc = ...	·186
Greece.—100 lepta = 1 drachme; 1 drachme (silver) =	·166
Holland.—100 cents = 1 florin or guilder; 1 florin (silver) = ...	·40

Hamburg.—12 pfenning = 1 schilling ; 16 schilling = 1 marc ; 3 marcs = 1 rix-dollar = ... ·84
Mexico.—8 rials = 1 dollar = ... 1·00
Portugal.—400 rees = 1 cruzado ; 1000 rees = 1 milree or crown = ... 1·12
Prussia.—12 pfennings = 1 grosch (silver) ; 30 groschen = 1 thaler or dollar = ... ·69
Russia.—100 copecks = 1 ruble (silver) = ... ·78
Sweden.—48 skillings = 1 rix-dollar specie = ... 1·06
Spain.—34 maravedis = 1 real of old plate* = ... ·10
8 reals = 1 piastre ; 4 piastres = 1 pistole of exchange ; 20 reals vellon = 1 Spanish dollar = ... 1·00
Turkey.—3 aspers = 1 para ; 40 paras = 1 piastre (variable), about .... ·096
Venice.—100 centisimi = 1 lira = ... ·186

VALUE OF FOREIGN COINS.

Guinea	$5.10
Sovereign of Great Britain	4.86⅔
Crown of England	1.216
Half-crown of England	.608
Shilling of England	.24⅓
Franc of France	.18½
Five-franc piece of France	.93
Livre Tournois of France	.18½
Forty-franc piece of France	7.66
Crown of France	1.06
Louis-d'Or of France	4.56
Florin of the Netherlands	.40
Guilder of the Netherlands	.40
Florin of South Germany	.40
Thaler or Rix-Dollar of Prussia and North Germany	.69
Rix-Dollar of Bremen	.78¾
Florin of Prussia	.22¼
Marc-Banco of Hamburg	.35
Florin of Austria	.48½
Florin of Saxony, Bohemia, and Trieste	.48
Florin of Nuremburg and Frankfort	.40
Rix-Dollar of Denmark	1.00
Specie-Dollar of Denmark	1.05
Dollar of Sweden and Norway	1.06
Milree of Portugal	1.12
Milree of Madeira	$1.00
Milree of Azores	.83⅓
Real-Vellon of Spain	.05
Real-Plate of Spain	.10
Pistole of Spain	3.97
Rial of Spain	.12
Pistareen	.18
Cross Pistareen	.16
Ruble (silver) of Russia	.75
Imperial of Russia	7.83
Doubloon of Mexico	15.60
Half-Joe of Portugal	8.53
Lira of Tuscany and Lombardy	.16
Lira of Sardinia	.186
Ounce of Sicily	2.40
Ducat of Naples	.80
Crown of Tuscany	1.05
Florence Livre	.15
Genoa Livre	.18¾
Geneva Livre	.21
Leghorn Dollar	.90
Swiss Livre	.27
Scudo of Malta	.40
Turkish Piastre	.05
Pagoda of India	1.84
Rupee of India	.44½
Tael of China	1.48

Ex. (1). A broker in Toronto sold a bill of exchange on London, the face of which was for £750 8*s.* ; what did he receive for the bill, exchange being quoted at 110¼ ?

---

*The old plate real is not a coin, but is the denomination in which exchanges are usually made.

Since $£1 = \$4\frac{4}{9} \times 1·10\frac{1}{4}$, *i.e.*, $\$4\frac{4}{9}$ increased by $10\frac{1}{4}$ per cent.,

$\therefore £750·4 = \$750·4 \times 4\frac{4}{9} \times 1·10\frac{1}{4}$

$= \$3676.96$;

$\therefore$ he got \$3676.96 for the bill.

Ex. (2). What is the value in English money of 4528·7 francs, when the course of exchange between Paris and London is at 25·3 francs per pound sterling?

Since 25·3 francs = £1,

1 franc $= £\frac{1}{25·3}$;

$\therefore$ 4528·7 francs $= £\frac{4528·7}{25·3}$, or £179.

Ex. (3). A merchant pays a debt of 4379 milrees in Portugal with £971 11*s.* $9\frac{3}{4}$*d.* What is the course of exchange in pence per milree?

£971 11*s.* $9\frac{3}{4}$*d.* = 932727 farthings;

Then since 4379 milrees = 932727 farthings,

1 milree $= \frac{932727}{4379}$ farthings, or 213 farthings;

$\therefore$ the course of exchange is $53\frac{1}{4}$ pence per milree.

Ex. (4). If 11·65 Dutch florins are given for 24·42 francs, 352 florins for 407 marks of Hamburg, and $58\frac{1}{4}$ marks for 32 silver rubles of St. Petersburg; how many francs should be given for 932 silver rubles?

Here 1 silver ruble $= \frac{58·25}{32}$ marks,

1 mark $= \frac{352}{407}$ florins,

1 florin $= \frac{24·42}{11·65}$ francs;

$\therefore$ 1 silver ruble $= \frac{58·25}{32} \times \frac{352}{407} \times \frac{24·42}{11·65}$ francs, or 3·3 francs;

$\therefore$ 932 silver rubles $= 932 \times 3·3$ francs, or 3075·6 francs.

Ex. (5). A New York merchant remits 27940 florins to Amsterdam by way of London and Paris, at a time when the exchange of New York on London is \$4·885 for £1, of London on Paris is 25·4 francs for £1, and of Paris on Amsterdam is 212 francs for 100 florins; $\frac{1}{8}$ per cent. brokerage being paid in London and in Paris, how many dollars will purchase the bill of exchange?

Since 100 florins = 212 francs,

∴ 1 florin = $\frac{212}{100}$ francs.

But to buy a bill of 100 fr. requires a bill of 100⅛ fr.;

∴ to buy a bill of 1 fr. requires a bill of $\frac{801}{800}$ fr.

Again, 25·40 fr. = £1,

∴ 1 fr. = £$\frac{1}{25{\cdot}40}$;

but to buy a bill of £100 requires £100⅛;

∴ " £1 " £$\frac{801}{800}$.

Again, £1 = $4·885;

$$\therefore 1 \text{ florin} = \$\tfrac{212}{100} \times \tfrac{801}{800} \times \frac{1}{25{\cdot}40} \times \tfrac{801}{800} \times 4{\cdot}885;$$

$$\therefore 27940 \text{ florins} = \$\frac{27940 \times 212 \times 801 \times 801 \times 4{\cdot}885}{100 \times 800 \times 25{\cdot}40 \times 800}$$

= $11420·317, sum required.

Ex. (6). A merchant of Toronto wishes to transmit 2400 marcs banco to Hamburg. He finds exchange between Toronto and Hamburg to be 35 cents for 1 marc. The exchange between Toronto and London is $4.83 for £1; that between London and Paris is 26 francs for £1; and that of Paris on Hamburg is 47 francs for 25 marcs. By what way should the Toronto merchant remit?

By direct exchange 1 marc = $0.35;

∴ 2400 marcs = $2400 × 0·35

= $840.

By circuitous exchange 25 marcs = 47 francs;

∴ 1 marc = $\frac{47}{25}$ francs;

but 26 francs = £1;

∴ 1 franc = £$\frac{1}{26}$,

and £1 = $4.83;

$$\therefore 1 \text{ marc} = \$4.83 \times \tfrac{1}{26} \times \tfrac{47}{25};$$

$$\therefore 2400 \text{ marcs} = \$\frac{2400 \times 4{\cdot}83 \times 47}{26 \times 25}$$

= $838.19.

By direct exchange the merchant pays $840 for his bill of exchange, and only $838.19 by the circuitous mode;

∴ the circuitous mode is better by $1.81.

Examples. (cxv.)

(1) When $7300 are paid in Toronto for a bill of exchange on Liverpool for £1500, how was sterling exchange quoted?

(2) What will be the cost of a bill on Paris for 236874 francs, exchange being 5·3 francs to the dollar?

(3) If £1 be worth 12 florins, and also be worth 25 francs 56 centimes, how many francs and centimes is one florin worth?

(4) If £1 be worth $25\frac{1}{2}$ francs, and be also worth 2244 copecks in Russian money, what is the value of the napoleon in Russian copecks? (N.B.—20 francs = 1 napoleon).

(5) The French franc is divided into 100 centimes, and the Frankfort florin into 60 kreutzers. When the pound sterling is worth 25·50 francs in Paris, and 11 fl. 54 kr. at Frankfort, what is the worth of the napoleon in florins and kreutzers?

(6) In 1869 exchange on Paris was quoted in New York at 5·$12\frac{1}{2}$ francs to the dollar and gold was at $135\frac{1}{3}$. If a New York merchant owed 12669 francs in Havre, how much would he have to pay in greenbacks for a bill of exchange to cover his indebtedness?

(7) A merchant in Toronto wishes to remit $2767.80 to Manchester, England, exchange being at 108; what will be the face of his bill in pounds, shillings, and pence?

(8) Find the par of exchange between the U.S. gold eagle, weighing 258 grains $\frac{9}{10}$ fine, and the sovereign, of which 1869 weigh 40 lbs. of gold $\frac{11}{12}$ fine.

(9) Find the arbitrated rate of exchange between London and Paris when the course of exchange between London and Amsterdam is 12·$16\frac{1}{4}$ florins for £1, and between Amsterdam and Paris $209\frac{1}{4}$ francs for 100 florins.

(10) If a merchant buys a bill in London, drawn in Paris, at the rate of 25·5 francs per pound sterling, and if this bill is sold in Amsterdam at the rate of 30 francs for 14 florins, and the money received be invested in a bill on Hamburg, at the rate of 18 florins for 20 marcs banco, what is the rate of exchange between London and Hamburg, or what is a pound sterling in London worth in Hamburg?

(11) If the exchange of London on Hamburg is 14 marcs banco per pound sterling; that of Hamburg on Amsterdam is 20 marks banco for 18 florins; that of Amsterdam on Paris is 28 florins for 60 francs; and that of Paris on Toronto is 4 francs for 72 cents, what is the rate of exchange between London and Toronto, or how many dollars are equal to £1 sterling?

(12) The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1 sterling, and the exchange at Vienna upon Paris is at the rate of $40\frac{1}{2}$ Austrian florins for 20 francs. Find how many Austrian florins should be paid at Vienna for a £50 note.

(13) What is the arbitrated rate of exchange between London and Lisbon, when bills on Paris, bought in London at 25·65 francs per £, are sold in Lisbon at 525 rees per 3 francs?

(14) Given that 1 ounce Troy equals 31·1 grammes; that 10 grammes of French standard gold are worth 31 francs; and that the worth of a given weight of English standard gold is to that of the same weight of French standard gold as 3151 to 3100, find what number of Troy ounces of English standard gold the franc is equal to, and what is the fixed number of francs equivalent to £1?—the English mint price of standard gold being 77*s.* $10\frac{1}{2}$*d.* per ounce.

Examination Papers.

I.

(1) If three fluids, whose volumes are as 3, 7, and 12, and their specific gravities ·95, 1·15, and 1·36, be mixed together, what will be the specific gravity of the compound?

(2) If $\frac{2}{3}$ of *A*'s money equals $\frac{3}{4}$ of *B*'s, and $\frac{2}{3}$ of *B*'s equals $\frac{3}{5}$ of *C*'s, and the interest of all their money at 8 per cent. for 4 years 6 months is $6291, how much money has each?

(3) A Toronto merchant wishes to pay a debt of £1200 in London. How many dollars must he pay to procure remittances through France and Hamburg if we allow that 21 francs = $4, 19 marcs banco at Hamburg = 35 francs at Paris, and £7 at London = 96 marcs banco at Hamburg?

(4) A merchant in Cincinnati wishes to remit $14331.60 to New York. Exchange on New York is $\frac{3}{4}$ per cent. premium, but in St. Louis $\frac{1}{2}$ per cent. premium, from St. Louis to New Orleans $\frac{1}{2}$ per cent. discount, and from New Orleans to New York 1 per cent. discount. What will be the value in New York by each method, and how much better is the circular?

(5) A merchant in Toronto purchased a draft on New York for $2660, drawn at 60 days, paying $2570.89. What was the course of exchange?

II.

(1) A merchant mixes 11 lbs. of tea with 5 lbs. of an inferior quality, and gains 16 % by selling the mixture at 87

cents per pound. Allowing that a pound of the one cost 12 cents more than a pound of the other, what was the cost of each kind per pound?

(2) *A* and *B* are in partnership in a concern in which *A* has $20000 engaged, and *B* $30000. The gross receipts for a year are $12800; of this one-eighth part is expended in salaries of clerks, and $120 in insurance. By an arrangement between the partners *A* is to receive 8 % upon his capital, and *B* 4 % upon his, and then the remainder of the profits is to be divided in proportion to the capital employed. Find the net receipts of *A* and *B*.

(3) Bills on Amsterdam, bought in London at 12 florins 15 cents per £1 sterling, are sold in Paris at $57\frac{1}{2}$ florins for 120 francs. What is the course of exchange between London and Paris?

(4) On the 1st Jan. *A* brought into a business $1400, and on 1st April $2000 more; on the 1st June he took out $1600, and 3 months after this he brought in $2400. *B* brought into the business $2000; 4 months after this he took out $600, and on the 1st Nov. brought in $2600. Their clear profit for the year is $4032. How much ought each to receive?

(5) A cask contains 12 gals. of wine and 18 gals. of water; another cask contains 9 gals. of wine and 3 gals. of water; how many gallons must be drawn from each cask so as to produce by their mixture 7 gals. of wine and 7 gals. of water?

III.

(1) A merchant has sugar at 8, 10, 12, and 20 cents a pound; with these he wishes to fill a cask that holds 200 lbs. How much of each kind must he take so that the mixture may be worth 15 cents a pound?

(2) A 15 days' draft on Montreal yielded $1190·234 when sold at $1\frac{1}{2}$ % discount, and interest off at 6 per cent. What was the face of the draft?

(3) If *A* gain $120 in 6 months, *B* $150 in 5 months, and *C* $210 in 9 months, what was the whole stock, *C*'s part of it being $400?

(4) From a cask of wine one-fourth is drawn off, and the cask is filled up with water; one-fourth of the mixture is then drawn off, and the cask again filled up with water; after this has been done four times altogether, what fraction of the original quantity of wine will be left in the cask?

(5) A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker at a time when the exchange between London and Paris is 25·15 francs for £1, and between Paris and St. Petersburg 1·2 francs for 1 rouble. The remittance is delayed until the rates are 25·35 francs for £1 and 1·15 francs for 1 rouble. What does the broker gain or lose by the delay?

IV.

(1) If, when the course of exchange between England and Spain is 38½*d.* per dollar of 20 reals, a merchant in Liverpool draws a bill of £354 16*s.* 3*d.* on Madrid, how many dollars and reals will pay the draft?

(2) I wish to pay a bill in Naples of 7500 lira; the direct exchange is $0.22 = 1 lira; the exchange on London is $4.95; of London on Paris is £1 = 26 francs; of Paris on Naples is 1⅛ francs = 1 lira. What is the difference between the direct and circuitous exchange?

(3) A merchant in New York gave $1000 for a bill on London of £200. What was the rate of exchange?

(4) A merchant in New York wishes to pay £3000 in London. Exchange on London is at par; on Paris 5 francs 25 centimes per $1, and on Amsterdam 40 cents to a guilder. The exchange between France and England at the same time is 25 francs to £1, and that of Amsterdam on England 12⅕ guilders to £1. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam?

(5) How many pounds of sugar at 8, 13, and 14 cents per pound may be mixed with 3 pounds at 9¼ cents, 2 pounds at 8½ cents, and 4 lbs. at 14 cents a pound, so as to gain 16 per cent. by selling the mixture at 14½ cents per pound?

V.

(1) Three districts are to provide according to their population a contingent of 182 men. The population of the districts is 2456, 735, and 4361 respectively. Find as exactly as possible the number of men to be provided by each district.

(2) A person mixes 4 gallons of gin at 15*s.* per gallon with 4 gallons of water and a gallon of base spirit worth 10*s.* What is his gain per cent. on his outlay by selling the mixture at 2⅚*s.* per bottle of 6 to the gallon?

(3) The stocks of three partners, *A*, *B*, and *C*, are $3500, $2200, and $2500 respectively; their gains are $1120, $880, and $1200 respectively. If *B*'s stock is in trade 2 months longer than *A*'s, what time was each stock in trade?

(4) A merchant every year gains 50 % on his capital, of which he spends £1200 per annum in house and other expenses. At the end of 4 years he finds himself in possession of 4 times as much as what he had at commencing business. What was his original capital?

(5) There are two mixtures of wine and water, the quantities of wine in which are respectively ·34 and ·46 of the whole. If a gallon of the first is mixed with two gallons of the second, what decimal part will the wine be in the compound, and how much per cent. will the first mixture be strengthened?

XXXI. Ratio and Proportion.

211. If *A* and *B* be quantities of the same kind, the relative greatness of *A* with respect to *B* is called the RATIO of *A* to *B*.

212. The ratio of one quantity to another quantity is represented in Arithmetic by the fraction which expresses the measures of the first when the second is taken as the unit of measurement.

Thus, if 5 shillings be the unit, the measure of 3 shillings is $\frac{3}{5}$, and the ratio of 3 shillings to 5 shillings is represented by the fraction $\frac{3}{5}$.

The words "the ratio of 3 shillings to 5 shillings" are abbreviated thus:

3 shillings : 5 shillings.

213. Ratios may be compared with each other by comparing the fractions by which they are represented.

Thus 2 pence : 5 pence is represented by $\frac{2}{5}$,
and 3 pence : 7 pence is represented by $\frac{3}{7}$,
Now $\frac{2}{5} = \frac{14}{35}$, and $\frac{3}{7} = \frac{15}{35}$,
$\therefore$ $\frac{3}{7}$ is greater than $\frac{2}{5}$,
and $\therefore$ 3 pence : 7 pence is greater than 2 pence : 5 pence.

When we thus compare the ratios existing between two pairs of quantities, it is not necessary that all *four* quantities should be of the same kind; it is only necessary that *each pair* should be of the same kind.

For example, we can compare the ratio of 4 shillings to 7 shillings with the ratio of 7 days to 12 days, and finding that $\frac{4}{7}$ is less than $\frac{7}{12}$, we may say that the ratio of 4 shillings to 7 shillings is less than the ratio of 7 days to 12 days.

214. When the ratio symbol (:) is placed between two *numbers*, we may substitute for it the fraction symbol.

Thus, if we have to compare the ratios 2 : 3 and 5 : 7, we effect it by comparing the fractions $\frac{2}{3}$ and $\frac{5}{7}$.

215. Ratios are *compounded* by multiplying together the fractions by which they are represented, and expressing the resulting fraction as a ratio.

Thus the ratio compounded of 2 : 3 and 5 : 7 is 10 : 21.

2 and 3 are called the TERMS of the ratio 2 : 3.

2 is called the ANTECEDENT and 3 the CONSEQUENT of the ratio.

216. Ratios are either *direct* or *inverse*.

A *direct* ratio is the quotient of the antecedent divided by the consequent.

An *inverse* ratio, or reciprocal ratio, is the quotient of the consequent divided by the antecedent.

Examples. (cxvi.)

(1) Compare the ratios 2 : 5 and 4 : 9.

(2) Compare the ratios 17 : 39 and 19 : 41.

(3) Compare the ratios 4 : 7, 8 : 15, and 13 : 24.

(4) Compound the ratios 5 : 7, 13 : 15, 21 : 91, and 45 : 52.

(5) Compound the ratios $3\frac{1}{2} : 4$, $3\frac{1}{5} : 7$, $1\frac{1}{3} : 3\frac{1}{4}$, $2\frac{1}{6} : 1\frac{2}{7}$.

(6) If the ratio be 25 and the consequent $1.25, what is the antecedent?

(7) How much does the ratio $36 \times 4 \times 3 : 12 \times 16 \times 2$ exceed that of $60 \div (3 \times 5) : 20 \times 2 \div 8$?

(8) What is the reciprocal ratio of $\frac{1}{3} : \frac{1}{6}$; of $2\frac{1}{3} : 7\cdot\dot{9}$?

(9) *A* owns a farm of 180 acres. There are 36 sq. miles in the township in which it is situated. What is the relation of the latter to the former?

(10) The ratio 63 : 52 results from compounding four ratios together; three of these are 7 : 8, 12 : 15, and $\frac{1}{2} : \frac{1}{5}$. Express the fourth ratio in its simplest form.

(11) What effect has adding the same quantity to both terms of a ratio?

PROPORTION.

217. PROPORTION consists in the equality of two ratios.

The Arithmetical test of Proportion is therefore *that the two fractions representing the ratios must be equal.*

Thus the ratio 6 : 12 is equal to the ratio 4 : 8, because the fraction $\frac{6}{12}$ = the fraction $\frac{4}{8}$.

The four numbers 6, 12, 4, 8, written in the order in which they stand in the ratios, are said to be *in proportion*, or *proportionals*, and this relation is thus expressed—

$$6 : 12 = 4 : 8.$$

The two terms 6 and 8 are called the EXTREMES.
" 12 and 4 " MEANS.

The sign of equality is usually expressed thus, :: and then the ratios read 6 is to 12 as 4 is to 8.

218. When four numbers are in proportion,
the product of the extremes = the product of the means

For example, if 6 : 12 :: 4 : 8,

$$6 \times 8 = 12 \times 4.$$

For, since $\frac{6}{12} = \frac{4}{8}$, by hypothesis,

and $\frac{6 \times 8}{12 \times 8} = \frac{6}{12}$,

and $\frac{4 \times 12}{8 \times 12} = \frac{4}{8}$;

$$\therefore \frac{6 \times 8}{12 \times 8} = \frac{4 \times 12}{8 \times 12}.$$

Now the *denominators* of these fractions are equal, and therefore the numerators must also be equal, that is,

$$6 \times 8 = 4 \times 12.$$

From this it is evident that if three out of the four numbers that form a proportion are given, we can find the fourth.

Ex. (1). Find a fourth proportional to 3, 15, 7.

$$3 : 15 = 7 : \text{number required};$$
$$\therefore 3 \times \text{number required} = 15 \times 7;$$
$$\therefore \text{number required} = \frac{15\times7}{3} = 35.$$

Ex. (2). What number has the same ratio to 9 that 3 has to 5?

$$3 : 5 = \text{number required} : 9;$$
$$\therefore 5 \times \text{number required} = 3 \times 9;$$
$$\therefore \text{number required} = \frac{27}{5} = 5\frac{2}{5}.$$

219. Three numbers are said to be in CONTINUED PROPORTION when the ratio of the first to the second is equal to the ratio of the second to the third.

Thus 3, 6, 12, are continued proportion,

$$\text{for } \frac{3}{6} = \frac{6}{12}.$$

The second number is called a MEAN PROPORTIONAL between the first and the third.

Ex. Find a mean proportional between 6 and 24.

$$6 : \text{required number} = \text{required number} : 24;$$
$$\therefore \text{required number} \times \text{required number} = 6 \times 24;$$
$$\therefore \text{square of required number} = 144;$$
$$\therefore \text{required number is } 12.$$

220. When two quantities are connected in such a way that when one is increased 2, 3, ... times, the other is also increased 2, 3, ... times, they are in *direct* proportion.

For example, if 1 lb. of sugar cost 9 cents,

2 lbs. will cost 2 × 9 cents,
3 lbs. " 3 × 9 cents;
hence 7 lbs. " 7 × 9 cents,
and 25 lbs. " 25 × 9 cents;
∴ 7 lbs. : 25 lbs. :: 7 × 9 cents : 25 × 9 cents.

That is, the *cost* of sugar is *directly* proportional to its *weight*.

221. When two quantities are connected in such a way, that when one is *increased* 2, 3, ... times, the other is *diminished* 2, 3, ... times, they are *inversely* proportional: thus, if one man can mow a field in 12 days, 2 men can mow it in half the time, or in $\frac{12}{2}$ days; 3 men in a third of the time, or in $\frac{12}{3}$ days, &c.

Hence 4 men can mow it in $\frac{12}{4}$ days;
and 12 " " $\frac{12}{12}$ days;
$\therefore$ 4 men : 12 men :: $\frac{12}{12}$ days : $\frac{12}{4}$ days;

that is, the *number of men* required to do a certain work is *inversely* proportional to the number of days, or *vice versa*.

Examples. (cxvii.)

(1) Arrange 4, 3, 9, and 12 so that they may be in proportion.

(2) Find the second term when 18, $2\cdot\dot{6}$, and $1\cdot\dot{8}$ are the other three terms of a proportional?

(3) Find a mean proportional to ·038 and ·00152.

(4) If $A = 3\frac{1}{3}$ of B, and $C = 5\frac{1}{5}$ of B, find the ratio of A to C.

(5) Find a fourth proportional to 5, 7, and 15.

(6) Find a fourth proportional to $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{5}{9}$.

(7) Find a fourth proportional to ·3, ·16, and ·09.

(8) Find a mean proportional to 14 and 56.

(9) Find a mean proportional to $\frac{5}{63}$ and $\frac{35}{81}$.

(10) Divide $1587 among A, B, C, D, so that A's share : B's share = 6 : 5, B's share : C's share = 4 : 3, and C's share : D's share = 3 : 2.

SIMPLE PROPORTION, OR RULE OF THREE.

222. When *three* terms of a proportion are given, to find the *fourth*, it is a SIMPLE PROPORTION. In a simple proportion we have two ratios given; one of these has both terms, the other is incomplete, having only one term. Two of the given terms must be of one kind, and the third and the answer of another kind.

Ex. (1). If 5 horses eat 20 bushels of oats in a given time, how many bushels will 8 horses eat in the same time?

Here the number of bushels consumed is *directly* proportional to the number of horses.

Hence 5 : 8 :: 20 bu. : bu. required;
$\therefore$ bu. required $= \frac{8\times 20}{5} = 32$.

Ex. (2). If 6 men can do a piece of work in 5 days, in what time can 9 men do the same work?

Here the time is *inversely* proportioned to the number of men.

Hence 9 : 6 :: 5 days : days required;

$$\therefore \text{days required} = \frac{6\times 5}{9} = 3\tfrac{1}{3}.$$

Ex. (3). If 3 cwt. 1 qr. of hay cost $2.21, what should 3 tons 5 cwt. cost?

Here the cost is *directly* proportional to the quantity.

Hence 3 cwt. 1 qr. : 3 tons 5 cwt. :: $2.21 : dollars required;

Here we reduce the 1st and 2nd terms to the common denomination, quarters, and the proportion becomes

13 : 260 :: $2.21 : dollars required;

$$\therefore \text{dollars required} = \frac{260\times 2.21}{13} = \$44.20.$$

From these examples we deduce the following rule:

Write the given number that is of the same kind as the required fourth term, for the third term of the proportion. Then consider from the nature of the question whether the answer is to be greater or less than the third term. If greater, place the larger of the two remaining numbers in the second place; if less, in the first. Then having reduced the first and second terms to the same denomination, multiply the second and third terms together, and divide the product by the first term. The quotient will be the answer required.

NOTE.—After the third term has been written down, the order of the other two may be ascertained by a question. Thus, in Ex. (1): "If 5 horses eat 20 bu., will 8 horses eat more or less than 20 bu.?" More; hence 5 : 8. In Ex. (2): "If 6 men do a piece of work in 5 days, will it take 9 men a longer or shorter period than 5 days?" Shorter; hence 9 : 6.

Examples. (cxviii.)

(1) A person after paying an income tax of 7*d.* in the £ has a net income of £1247 10*s.* 5*d.* What was his gross income?

(2) A watch which is 10 minutes too fast at 12 o'clock noon on Monday, gains 3m. 10s. a day; what will be the time by the watch at a quarter past 10 a.m. on the following Saturday?

(3) In running a 3 mile race on a course $\frac{1}{3}$ of a mile round, *A* overlaps *B* at the middle of the 7th round. By what distance will *A* win at the same rate of running?

(4) A watch was $6\frac{7}{11}$ min. slow at noon; it loses 12 min. in $20\frac{1}{2}$ hours; find the true time when its hands are together for the fourth time after noon.

(5) If 4 men or 6 women or 9 boys can perform a piece of work in $27\frac{1}{2}$ days, in what time can (*a*) 5 men and 9 women perform it? and (*b*) 5 men and 8 boys perform it?

(6) If $14\frac{3}{8}$ shares of a property are worth $116.15, what are $5\frac{5}{6}$ shares worth?

(7) A floor can be covered by $32\frac{1}{2}$ yards of carpet 7 quarters wide; how many yards of Brussels carpet 26 in. width will cover the same room?

(8) Two clocks, of which one gains 4m. 15s. and the other loses 3m. 15s. in 24 hours, were both within $2\frac{1}{2}$m. of the true time, the former fast and the latter slow, at noon on Monday; they now differ from one another by half an hour. Find the day of the week and the hour of the day.

(9) If 6336 stones $3\frac{1}{4}$ ft. long complete a certain quantity of wall, how many similar stones of $2\frac{2}{3}$ feet long will raise a like quantity?

(10) A beseiged town, containing 22400 inhabitants, has provisions to last 3 weeks. How many must be sent away that they may be able to hold out 7 weeks?

COMPOUND PROPORTION.

223. Where *five*, *seven*, *nine*, &c., terms of a proportion are given, to find a *sixth*, *eighth*, *tenth*, &c., term, it is called COMPOUND PROPORTION or the DOUBLE RULE OF THREE.

In Compound Proportion there are three or more ratios given, all being complete but one.

A Compound Proportion is produced by multiplying together the corresponding terms of two or more simple proportions.

Thus, 12 : 6 :: 4 : 2
9 : 3 :: 6 : 2
5 : 4 :: 10 : 8 multiplied together produce the proportion 540 : 72 :: 240 : 32.

Ex. If 6 men in 8 days, working 10 hours a day, can reap 24 acres of wheat, how many acres could 10 men reap in 15 days of 12 hours each?

6 : 10 :: 24 : acres required.
8 : 15
10 : 12

---

480 : 1800 :: 24 : acres required ;
∴ acres required $= \frac{1800 \times 24}{480} = 90$.

24, the term of the imperfect ratio, is put in the 3rd place; the other ratios are then considered separately and treated as in Simple Proportion. After all the ratios have been stated, all the first terms are multiplied together for a new first term and similarly with the second terms. The answer is then got as in Simple Proportion.

NOTE I.—Before compounding the complete ratios it is convenient to cancel all the factors common to the 1st terms, and to the 2nd or 3rd terms. When any of the 1st and 2nd terms are not of the same denomination, they must be reduced to a common denomination before proceeding with the solution.

NOTE II.—Before stating the question it is convenient to write down the terms of the supposition under one another and opposite these to place the corresponding terms of the demand with an x opposite the term of the same name as the answer required.

Thus, in the above example, 6 men 10,
8 days 15,
10 hours 12,
24 acres x.

Examples. (cxix.)

(1) If 18 men in 12 days build a wall 40 feet long, 3 feet thick, and 16 feet high, how many men must be employed to build a wall 120 yards long, 8 feet thick, and 10 feet high, in 60 days?

(2) An engineer engages to complete a tunnel $3\frac{3}{4}$ miles long in 2 years 10 months; for a year and a half he employs 1200 men, and then finds he has completed only three-eighths of his work. How many additional men must he employ to complete it in the required time?

(3) Two sets of men perform the same amount of work. Each man in the first set is stronger than each man in the second in the ratio of 7 to 6; the first set works 6 days a week for 10 weeks, and the second set 5 days a week for 7 weeks. If there are 9 men in the first set, how many are there in the second?

(4) If 20 men can excavate 185 cubic yards of earth in 9 hours, how many men could do half the work in a fifth of the time.

(5) At the siege of Sebastopol it was found that a certain length of trench could be dug by the soldiers and navvies in 4 days, but that when only half the navvies were present it required 7 days to dig the same length of trench. Compare the amount of work done by the navvies with that done by the soldiers.

(6) Two elephants which are 10 in length, 9 in breadth, 36 in girth, and 7 in height, consume one *drona* of grain; how much will be the rations of 10 other elephants, which are a quarter more in length and other dimensions.

(7) How many revolutions will be made by a wheel which revolves at the rate of 360 revolutions in 7 minutes, while another wheel, which revolves at the rate of 470 in 8 min., makes 658 revolutions?

(8) A piece of work is to be done in 36 days; 15 men work at it 15 hours a day, but after 24 days only $\frac{3}{5}$ of it is done; if three more men are put on, how many hours a day must all work to finish it in the given time?

(9) If 248 men, in $5\frac{1}{2}$ days of 12 hours each, dig a ditch of 7 degrees of hardness, $232\frac{1}{2}$ yds. long, $3\frac{2}{3}$ yds. wide, and $2\frac{1}{3}$ yds. deep; in how many days of 9 hours each, will 24 men dig a ditch of 4 degrees of hardness, $387\frac{1}{2}$ yds. long, $5\frac{1}{4}$ yds. wide, and $3\frac{1}{2}$ yds. deep?

(10) If 5 compositors in 16 days of 11 hours each, can compose 25 sheets of 24 pages in a sheet, 44 lines in a page,

and 40 letters in a line, in how many days of 10 hours each can 9 compositors compose a volume (to be printed in the same kind of type), consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters to a line?

XXXII. The Metric System.

224. The Metric System of Weights and Measures is now in use in many countries of Europe. The following is an account of the system as it is established in France, where it originated at the end of the last century.

The basis of all measurement is the METRE, a measure of length equal to the ten-millionth part of the distance from the North Pole to the Equator.

The length of the Metre in English measure is 39·37 inches, nearly.

Units of Metric Measures.

1. LENGTH.—The METRE.
2. SURFACE.—The ARE = 100 square metres.
3. SOLIDITY.—The STERE = 1 cubic metre.
4. CAPACITY.—The LITRE = the cube of the tenth part of a metre.
5. WEIGHT.—The GRAMME, which is the weight of a quantity of distilled water which fills the cube of the hundredth part of a metre.

The tables of Weights and Measures under the Metric System are constructed upon one uniform principle. Prefixes derived from Greek and Latin are attached to each of the units.

Greek Prefixes.

Deca stands for 10 times
Hecto stands for 100 times
Kilo stands for 1000 times
Myria stands for 10000 times
} the unit.

Latin Prefixes.

Deci stands for the 10th part
Centi stands for the 100th part
Milli stands for the 1000th part
} of the unit.

Thus,

A decametre = 10 metres.
A hectolitre = 100 litres.
A kilogramme = 1000 grammes.
A myriametre = 10000 metres.

Also,

A decilitre = ·1 litre.
A centimetre = ·01 metre.
A milligramme = ·001 gramme.

NOTE.—In English measures the following are rough approximations of some of the French measures:

The kilogramme is about 2$\frac{1}{5}$ lb. Avoird.
The litre is about 1$\frac{3}{4}$ pints.
The kilometre is about 5 furlongs.
The hectare is about 2$\frac{1}{2}$ acres.

MEASURES OF LENGTH.

10 decimetres (dcm.)..........1 metre (m.).
100 centimetres (cm.).......... "
1000 millimetres (mm.).......... "
1000 metres....................1 kilometre.

1 inch = 2·539954 centimetres.
1 foot = 3·047945 decimetres.
1 yard = 0·914383 metres.
1 mile = 1·609315 kilometres.

NOTE.—A rough rule for converting French metres into English yards is to add 10 per cent. to them. Thus 40 metres are nearly equal to 44 yards.

MEASURES OF SURFACE.

100 square decimetres (sq. dcm.) = 1 square metre or centiare (sq. m.)
10000 " centimetres (sq. cm.) = "
1000000 " millimetres (sq. mm.) = "
100 square metres..................1 are.
10000 " ....1 hectare.

1 square inch = 6·4513669 sq. cm.
1 " foot = 9·2899683 sq. dcm.
1 " yard = 0·83609715 sq. m.
1 " acre = 0 40467101 hectare.

MEASURES OF CAPACITY.

1000 cubic decimetres (cb. dcm.)..1 cubic metre or stere.
1000000 cubic centimetres (cb. cm.) “
1000000000 cubic millimetres (cb. mm.)
1 cubic decimetre......1 litre.
1 “ inch = 16·386176 cb. cm.
1 “ foot = 28·315312 dcm.
1 gallon = 4·54345797 litres.

MEASURES OF WEIGHT.

1 cubic centimetre of distilled water at 4°C. at the sea's level in the latitude of Paris is 1 gram (grm.).

1000 cubic centimetres of distilled water weighed under the same conditions............1 kilogram (kilo.).

1000 grams (grms.) 1 kilogram.
10000 decigrams .. “
100000 centigrams .. “
1000000 milligrams .. “
1 grain = 0·06479895 gram.
1 Troy oz. = 31·103496 grams.
1 lb. Avd. = 0·45359265 kilo.
1 cwt. = 50·80237689 kilos.

Examples. (cxx.)

(1) What is the fundamental unit in this system? Whence and why was it chosen?

(2) Name the units of weight and capacity, and show how larger and smaller measures are attained.

(3) Give the English equivalents of a kilometre and kilogram.

(4) How many millimetres are contained in 5 metres?

(5) How many decimetres are equivalent to 106725 millimetres?

(6) Required the number of milligrams in 15 cb. cm. of water measured at 4°C.?

(7) How many millimetres and centimetres are respectively contained in 0·437 of a decimetre?

(8) How many square centimetres are there in 15·5 square metres?

(9) How many square decimetres are contained in 108612 square centimetres?

(10) Define the gram and litre. How many grams are contained in 1·725 kilograms?

(11) How many milligrams are there in a decigram? How many decigrams in a kilogram?

(12) How many centigrams are contained in 2·567 kilograms?

(13) Required, the number of milligrams contained in 5 cubic centimetres of water measured at 4°C.

(14) In an English inch are contained 25·3995 millimetres. How many kilometres are there in a mile?

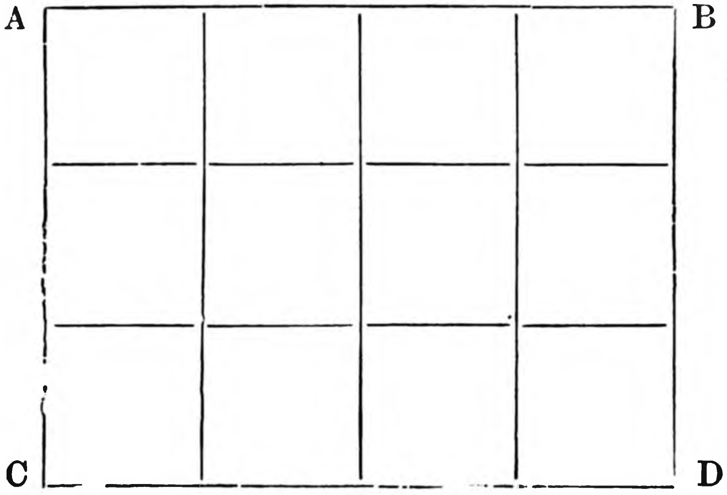
(15) A gallon is equal to 4·543 litres. How many cubic centimetres are contained in one pint?

(16) Three pipes furnish respectively 30 litres, 45 litres, and 80 litres an hour. What quantity of water do they supply together in 24 hours.

XXXIII.—Measurement of Area.

225. The unit of measurement, by which we measure Area or Surface, is derived from the unit of Length. Thus, if we take an inch as the unit of length, and construct a square whose side is an inch, this Square Inch may be taken as the Unit of Area, and the *measure* of any given area will be the number of times it contains this unit, in accordance with the remarks in Art. 58.

Let ABDC be a rectangle, and let the side AB be 4 inches in length and the side AC 3 inches in length,



Then, if the Unit of Length be an inch, the *measure* of AB is 4, and the *measure* of AC is 3.

Divide AB, AC into four and three equal parts respectively, and draw lines through the points of division parallel to AC, AB respectively. Then the rectangle ABDC is divided into a number of equal *squares*, each of which is a square inch.

If one of these squares be taken as the Unit of Area, the *measure* of the area of ABDC will be the number of these squares.

Now, this number is the same as that obtained by multiplying the measure of AB by the measure of AC:

that is, measure of ABDC $= 3 \times 4 = 12$;

$\therefore$ the area of ABDC is 12 square inches.

Hence, to find the area of a rectangle we multiply the measure of the length by the measure of the breadth, and the product will be the measure of the area.

Ex. (1). A rectangular garden is 48 feet long and 25 feet broad, what is its area?

Taking a foot as the unit of length, and therefore a square foot as the unit of area,

measure of the area $= 48 \times 25 = 1200$;

$\therefore$ the area is 1200 square feet.

Ex. (2). A rectangular board is 2 ft. 7 in. long and 1 ft. 4 in. broad, what is the area of its surface?

Taking 1 inch as the unit of length, and therefore 1 square inch as the unit of area,

measure of the area $= 31 \times 16 = 496$;

$\therefore$ the area is 496 square inches.

Or, we might take 1 foot as the unit of length, and then

measure of area $= 2\frac{7}{12} \times 1\frac{1}{3} = \frac{31 \times 4}{12 \times 3} = \frac{31}{9} = 3\frac{4}{9}$;

$\therefore$ the area is $3\frac{4}{9}$ square feet.

Ex. (3). The length of the side of a square croquet-ground is 49 yards, what is its area?

Taking 1 yard as a unit of length,

area $= (49 \times 49)$ sq. yds. $= 2401$ sq. yds.

NOTE.—Observe the difference between the expressions 49 *yards square* and 49 *square yards*. The former refers to a square whose side is 49 yards, and whose area is 2401 square yards; the latter to a surface whose area is 49 square yards.

Ex. (4). A rectangular bowling-green is 56 yards long and 42 yards broad. Find the distance from corner to corner.

By Euclid I. 47, we know that in a right-angled triangle the square *on* the side opposite the right angle is equal to the sum of the squares *on* the sides containing the right angle.

Hence the square *of* the measure of the side opposite the right angle is equal to the sum of the squares *of* the measures of the sides containing the right angle.

Thus, in our present example,

square of measure of distance from corner to corner

$$= (56 \times 56) + (42 \times 42) = 4900;$$

$\therefore$ distance is 70 yards.

Examples. (cxxi.)

Find the area of the rectangles having the following dimensions:

(1) 7 ft. by 5 ft. (2) $13\frac{1}{2}$ ft. by 10 ft.
(3) $22\frac{1}{4}$ ft. by $13\frac{1}{2}$ ft. (4) 5 ft. 4 in. by 2 ft. 3 in.
(5) 17 ft. 5 in. by 8 yd. 2 ft. (6) 5 yd. 1 ft. by 4 yd. 2 ft.
(7) 12 yd. 2 ft. by 5 yd. 1 ft.
(8) 6 yd. 2 ft. 3 in. by 2 yd. 1 ft. 5 in.
(9) 7 yd. 2 ft. by 5 yd. 2 ft. 6 in.

Find the area of the squares whose sides have the following lengths:

(10) $5\frac{1}{2}$ yd. (11) $37\frac{1}{2}$ yd. (12) $17\frac{3}{4}$ ft.
(13) $29\frac{1}{2}$ ft. (14) 9 ft. 7 in. (15) 3 ft. 4 in.
(16) 7 yd. 1 ft. 5 in. (17) 15 yd. 2 ft. 3 in.

Find the breadth of the following rectangles, having given the area and length:

(18) Area 176 sq. ft., length 11 ft.
(19) Area 71 sq. ft. 100 sq. in., length 9 ft. 8 in.

(20) Area 854 sq. ft. 84 sq. in., length 97 ft. 8 in.
(21) Area 1 acre, length 440 yd.
(22) Area 5 acres, length 275 yd.
(23) Area 5 ac. 1 ro. 36 po., length 267 yd. 2 ft.

What are the sides of the squares whose areas are

(24) 81 sq. ft.
(25) 256 sq. ft.
(26) 1178 sq. yd. 7 sq. ft.
(27) 33 ac. 4305 sq. yd.

(28) A rectangular field is 225 yards in length and 120 yards in breadth; what will be the length of a straight path from corner to corner?

(29) A rectangular field is 300 yards long and 200 yards broad. Find the distance from corner to corner.

(30) A rectangular plantation, whose width is 88 yards, contains $2\frac{1}{2}$ acres. Find the distance from corner to corner.

(31) What is the length of the diagonal of a square whose side is 5 inches?

(32) The area of a square is 390625 square feet. What is the length of the diagonal?

CARPETING ROOMS.

226. If we know the area of the floor of a room, we know how many square inches of carpet will be required to cover it. Carpets are sold in strips, and when the *width* of a strip is known, we shall know how much *length* of carpet will be required to cover a given surface.

For instance, if the surface be 162 square feet, and the carpet selected be 27 inches wide, we reason thus:

$$162 \text{ sq. ft.} = 162 \times 144 \text{ sq. inches};$$

$$\therefore \text{length of carpet required} = \frac{162 \times 144}{27} \text{ in.} = 864 \text{ in.} = 24 \text{ yds.}$$

Then we find the cost of 24 yards at $1.20 per yard to be $28.80.

Examples. (cxxii.)

How many yards of carpet, 27 inches wide, will be required for rooms whose dimensions are:

(1) 15 ft. by 13 ft.
(2) 25 ft. by 12 ft. 6 in.
(3) 22 ft. 4 in. by 20 ft. 3 in.
(4) 27 ft. by $14\frac{1}{2}$ ft.
(5) 35 ft. 4 in. by 27 ft. 3 in.

Find the expense of carpeting rooms whose dimensions are:

(6) 18 ft. by 14 ft., with carpet 30 inches wide, at $1 a yard.

(7) 22 ft. by 15½ ft., with carpet 27 inches wide, at $1.80 a yard.

(8) 29 ft. 9 in. by 23 ft. 6 in., with carpet a yard wide, at $1.08 a yard.

(9) 34 ft. 8 in. by 13 ft. 3 in., with carpet ¾ yard wide, at 3*s.* 4½*d.* a yard.

PAPERING THE WALLS OF A ROOM.

227. To find the quantity of paper required to cover *one* wall of a room, we find the area of the surface of the wall by taking the product of the measures of the length and breadth of that wall, the latter being the same as the height of the room. Hence, we shall find the area of the *four* walls of the room *if we take the measure of the compass of the room and multiply it by the measure of the height.*

By the compass of a room we mean the length of a string stretched tight on the floor, and going all round the room.

Deductions for doors, windows, and fire-place must be made in practice.

Suppose, then, we have to find how much paper is required for the walls of a room whose length is 22 ft. 3 in., breadth 17 ft. 4 in., and height 9 ft. 6 in.

We first find the compass of the room, thus:

ft.	in.	
22	3	dimensions of the four sides.
17	4	
22	3	
17	4	
79	2	compass of the room.

To get the area of paper required, we multiply the measure of the compass of the room by the measure of the height, thus:

$$\text{area} = (9\tfrac{1}{2} \times 79\tfrac{1}{6}) \text{ sq. ft.} = \frac{19 \times 475}{12} \text{ sq. ft.} = 752\tfrac{1}{12} \text{ sq. ft.}$$

NOTE.—Papers, like carpets, are sold in strips, and if we know the width of a strip we shall know how many feet in length will be required to cover a given surface.

Thus, in the room under consideration, if the paper be 20 inches wide,

length of paper required $= (752\frac{1}{12} \div \frac{20}{12})$ ft. $= \frac{9025}{20}$ ft. $= 451\frac{1}{4}$ ft.

Examples. (cxxiii.)

How many square feet of paper will be required for rooms whose dimensions are:

(1) Length, 19 ft.; breadth, 16 ft.; height, 9 ft.?

(2) Length, $24\frac{1}{2}$ ft.; breadth, $18\frac{1}{4}$ ft.; height, 10 ft.?

(3) Length, 25 ft. 7 in.; breadth, 19 ft. 4 in.; height, 9 ft. 9 in.?

(4) Length, 23 ft. 5 in.; breadth, 18 ft. 7 in.; height, 9 ft. 6 in.?

Find the expense of papering rooms whose dimensions are:

(5) Length, 18 ft.; breadth, 14 ft.; height, 8 ft.; with paper 16 inches wide, at 20 cents a yard.

(6) Length, 20 ft. 6 in.; breadth, 17 ft. 4 in.; height, 9 ft.; with paper 20 inches wide, at 10 cents a yard.

(7) Length, 30 ft. 8 in.; breadth, 26 ft. 5 in.; height, 10 ft. 6 in.; with paper 2 ft. wide, at 8*d.* a yard.

(8) Length, 26 ft.; breadth, 21 ft.; height, 10 ft.; with paper 20 in. wide at 9*d.* a yard, allowing for a fireplace which is 5 ft. 3 in. by 4 ft., a door which is 7 ft. by $4\frac{1}{2}$ ft., and two windows, each 6 ft. by $3\frac{1}{2}$ ft.

Miscellaneous Examples. (cxxiv.)

(1) Find the cost of varnishing the floor of a room 14 ft. 4 in. broad, and 15 ft. 6 in. long, at 20 cents per square yd.

(2) What will it cost to pave an area 146 ft. 9 in. long and 88 ft. 9 in. broad, at 36 cents per square yard?

(3) The area of a square garden is 4 roods 1 pole 29 sq. yd. $6\frac{3}{4}$ sq. ft. Find the length of its side?

(4) Find the length of the side of a square whose area is 1 ro. 26 po. 28 sq. yd. $4\frac{1}{2}$ sq. ft.

(5) Find the expense of turfing a plot of ground which is 40 yd. long and 100 ft. wide with turfs each a yard in length and 1 ft. in breadth, the turfs, when laid, costing 6*s.* 9*d.* per hundred.

(6) A square room, whose floor measures 32 sq. yd. 1 sq. ft., is 11 ft. 6 in. in height. Find the expense of whitewashing its ceiling and walls at 5 cents per square yard.

(7) It costs $99 to cover the floor of a room $8\frac{1}{4}$ yd. long by $6\frac{2}{3}$ yd. wide, with carpet 2 ft. wide. Find the price of the carpet per yard.

(8) If the cost of papering a room $8\frac{1}{4}$ yd. long and $6\frac{2}{3}$ yd. wide, with paper 2 ft. wide at 4*d.* per yard, be £2 19*s.* 8*d.*, find the height of the room.

(9) A rectangular field, whose length is 997 yd. 1 ft., contains 12 acres 4087 sq. yd. 1 sq. ft. Find the breadth of the field.

(10) How many acres are there in a square field each side of which is 330 yd.?

(11) The length of a room is 21 ft. and its height 10 ft. 6 in., and the area of the floor is $\frac{5}{11}$ of the area of the four walls. Find the breadth of the room.

(12) What length must be cut off a board which is $6\frac{3}{4}$ in. broad, that the area may contain a square foot?

(13) What is the expense of papering a room 4 yd. $6\frac{3}{4}$ in. long, 3 yd. $11\frac{1}{4}$ in. wide, and 3 yd. 1 ft. high, with paper half a yard wide, at 12 cents a yard?

(14) How many stones, each 2 feet long and $15\frac{1}{2}$ inches wide, would be required to pave a square courtyard whose side is 124 feet?

(15) What is the cost of papering a room 15 ft. long, 12 ft. wide, and 10 ft. high, with paper $\frac{5}{6}$ yd. wide, at $12\frac{1}{2}$ cents a yard?

(16) Find the cost of papering a room 21 ft. long, 15 ft. wide, and 12 ft. high, with paper $2\frac{1}{2}$ ft. wide, at 15 cents a yard, allowing for a door 7 ft. high and 3 ft. wide, two windows each 5 ft. high and 3 ft. wide, and a panelling 2 feet high round the floor.

(17) The length of one side of a rectangular field is 572 yards, and the area of the field is 50 ac. 2 ro. 32 po. Find the length of the other side and of the diagonal.

(18) A rectangular field, 300 yards long and 150 broad, is separated into 4 equal parts by 2 bands of trees, 20 feet wide, parallel to the sides. How large will each part be, and what will be the area covered by the trees?

(19) A room, whose height is 11 feet, and length twice its breadth, takes 143 yards of paper 2 feet wide for its four walls. How many yards of gilt moulding will be required?

(20) What will be the cost of painting the walls and ceiling of a room whose height, length, and breadth are 12 ft. 6 in., 27 ft. 4 in., and 20 ft. respectively, at 36 cents per square yard?

(21) Find the expense of carpeting a room 15 ft. 9 in. long and 13 ft. 4 in. broad, with carpet 27 inches wide, at 95 cents per yard.

(22) Find the cost of carpeting a room 10 yd. 2 ft. long and 7 yd. 1 ft. broad, with carpet $\frac{3}{4}$ yd. wide, at $1.08 a yard.

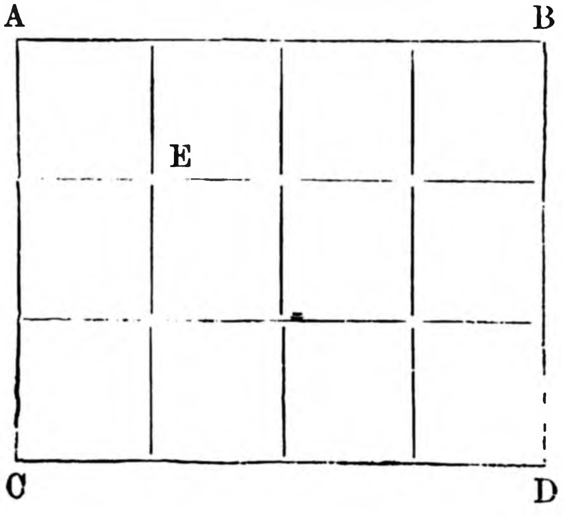
(23) If the cost of carpeting a room 11 yd. long and 8 yd. wide, with carpet at 3*s*. a yard be £19 16*s*., find the width of the carpet.

(24) How many flag-stones, each 5·76 ft. long and 4·15 ft. wide, are requisite for paving a cloister which incloses a rectangular court 45·77 yd. long and 41·93 yd. wide, the cloister being 12·45 ft. wide?

XXXIV. Measurement of Solidity.

228. The Unit of Measurement, by which we measure the Volume of a Solid body, or the Capacity of a vessel, is derived from the Unit of Length. Thus, if we take an inch as the unit of length, and construct a *cube*, each of whose edges is an inch in length, this Cubic Inch may be taken as the Unit of Volume, and the measure of any given volume will be the number of times it contains this unit.

229. Let ABDC be a rectangle, and let the side AB be 4 inches in length, and the side AC 3 inches in length.



Then ABCD will contain 12 square inches (Art. 225).

Now, suppose we construct a number of blocks of wood, perfect cubes, whose volume is a cubic inch, and place one of these on each side of the squares in ABDC, and then place another of the blocks on the top of each of the first set, and so on till we have piled up 5 layers. Then we shall have constructed a rectangular solid, whose length is 4 inches, breadth 3 inches, and depth or thickness 5 inches.

Now the number of cubic inches in this solid we estimate in the following way: for each of the squares in ABDC we shall have a pile of 5 cubic inches; therefore the number of cubic inches in the solid will be 5 × 12, or 60.

Hence we obtain the following Rule:

To find the cubic content of a rectangular solid, find the continued product of the measures of the length, breadth, and thickness, and the result is the measure of the cubic content.

Ex. (1). Find the cubic content of a rectangular piece of timber whose length is 47 ft., breadth 4 ft., and thickness 3 ft.

Taking a foot as the unit of length, and therefore a cubic foot as the unit of cubic content,

measure of cubic content is $47 \times 4 \times 3 = 564$;

$\therefore$ the cubic content is 564 cubic feet.

Ex. (2). What is the cubic content of a room whose length is 22 ft. 6 in., breadth 18 ft. 3 in. and height 10 ft.?

$$\text{Cubic content} = (22\tfrac{1}{2} \times 18\tfrac{1}{4} \times 10) \text{ cub. ft.}$$
$$= \frac{45 \times 73 \times 10}{2 \times 4} \text{ cub. ft.} = 4106\tfrac{1}{4} \text{ cub. ft.}$$

Ex. (3). A rectangular sheet of water, of uniform depth, is 430 yards long, 270 yards broad, and contains 7314300 cubic feet of water. What is its depth?

Reducing the length and breadth to feet,

$$\text{area of surface} = (430 \times 3 \times 270 \times 3) \text{ sq. ft.};$$

$$\therefore \text{depth} = \frac{7314300}{430 \times 3 \times 270 \times 3} \text{ ft.} = 7 \text{ ft.}$$

Examples. (cxxv.)

Find the cubic content of the rectangular solids whose dimensions are :

(1) 8 ft., 7 ft., 6 ft. (2) 10½ ft., 8¼ ft., 6⅓ ft.

(3) 5 ft. 6 in., 4 ft. 3 in., 3 ft. 7 in.

(4) 11 ft. 8 in., 9 ft. 10 in., 7 ft. 5 in.

(5) 6 yd. 2 ft. 4 in., 3 yd. 1 ft. 7 in., 4 ft. 11 in.

(6) How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 18 in. thick, each brick being 9 in. long, 4½ in. wide, and 3 in. deep?

(7) A lake, whose area is 45 acres, is covered with ice 3 inches thick. Find the weight of the ice in tons, if a cubic foot of ice weigh 920 oz. avoir.

(8) If 500 men excavate a basin 800 yd. long, 500 yd. wide, and 40 yd. deep, in 4 months, how many men will be required to excavate a basin 1000 yd. long, 400 yd. wide, and 50 yd. deep, in 5 months?

(9) A square block of stone, 2 ft. in thickness, is in cubic content 5 cub. ft. 24 in. What is the length of its edge?

(10) What weight of water will a rectangular cistern contain, the length being 4 ft., the breadth 2 ft. 6 in., and the depth 3 ft. 3 in., when a cubic foot of water weighs 1000 oz.?

(11) A block of stone is 4 ft. long, 2½ ft. broad, and 1¼ ft. thick; it weighs 27 cwt. Find the weight of 100 cubic inches of the stone.

(12) A cubic foot of water weighs 1000 oz. Find the length of the side of a cubic vessel whose contents (water) weigh 4 tons 12 cwt. 3 qr. 10 lbs. 7 oz. (112 lbs. = 1 cwt.)

(13) If 120 men can make an embankment ¾ of a mile long, 30 yards wide, and 7 yards high, in 42 days, how many men would it take to make an embankment 1000 yards long, 36 yards wide, and 22 feet high, in 30 days?

(14) A rectangular cistern, 9 ft. long, 5 ft. 4 in. wide, and 2 ft. 3 in. deep, is filled with liquid which weighs 2520 pounds. How deep must a rectangular cistern be which will hold 3850 pounds of the same liquid, its length being 8 ft., and its width 5 ft. 6 in.?

(15) Find the cost of making a road 110 yards in length and 18 feet wide; the soil being first excavated to the depth of 1 foot, at a cost of 1s. per cubic yard; rubble being then laid 8 inches deep, at 1s. per cubic yard, and gravel placed on the top, 9 inches thick, at 2s. 6d. per cubic yard.

EXAMINATION PAPERS.

(1) Find a number such, that if it be added twenty-three times to 37601, the sum will be 40200.

(2) A person bought 500 yards of cloth at $3.20 per yard, and retailed it at $3.35 per yard. What was his profit?

(3) Find the H. C. F. of 372, 837, 248; and arrange the three fractions $\frac{7}{9}$, $\frac{11}{13}$, $\frac{24}{29}$ in order of magnitude.

(4) A soldier takes 7920 paces in a march of $3\frac{3}{4}$ miles. Find his length of pace.

(5) Divide 9366 farthings into an equal number of sovereigns, half-sovereigns, half-crowns, and farthings.

(6) Divide ·14 by 7, 140 by ·07, and ·014 by 7000; add the results together, and turn the decimal into a vulgar fraction.

(7) Simplify the expression $7\cdot\dot{5}\dot{7} \times \cdot 3\dot{6} - 2\cdot 3\dot{4}\dot{5}$.

(8) The polar diameter of the earth is 41707796 feet. Reduce this to miles.

(9) If telegraph posts are placed 66 yards apart, and a train passes one in every three seconds, how many miles an hour is the train running?

(10) If a person spends in four months as much as he earns in three, how much can he lay by annually, supposing that he earns $420 every six months?

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(11) How many steps does a man, whose length of pace is 32 inches, take in $4\frac{3}{4}$ miles?

(12) Divide $13230 between 2 men, so that one may receive a third as much again as the other.

(13) Divide $\frac{41}{162} - \frac{9}{49} - \frac{3}{54}$ by $\frac{4}{9} + \frac{1}{2} - \frac{3}{14}$, and express the result as a decimal.

(14) Find the value of

$$\frac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{8} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$$

and express the result as a decimal.

(15) Simplify the expression $1\cdot\dot{3} \times (2\cdot\dot{4} + 7\cdot\dot{5}) + 2\cdot 3\dot{6}\dot{4} - 1\cdot 6\dot{9}\dot{7}$.

(16) Reduce 11 ro. 11 po. 11 yd. to inches, and find what fraction the result is of 3 acres.

(17) *A* is to receive $1.25 a day every day he works.

and to forfeit $.80 every day he is idle. At the end of 75 days his wages amount to $69.15. How many days was he idle?

(18) If 24 men can do a piece of work in 12 days of 10 hours each, how many men can do three times as much in 10 days of 8 hours each?

(19) If ·3 of an estate is worth $7500, what is the value of ·48 of the estate?

(20) *A*, *B*, and *C* start on a tour, each with $200 in his pocket, and agree to divide their expenses equally. When they return *A* has $37.50, *B* $50.82, and *C* $16.71. What ought *A* and *B* to pay *C* to settle their accounts?

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(21) Find the value of

$$\frac{1\frac{1}{4}-\frac{5}{12}}{1\frac{1}{4}+\frac{5}{12}}+\frac{7}{6}\text{ of }\frac{9\times 5}{14\times 3}-\frac{11\frac{1}{4}}{15},$$

and reduce to its lowest terms $\frac{2622}{3381}$.

(22) Express as vulgar fractions in the lowest terms 24·0025 and ·0008125; and divide 1·1214 by 5·34 and 1121·4 by ·534.

(23) What fraction is 7 cwt. 4 lb. of 3 tons 1 qr. (long ton)? How often must one go round a square field of 10 acres to run 1 mile?

(24) A gunboat's crew, consisting of a lieutenant, a gunner, and 15 seamen, captured a prize worth £399 7*s.*; the lieutenant's share is 10 times and the gunner's share 3 times as much as that of each seaman. What is the value of each person's share?

(25) Extract the square root of 167·9616, and of $\frac{529}{2401}$.

(26) A clock which loses 4 minutes in 12 hours is 10 minutes fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

(27) The distance between two wickets was marked out for 22 yards, but the yard measure was $\frac{5}{12}$ of an inch too short. What was the actual distance?

(28) What is the difference between simple interest, compound interest, and discount? Find the difference between the simple interest and the true discount on $1900 for $1\frac{3}{4}$ years at 8 per cent.

(29) What is the present worth of a bill of $170, due in 4 months, reckoning money at 6 % per annum?

(30) Find the interest on \$880 for $1\frac{1}{4}$ years at $4\frac{1}{2}$ per cent., and the discount on \$929.50 for $2\frac{1}{4}$ years, at $2\frac{1}{2}$ per cent.

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(31) Simplify $\dfrac{7\left(1\frac{1}{2} \text{ of } \dfrac{3}{14}\right)}{\dfrac{1}{6}\left(\dfrac{3}{3\frac{1}{2}} \text{ of } 7\right)} \div \dfrac{9}{14}.$

(32) Find the vulgar fraction equivalent to $\dfrac{1\cdot010\dot{1}\dot{5}}{\cdot55}$

(33) Which is the better investment, the $3\frac{1}{2}$ per cents at 91 or the 4 per cents. at 103?

How much must a man invest in the former that he may have a yearly income of \$4851, after paying an income tax of 2 cents in the dollar?

(34) Two ships get under weigh at the same time for the same port, distant 1200 miles. The faster vessel averages 10 knots an hour, and arrives at the port a day and a half before the other. What will the latter vessel average an hour?

(35) Divide \$87.50 between two men, so that one may receive half as much again as the other.

(36) A man has \$3430 stock in the $3\frac{1}{2}$ per cents. at $83\frac{1}{2}$; when the stock rises 2 per cent. he transfers his capital to the 4 per cents. at 98. Find the alteration in his income?

(37) The weight of the water contained in a rectangular cistern 8 ft. long, 7 ft. wide, is $93\frac{3}{4}$ cwt. If a cubic foot of water weigh 1000 oz., find the depth of water in the cistern.

(38) If \$3 is the discount off \$333 for 2 months, what was the rate per cent.? What should be the discount off \$333 for 1 year?

(39) The height of a tower on a river's bank is 55 feet, the length of a line from the top to the opposite bank is 78 feet. What is the breadth of the river?

(40) How many yards of matting, 2·4 feet broad, will cover a floor that is $27\cdot\dot{3}$ feet long and $20\cdot1\dot{6}$ feet broad?

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(41) Simplify the fraction

$$\frac{1\frac{1}{2} \text{ of } 1\frac{4}{5} - \frac{5}{3} \text{ of } 1\frac{1}{5}}{\dfrac{3\frac{1}{2}}{2} + \dfrac{4\frac{1}{4}}{11\frac{1}{3}}} \div \frac{2}{17}.$$

(42) If $\frac{2}{5}$ of $1\frac{1}{4}$ of an estate be worth \$300, what will be the value of $\frac{2\frac{1}{2}}{\frac{5}{14}}$ of the estate ?

(43) Of an electric cable $\frac{11}{13}$ rests on the bottom of the sea, $\frac{1}{15}$ hangs in the water, and $234\frac{2}{3}$ yards are employed on land ; what is the length of the cable ?

(44) Extract the cube root of 16777216.

(45) At what price must an article, which cost 15*s.*, be sold so as to gain 10 per cent. ?

(46) The number of disposable seamen at Portsmouth is 800, at Plymouth 756, and at Sheerness 404. A ship is commissioned, whose complement is 490 seamen. How many must be drafted from each place so as to take an equal proportion ?

(47) (*a*) Find the difference between the simple and compound interest of \$416.66$\frac{2}{3}$ for 2 years at 8 per cent.

(*b*) Find the rate of interest when the discount on \$211.60 due at the end of $1\frac{1}{2}$ years is \$27.60.

(48) What sum will amount to \$3213 in ten years at 8 per cent. simple interest ?

(49) The length of a rectangular field which contains 4 ac. 3 ro. 14 po. $26\frac{1}{2}$ sq. yd. is 260 yd. 1 ft. 4 in.; what is its breadth ?

(50) A room is 14 ft. 3 in. high, 20 ft. wide, 24 ft. long; what will it cost to paper it with paper $2\frac{1}{2}$ ft. wide, whose price is $11\frac{1}{4}$*d.* per yard ; allowing 8 ft. by 5 ft. 3 in. for each of four doors, 10 ft. by 6 ft. 8 in. for each of two windows, and 6 ft. 6 in. by 5 ft. for a fireplace ?

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(51) Simplify the fraction

$$\frac{\dfrac{3-\frac{1}{3}}{3+\frac{1}{3}} \text{ of } \dfrac{2-\frac{1}{2}}{2+\frac{1}{2}}}{\dfrac{3+\frac{1}{3}}{3-\frac{1}{3}} \text{ of } \dfrac{2+\frac{1}{2}}{2-\frac{1}{2}}}$$

(52) Find the value of

$\cdot 00\dot{3}$ of £1 5*s.* + $\cdot 06\dot{9}$ of £5 − ·8 of 2*s.* 3*d.*

(53) If $\frac{5}{7}$ of the cargo of a ship be worth \$16000, what will be the value of $\frac{2}{3}$ of $\frac{7}{8}$ of the remainder ?

(54) *A* can mow 5 acres of grass in 3 days, *B* 7 acres in 9 days, *C* 11 acres in 12 days ; in how many days can they jointly mow 121 acres ?

(55) A watch, which is 5 m. 40 s. fast on Monday at noon, is 2 m. 51 s. fast at midnight on the following Sunday: what did it lose in a day?

(56) The rent of a farm is $720, and the taxes are $14\frac{3}{5}$ per cent. on the rent: find the amount of rent and taxes together?

(57) Three persons divide the cost of an entertainment amongst them in such a manner that the first pays $\frac{1}{3}$ of the whole, and the second $\frac{6}{5}$ of what the first pays, and the third pays the remainder, which is $2.50: what is the amount of the bill?

(58) If an income of $1200 pays $18 for income tax, how much must be paid on an income of $750 when the tax is half as much again?

(59) *A* invests $552 in the $3\frac{1}{4}$ per cents. when they are at 92; *B* invests $679 in the 3 per cents. when they are at 97. Find the difference of their incomes.

(60) What is the cost of the carpet for a room, the dimensions of which are 21 feet long, $15\frac{5}{8}$ feet wide, at $42\frac{2}{3}$ cents per square yard?

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(61) Simplify:

$$\left(\frac{2\frac{3}{4}+3\frac{2}{3}}{4\frac{1}{5}+5\frac{1}{4}}+\frac{3\frac{2}{5}}{10\frac{1}{2}}\right)\times\left(\frac{2\frac{4}{11}}{2\frac{3}{5}}\div\frac{2\frac{7}{11}}{8\frac{7}{10}}\right)-\frac{\cdot281}{1\cdot405}$$

(62) A regiment marching $3\frac{1}{2}$ miles an hour makes 110 steps a minute: what is the length of the step?

(63) How long would a column of men, extending 3420 feet in length, take to march through a street a mile long at the rate of 58 paces in a minute, each pace being $2\frac{1}{2}$ feet?

(64) A street being 850 feet long, and the width of the pavement on each side being 5 feet 3 in. find the cost of paving it at $37\frac{1}{2}$ cents a square foot?

(65) Two pipes together fill a cistern in 1 hour: one of them alone fills it in $1\frac{1}{2}$ hour. How long will it take the other to fill it?

(66) How many hours a day must 42 boys work, to do in 15 days what 27 men can do in 28 days of 10 hours long; the work of a boy being half that of a many?

(67) At what rate will the simple interest on $125 amount to $13.12\frac{1}{2}$ in $1\frac{1}{2}$ years?

(68) What principal will give $616 simple interest in $5\frac{1}{3}$ years at $6\frac{2}{3}$ per cent.?

(69) A log of timber is 18 ft. long, 1 ft. 4 in. wide, and 15 in. thick. If a piece containing $2\frac{1}{2}$ solid feet be cut off the end of it, what length will be left?

(70) If 8 guineas be expended in purchasing Brussels carpet $\frac{3}{4}$ yd. wide, at 3*s.* 6*d.* a yard, for a room 20 ft. long and 16 ft. 9 in. broad, how much of the floor will remain uncovered?

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(71) Simplify $\dfrac{1\frac{1}{2} + 2\frac{1}{3}}{2\frac{1}{3} + 3\frac{1}{4}} \div \dfrac{\frac{5}{7}}{1 + \dfrac{1}{2 + \frac{1}{3}}} - \dfrac{\cdot 06}{\cdot 6}$.

(72) Find the value of

$\cdot 02$ of £1 + $\cdot 03$ of 7*s.* 6*d.* + $\cdot 01\dot{4}$ of 2*s.* 9*d.*

(73) Extract the square root of 30712·5625 of $\frac{625}{2401}$, and of ·000000133225.

(74) A bankrupt owes $7850, and pays $37\frac{1}{2}$ cents in the dollar. How much did his creditors jointly lose?

(75) If 14 men can mow 35 acres of grass in 6 days of 10 hours each, in how many days of 12 hours each can 3 men mow 24 acres?

(76) If 9 men or 16 women could do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together?

(77) Divide $2849 among *A*, *B*, and *C*, in the proportion of ·7, ·28, and ·056.

(78) The mathematical discount on a sum of money for 2 years is $360; the interest on the same sum for the same time is $400. Find the sum and the rate per cent.

(79) Find the gain or loss per cent. in buying oranges at $2.50 per hundred and selling them at 8 for 12 cents.

(80) What will be the cost of papering a room 21 ft. long by 15 ft. broad and 11 ft. high, which has two windows, each 9 ft. high and 3 ft. wide, a door 7 ft. high and 3 ft. 6 in. wide, and a fire-place 4 ft. high by 4 ft. 6 in. wide, with paper 2 ft. 3 in. wide at 9*s.* a piece; the price of putting it on being 6*d.* per piece, and each piece containing 12 yards?

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(81) Simplify

(1) $\dfrac{2\frac{4}{5} - 1\frac{1}{2} + 9\frac{1}{11}}{4\frac{1}{5} - 2\frac{1}{4} + 13\frac{7}{11}}$

(2) $\dfrac{(3\cdot 71 - 1\cdot 908) \times 7\cdot 03}{2\cdot 2 - \frac{74}{333}}$

(82) A man owns $\frac{3}{16}$ of a mine, and sells $\cdot 1\dot{3}5\dot{1}$ of his share. What fraction of the mine has he left?

(83) *A* and *B* can do a piece of work in 8 days, *B* and *C* can do it in 12 days, and *A*, *B*, and *C* can do it in 6 days. In how many days can *A* and *C* do it?

(84) A clock which gains $7\frac{1}{2}$ minutes in 24 hours is 12 minutes fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

(85) Gunpowder being composed of 33 parts of nitre, 7 of charcoal, and 5 of sulphur, find how many pounds of each will be required to make 30 lbs. of powder.

(86) What is the difference between Interest and Discount? Which of the two is greater?

Find the difference between the interest and discount on \$1639 for $4\frac{3}{4}$ mos. at $6\frac{3}{19}$ per cent.

(87) Find the difference between the true and bank discounts on a note of \$10400 due in 6 months (days of grace included), at 8 % per annum.

(88) $\frac{4}{5}$ of *A*'s stock was destroyed by fire, $\frac{7}{8}$ of the remainder was injured by water and smoke; he sold the uninjured goods at cost price, and the injured goods at a third of cost price. He realized \$1155. What did he lose by the fire?

(89) Having given that the weight of a cubic foot of water is 1000 oz., and that the imperial gallon contains 277·274 cubic inches, find the weight of a pint of water.

(90) A room is 22 ft. 6 in. long, 20 ft. 3 in. wide, and 10 ft. 9 in. high. Find the cost of carpeting the room at \$1.20 a square yard, and of papering the walls at 20 cents a square yard.

(91) Simplify

$$\frac{\cdot 004 \div \cdot 0005}{2\cdot 4\dot{2}\dot{3} + 3\cdot\dot{5}7\dot{6} + 2\cdot 0\dot{0}0191\dot{1}}$$

(92) The quotient in a division question equals six times the divisor, and the divisor equals six times the remainder; the three amount together to 516. Find the dividend.

(93) Add together ·60625 of £1 + $\cdot\dot{1}4285\dot{7}$ of 14*s.* $10\frac{1}{2}$*d.*, and $\frac{23}{11}$ of $\frac{3}{71}$ of £3 5*s.* 1*d.*, and express the result as the decimal of 27 shillings.

(94) A clock gains $3\frac{1}{2}$ minutes a day. How must the

hands be placed at noon so as to point to true time at 7 h. 30 m. P.M.?

(95) A person invests $750 at simple interest, and at the end of 3 years and 8 months he finds that he possesses $956.25; at what rate per cent. per annum was his profit?

(96) A person's half-yearly income is derived from the proceeds of $4550 at a certain rate per cent., and $5420 at 1 per cent. more than the former. His whole income is $453. Determine the rates.

(97) What will be the cost of enclosing a rectangular garden, 90 yd. long and 30 yd. 2 ft. 3 in. broad with a wall 8 ft. 4 in. high, at the rate of $1.20 per superficial square yard?

(98) A person invests £10000 in 3 per cents. at 75, and when they rise to 78 he sells out and invests the produce in bank shares at £208 each, which pay a dividend of £8 per share. Show that his income is not altered.

(99) What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

(100) If $40 is a proper discount off $360 for 8 months, what should be the 12 months' interest on $360?

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(101) Multiply 57875 by 729819 with three lines of multiplication, and divide 123456 by 63, using short division.

(102) A French metre = 1·0936 of a yard, and a centimetre is the hundredth part of a metre. Find a centimetre in decimals of an inch to 4 places.

(103) *A* and *B* can do a piece of work in 4 days, *B* and *C* in $5\frac{3}{4}$ days, and *A* and *C* in $4\frac{3}{5}$ days. In what time can each do the work separately?

(104) *M* starts from C and travels towards D at a rate of ·6 miles per hour; two hours afterwards *N* starts from C, and going 10 miles per hour reaches D 4 hours before *M*. Find the distance from C to D.

(105) Find the simple interest on $2733⅓ at 4 per cent. for 3 years and 9 months; and determine what sum will amount to $926.10 in 3 years at 5 per cent. compound interest.

(106) Find the difference between the discount on $1622.50 for 14 months at 7 per cent. per annum and the interest on $1760 for 15 months at 6 per cent. per annum.

(107) A women buys a certain number of apples at 3 a

penny, and the same number at 2 a penny; she then mixes them and sells them at 5 for twopence. How much does she gain or lose per cent.?

(108) A person, by disposing of goods for $182, loses 9 per cent. What ought they to have been sold at to realize a profit of 7 per cent.?

(109) Find the cost of papering a room 14 ft. 5 in. long, 13 ft. 7 in. broad, and 12 ft. 3 in. high, with paper at $14\frac{2}{5}$ cents per square yard. In the room are 4 windows 4 ft. by 3 ft., 2 doors 6 ft. 6 in. by 2 ft. 5 in., and a fireplace 5 ft. by 4 ft.

(110) The external dimensions of a box without a lid are, length 4 feet, breadth 3 feet, depth 2 feet, and the thickness of the sides and bottom is the same, namely 1 inch. If the cost of a cubic yard of the material is 9*s.*, and the cost of making the box $= \frac{1}{11}$ of the cost of the material, what will the box cost?

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(111) Eight bells begin tolling together at the same instant, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively. After what time will they be again tolling at the same instant?

(112) Simplify

$$\frac{1\frac{3}{4} - \frac{7}{6} \text{ of } \frac{18}{28}}{\frac{5}{6} \text{ of } \frac{12}{20} + 5\frac{1}{2}} \div \frac{1}{6} - \left(\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \text{ of } \frac{4}{7}\right) \div \frac{4}{7}.$$

(113) *A*, *B*, and *C* are partners; *A* receives two-fifths of the profits, *B* and *C* dividing the remainder equally; *A*'s income is increased by $220 when the rate of profit rises from 8 to 10 per cent. Find the capital of *B* and *C*.

(114) A railway train, having left a terminus at noon, is overtaken at 6 P.M. by another train which left the same terminus at 1 P.M. If the former train had been 10 miles farther on the road when the latter started, it would not have been overtaken till 8 P.M. Find the rates of the trains.

(115) A person invests £5000 in Turkish 6 per cent. stock at 80; find the rate of interest he gets for his money. When his stock has risen to 104 he sells out, and buys £20 railway shares at £18, which pay dividend at the rate of $4\frac{1}{2}$ per cent. Find the alteration in his income.

(116) If 6 men and 2 boys can reap 13 acres in 2 days, and 7 men and 5 boys can reap 33 acres in 4 days, how long will it take 2 men and 2 boys to reap 10 acres?

(117) The cost price of a book is $4.75, expense of the sale 6 %, profit 24 % ; what is the retail price?

(118) Show that the simple interest on $625 for 8 months at 7 % is equal to that on $1093.75 at 8 % for 4 months.

(119) One clock gains 4 minutes in 12 hours, and another loses 4 minutes in 24 hours. They are set right at noon on Monday. Determine the time indicated by each clock when the one appears to have gained 16½ minutes on the other.

(120) A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth, and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 12½ cents per square foot, and of the grass 70 cents per square yard, find the whole cost of laying out the court.

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(121) How many times does the 29th day of the month occur in 400 consecutive years?

(122) A creditor, agreeing to receive $281.25 for a debt, finds that he has been paid at the rate of 62½ cents in the dollar; how much was the debt?

(123) *A*, *B*, and *C* rent a meadow for $43. *A* puts in 10 horses for 1 month, *B* 12 oxen for 2 months, and *C* 20 sheep for 3 months. How should the expense be divided if the quantities eaten by a horse, an ox, and a sheep during the same time be in the ratio of 4, 3, and 1?

(124) If the price of 9760 bricks, of which the length, breadth, and thickness are 20 inches, 10 inches, and 12½ inches respectively, be $213.50, what will be the price of 100 bricks which are one-fifth smaller in every dimension?

(125) How many years' purchase should I give for an estate so as to get 3⅓ per cent. interest for my money?

(126) How often between 11 and 12 are the hands of a clock an integral number of minute spaces apart?

(127) *A* and *B* walk a race of 25 miles; *A* gives *B* 45 minutes' start; *A* walks uniformly a mile in 11 minutes, and catches *B* at the 20th milestone; find *B*'s rate, and by how much he lost in time and space.

(128) A debt is due at the end of 4½ months; ¼ is paid immediately, and ¼ at the end of 3 months; when ought the remainder to be paid?

(129) A man, by selling out of a 3 per cent. stock at 99, gains 10 per cent. on his investment. At what price did he buy, and what was his income, supposing that he realized $15345?

(130) A tank is 8 ft. long, 5 ft. 4 in. wide, 4 ft. 6 in. deep. Find the number of gallons it contains, having given that 1 cub. ft. of water weighs 1000 oz., and that a pint of water weighs a pound and a quarter.

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(131) Simplify

$$\frac{1}{7\frac{1}{4} \text{ of } 3\frac{3}{11} + 3\frac{3}{11}} \div \left(\frac{3}{13} - \frac{2}{9}\right) - \left(\frac{13}{3} + \frac{1}{6}\right) \div \frac{2}{3} \text{ of } \frac{3}{8} \text{ of } 63.$$

(132) In a dormitory $\frac{11}{23}$ of the boys are in the upper school, $\frac{2}{3}$ of the remainder in the middle, and the rest, 8 in number, in the lower. Find the number in the dormitory.

(133) The circumference of the fore-wheel of a carriage is 8 feet, and that of the hind-wheel is 10 feet. In what distance will the fore-wheel make 100 revolutions more than the hind-wheel?

(134) *A* and *B* receive $1.37½ for digging a garden. They work at it together for 4½ hours; *B* then left, and *A* finished the work in 3½ hours. How should the pay be divided?

(135) What are the two exact times when the hands of a watch are equally distant from fig. III.?

(136) In how many years will $320 double itself at 7½ per cent. simple interest?

(137) A person invests the present value of £2358, due two years hence at 4 per cent., in gas shares, which pay at the rate of 9 per cent.; he gives £144 for each share of £100. What is his annual income, and what rate per cent. does he make of his money invested in the gas shares?

(138) At billiards *A* can give *B* 5 points in a game of 50, and *C* 10 points in 50. How many points can *B* give *C* in a game of 90?

(139) How much money must one invest in 3 per cent. Consols, when they are at 10 per cent. below par, in order to have an income of £2000 a year?

(140) A level reach in a canal, 14 miles 6 furlongs long and 48 feet broad, is kept up by a lock 80 feet long, 12 feet broad, and having a fall of 8 ft. 6 in. How many barges

might pass through the lock before the water in the upper canal was lowered one inch ?

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(141) Find the value of $\dfrac{3\frac{1}{7}}{7\frac{1}{5} \text{ of } \frac{3}{4}} \times \dfrac{\frac{5}{7} - \frac{1}{4}}{\frac{3}{8} + \frac{3}{7}}$ of $5.67.

(142) *A* can do a piece of work in 6 days, which *B* can destroy in 4. *A* has worked for 10 days, during the last 5 of which *B* has been destroying; how many days must *A* now work alone, in order to complete his task ?

(143) Two cisterns of equal dimensions are filled with water, and the taps for both are opened at the same time. If the water in one will run out in 5 hours, and that in the other in 4 hours, find when one cistern will have twice as much water in it as the other has.

(144) If 3 men, 4 women, 5 boys, or 6 girls, can perform a piece of work in 60 days, how long will it take 1 man, 2 women, 3 boys, and 4 girls, all working together?

(145) Two trains start at the same time from London and Edinburgh, and proceed towards each other at the rates of 30 and 50 miles per hour respectively. When they meet, it is found that one train has run 100 miles farther than the other. Find the distance between London and Edinburgh.

(146) Two persons buy respectively with the same sums into the 3 and 3½ per cents., and get the same amount of interest. The 3 per cents. are at 75 : at what price are the 3½ per cents. ?

(147) Divide $1986.50 among *A*, *B*, and *C*, in the proportion of $2{\cdot}\dot{3}$, $1{\cdot}1\dot{5}$, and ${\cdot}5\dot{2}\dot{4}$ respectively.

(148) If for a sovereign one can buy 11 gulden 12 kreutzers or 25·5 francs, and for one 20-franc piece 9 gulden 20 kreutzers, how much per cent. is gained by buying French gold with English gold before buying German money ?

(149) Express 69½ miles in metres, 32 metres being taken to be equivalent to 35 yards.

(150) Find the cost of painting the walls of a square room 14 ft. high and 18 ft. long, with two doors 8 ft. by 4, and three windows 10 ft. by 5, the amount saved by each window being £2 16*s*. 3*d*. What additional height would increase the cost by nine shillings.

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(151) Simplify $\dfrac{2\frac{1}{4}}{2\frac{2}{3}} + \dfrac{2\frac{1}{2} + 3\frac{1}{5}}{3\frac{1}{4} + 9\frac{1}{2}} \div \frac{1}{2} + \frac{3}{8}$ of $\frac{3}{10}$.

(152) Two lines are 41·06328 inches and ·0438 of an inch long respectively. How many lines as long as the latter can be cut off from the former, and what will be the length of the remaining line?

(153) *A* and *B* start to run a race; their speeds are as 17 to 18. *A* runs $2\frac{1}{3}$ miles in 16 min. 48 sec.; *B* finishes the course in 34 minutes: determine the length of the course.

(154) A boat's crew row over a course of a mile and a quarter against a stream which flows at the rate of 2 miles an hour, in 10 minutes. The usual rate of the stream is half a mile an hour. Find the time which the boat would take in the usual state of the river.

(155) A person pays one tax of 10*d.* in the £, and another of 5 per cent. on his income. His remaining income is £545. What was his original income?

(156) A man invested $14350 in the U. S. 6's at $107\frac{1}{2}$, the brokerage being $\frac{1}{8}$%; what will be his clear income after an income tax of 5% is deducted?

(157) A soldier has 5 hours' leave of absence: how far may he ride on a coach which travels 10 miles an hour, so as to return to the camp in time, walking at the rate of 5 miles an hour?

(158) Two trains start at the same time, the one from London to Norwich, the other from Norwich to London. If they arrive in Norwich and London respectively 1 hour and 4 hours after they pass each other, show that one travels twice as fast as the other.

(159) When £170 will purchase 4233 francs, what is the *course* of exchange between London and Paris? And if 503 gold pieces of 20 francs contain as much pure gold as 400 sovereigns, what is the *par* of exchange between London and Paris?

(160) A hollow cubical box, made of material which is $1\cdot\dot{3}$ inches in thickness, has an interior capacity of 50·653 cubic feet: determine the length of the outside edge of the box.

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(161) Simplify $\left(6\frac{3}{7} \text{ of } \frac{5\frac{1}{6} - 4\frac{7}{12}}{12\frac{2}{3} - 7\frac{5}{12}}\right) \div \frac{1 + \frac{1}{2\frac{1}{3}}}{2}$

(162) Gold of the value of £423267 arrives from Australia; what is its weight in lbs. avoirdupois, the price being £3 18*s.* per oz. troy?

(163) *A* can do one-half of a piece of work in 1 hour, *B* can do three-fourths of the remainder in 1 hour, and *C* can finish it in 20 minutes; how long would *A*, *B*, and *C* together take to do it?

(164) If I pay $750 now for a debt of 771.09\frac{3}{8}$ not yet payable, and money be considered worth 7$\frac{1}{2}$ per cent. per annum, when will the debt be due?

(165) Two equal wine-glasses are filled with mixtures of spirit and water in the ratios of 1 of spirit to 3 of water and 1 of spirit to 4 of water; when the contents are mixed in a tumbler, find the strength of the mixture.

(166) At what per cent. in advance of cost must a merchant mark his goods so that after throwing off 20 per cent. of the marked price he may make a profit of 25 per cent.?

(167) A man receiving a legacy of $34510 invested one-half in Dominion 6 per cents. at 101, and the other half in U. S. 5 per cents. at 84$\frac{1}{2}$, paying brokerage at $\frac{1}{2}$ %; what annual income did he secure from his legacy?

(168) A piece of work must be finished in 36 days, and 15 men are set to do it, working 9 hours a day; but after 24 days it is found that only three-fifths of the work is done. If 3 additional men be then put on, how many hours a day will they all have to labor in order to finish the work in time?

(169) Of two stalactites hanging from the flat roof of a cavern, one is 1·02 inches longer than the other, and the shorter one increases in length at the rate of 3·014 inches in a century. Find the rate of increase of the other, in order that they may be of the same length at the end of 125 years.

(170) Two men, *A* and *B*, start from Cambridge, at 4 and 5 o'clock a.m. respectively, to walk to London, a distance of 50 miles; *B* passes *A* at the twentieth milestone, and reaches London at 5 p.m. When will *A* arrive there?

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(171) Find the square root of 10747·4689, and the cube root of 189119224.

(172) A person can read a book containing 220 pages, each of which contains 28 lines, and each line on an average 12 words, in 5$\frac{1}{2}$ hours; how long will it take him to read a book containing 400 pages, each of which contains 36 lines, and each line on an average 14 words?

(173) The whole time occupied by a train 120 yards long,

travelling at the rate of 20 miles an hour, in crossing a bridge is 18 seconds. Find the length of the bridge.

(174) If 20 men, 40 women, and 50 children receive $4200 among them for seven weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

(175) Two clocks begin to strike 12 together; one strikes in 35 seconds, the other in 25. What fraction of a minute is there between their seventh strokes?

(176) A speculator bought 43 shares in a mine at $35\frac{1}{4}$, and kept them till they dropped to $11\frac{1}{2}$, when he sold out and bought with the proceeds 6 per cent. railway stock at 28 premium. Find his annual income from the latter investment.

(177) Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 minutes to 11 when the other strikes 11. How much must the faster be put back that they may strike 9 together on Wednesday evening?

(178) How much ore must one raise that on losing $\frac{17}{40}$ in roasting and $\frac{8}{19}$ of the residue in smelting, there may result 506 tons of pure metal?

(179) If a population is now ten millions, and the births are 1 in 20 and the deaths 1 in 30 annually, what will the population become in 5 years?

(180) There are two rectangular fields equal in area; the sides of one are 945 yards and 1344 yards in length, and the longer side of the second is 1134 yards. What is the length of its shorter side, and how many acres are there in each field?

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(181) The masters of a school are ·0416 of its whole number, but after 40 new boys have been added the masters became ·0375 of the whole. How many boys and masters were there before the new boys came?

(182) Divide $350 among 4 persons, so that *B* may have three times as much as *A*, *C* half as much again as *A* and *B* together, and *D* as much as *A*, *B*, and *C* together?

(183) By selling a house for $3700 I lost $7\frac{1}{2}$ per cent. What must I have sold it for to have gained $12\frac{1}{2}$ per cent.?

(184) Find the difference between the interest and discount on $1265 for 73 days at 6 %.

(185) A merchant sells tea to a tradesman at a profit of 60 per cent., but the tradesman, becoming a bankrupt, pays

$37\frac{1}{2}$ cents in the dollar. How much does the merchant gain or lose by the sale?

(186) What sum must a man invest in the 6 per cent. County bonds at $101\frac{1}{2}$ in order to have a clear income of $1424.40, after paying an income tax of $1\frac{1}{2}$ % on all over $400?

(187) A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses 20 per cent. The price of flour falls 50 per cent., and other trade expenses are thereby reduced 25 per cent. What reduction should he make in the price of a 15c. loaf, allowing him still to realize the same amount of profit?

(188) What is the average annual profit of a business when a partner, entitled to $\frac{2}{7}$ of the profits, receives as his share for 2 years and 4 months the sum of $7890.50?

(189) If a tradesman adds to the cost price of his goods a profit of $12\frac{1}{2}$ per cent., what is the cost price of an article which he sells for $3.82\frac{1}{2}$?

(190) A rectangular piece of ground 72 yards by 45 yards is to be laid out in 4 plots of grass, each 27 feet by $13\frac{1}{2}$ feet, and a pond in the centre 6 yards square, to contain 252 cubic yards of water. Find the expense of gravelling the remainder at $2\frac{2}{3}$ cts. per square yard, and the depth of the pond.

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(191) Find the value of

$$\frac{5\frac{1}{2} \text{ of } \frac{2}{9} \text{ of } 2\frac{4}{7} - 1 \div \left(\frac{1}{5} + \frac{1}{2}\right)}{1 - \frac{3}{14} \text{ of } \left\{\frac{1}{2} + \frac{1}{2} \text{ of } \dfrac{\frac{1}{20}}{\frac{1}{7} \text{ of } 1\frac{1}{20}}\right\}}$$

(192) If 12 men or 18 boys can do $\frac{3}{4}$ of a piece of work in $6\frac{1}{2}$ hours, in what time will 11 men and 9 boys do the rest?

(193) Find the principal sum on which the simple interest in $2\frac{1}{4}$ years at $6\frac{1}{3}$ % per annum is $1068.75.

(194) The compound interest on a certain sum at 4 per cent. for 2 years exceeds the simple interest for the same time at the same rate by $6. What is the sum?

(195) Two ships are built. Twice as many ship-carpenters are employed about the first as about the second; the first is built in 9 months, the second in 8 months; the wages of each man of the first set are 25 cents per hour, and they work 12 hours a day; the wages of each of the second set are 18 cents per hour, and they work $10\frac{1}{2}$ hours a day. The cost of the first in carpenters' wages was $300.00; what was that of the second?

(196) A person leaves \$12670 to be divided among his five children and three brothers, so that, after the legacy duty has been paid, each child's share shall be twice as great as each brother's. The legacy duty on a child's share being one per cent., and on a brother's three per cent., find what each will receive.

(197) Two persons, *A* and *B*, meet to settle their accounts. *A* had $3\frac{1}{4}$ years previously lent *B* \$500, and *B* has a bill of \$360.50 against *A*, for which he is to allow nine months' discount. If the *interest* in each case is 4 per cent. per annum, what has *B* to pay *A*?

(198) A grocer buys 4 cwt. 3 qr. 14 lb. of sugar at £1 16s. 8*d.* per cwt. (long cwt.), and sells it at $4\frac{1}{2}d.$ per lb. How much does he gain or lose per cent.?

(199) If when 25 per cent. is lost in grinding wheat a country has to import 10000000 quarters, but can maintain itself on its own produce if only 5 per cent. be lost, find the quantity of wheat grown in the country.

(200) A man rows down a river 18 miles in 4 hours, with the stream, and returns in 12 hours. Find the rate at which he rows, and the rate at which the stream flows.

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(201) Show that

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{5}}}}}+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1}}}}}=\frac{8}{9}.$$

(202) *A* and *B* can do a piece of work in 6 days, *B* and *C* in 8 days, *A*, *B*, and *C* in 4 days. How long would *A* and *C* take to do it?

(203) If, by selling an article for \$38.25, 8 per cent. is lost, what per cent. is gained or lost by selling it for \$57.

(204) A French metre contains 39·371 English inches. Express to three decimal places an English mile in metres.

(205) A tradesman marks his goods with two prices: the one for ready money, the other for 6 months' credit, the rate of interest being 5 per cent. per annum. If the credit price of an article be \$2.05, what ought its ready-money price to be?

(206) Compound interest reckoned quarterly at 2% is equal to what interest reckoned yearly?

(207) A person having $9790 in the Toronto city 6 per cent. bonds sells out at 98½, and invests the proceeds in Bank of Montreal stock at 177½, which pays a dividend of 12 per cent. per annum. Find the change in his income, brokerage in each transaction being ½ %.

(208) I buy wheat at 39*s.* a quarter, and some of a superior quality at 6*s.* per bushel; in what proportion must I mix them so as to gain 25 per cent. by selling the mixture at [illegible]*s.* 6*d.* per quarter?

(209) The weight of a cubic foot of water being 1000 oz., find the weight of a rectangular block of gold 8 inches in length, 2 inches in thickness, and 3 inches in breadth, the weight of a mass of gold being 19·26 times the weight of an equal bulk of water.

(210) The contents of a cistern is the sum of two cubes whose edges are 10 inches and 2 inches, and the area of its base is the difference between two squares whose sides are $1\frac{1}{9}$ and $1\frac{2}{9}$ feet. Find its depth.

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(211) Find the value of $\cdot 857\dot{1}4285\dot{7}$ of £10 14*s.* 1*d.* accurately; and show that the error committed by neglecting all decimals of an order higher than the fifth is less than $\frac{1}{125}$ of a penny.

(212) The sum of $327 is borrowed at the beginning of a year at interest, and after 9 months have passed $400 more is borrowed at a rate of interest double that which the former sum bears. At the end of the year the interest on both loans is $26.35. What is the rate of interest in each case?

(213) A dealer purchases a liquid at 4*s.* per gallon, and dilutes it with so much water that, when he sells the compound at 3*s.* a gallon, he gains 20 per cent. on his outlay. How much water is there in every gallon of the compound sold?

(214) The discount on $566.50 for 9 months is $16.50; find the rate of interest.

(215) A merchant lost a cargo at sea which he had insured; the broker offered him a sum of money for his loss, which the merchant refused as being 10 per cent. below the estimated value of his loss; the broker then offered $379.75 more than he offered at first, and the whole amount of the

second offer was 5½ per cent. in excess of the estimated value. What was that value?

(216) A man wishing to sell a horse asked 25 per cent. more than it cost; he finally sold it for 15 per cent. less than his asking price, and gained $5.75. How much did the horse cost, and what was the asking price?

(217) If 15 masons, working 10 hours a day, can build a wall 6 feet high and 100 yards long in 6 days, how long will it take 7 masons, working 9 hours a day, to build a wall 9 feet high and 140 yards long?

(218) A bankrupt's assets are $2700, out of which he pays 75 cents in the dollar on half his debts, and 60 cents on the other half. What is the amount of his debts?

(219) If a ship containing 150 hhd. of wine pays for toll at the Suez Canal the value of 2 hhd., wanting $30; and another, containing 240 hhd., pays at the same rate, the value of 2 hhd. and $90 besides; what is the value of the wine per hhd?

(220) A picture-gallery consists of three large rooms; the first is 20 yd. long, 20 yd. broad, and 6 yd. high; the other two are 20 yd. long, 20 yd. broad, and 5 yd. high. Supposing the walls to be covered with pictures, except the doors, which are 8 ft. high and 3 ft. wide, and of which each room has two, what will be the number of pictures, the average size being 8 feet by 3 feet?

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(221) Simplify $\dfrac{3{\cdot}\dot{5}-1{\cdot}8\dot{3}}{9{\cdot}\dot{7}-6{\cdot}\dot{4}}\cdot\dfrac{1}{71}\div\dfrac{3{\cdot}1\times{\cdot}\dot{1}0\dot{1}}{2{\cdot}\dot{1}\dot{5}}$

(222) Find the square roots of 15376·248001 and $\dfrac{31{\cdot}36}{39{\cdot}69}$

(223) A general, after losing a battle, found that he had only two-thirds of his army left fit for action; one-ninth of the army had been wounded, and the remainder, 2000 men, killed or missing; of how many did the army consist before the battle?

(224) A contractor sends in a tender of $20,000 for a certain work; a second sends in a tender of $19,000, but stipulates to be paid $2000 every three months; find the difference between tenders, supposing the work in both cases to be finished in two years, and money to be worth 7½ per cent. simple interest.

(225) What sum of money must be left in order that, after a legacy duty of 10 % has been paid, the remainder being invested in the Dominion 5 per cents. at $91\frac{1}{8}$ may give a yearly income of $450, brokerage at $\frac{1}{8}$ %.

(226) If two boys and one man can do a piece of work in 4 hours, and two men and one boy can do the same in 3 hours, find in what times a man, a boy, and a man and a boy together, respectively, could do the same.

(227) Show that the interest on $15840 for 3 months at 8 per cent. is equal to the discount on $3696 for 15 months at $7\frac{1}{2}$ per cent.

(228) A piece of work has to be finished in 36 days, and 15 men are set to do it, working 9 hours a day; but after 24 days it is found that only three-fifths of the work is done; if 3 additional men be then put on, how many hours a day will all have to work so as to finish the task in time?

(229) The interest on a certain sum at simple interest is $360, and the discount $340 for the same time and rate. What is the sum?

(230) The breadth of a room is twice its height and half its length, and the contents are 4096 cubic feet. Find the dimensions of the room.

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(231) If 1 lb. of tea be worth 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea when a lemon is worth a penny?

(232) At a certain battle two-thirds of the defeated army ran away with their arms, five-sevenths of the remainder left their arms on the field, and of the rest seven-eighths were missing, the remaining 500 being either killed or wounded. Find the whole number of the army.

(233) If gold be at a premium of 20 per cent., and a person buy goods marked $135, and offers gold to the amount of $135, what change ought he to receive in notes, 5 per cent. being abated for ready payment.

(234) Show that the difference between the interest and the discount on the same sum for the same time is the interest of the discount.

(235) I bought 20 lbs. of opium by Avoirdupois weight at 55 cents per ounce, and sold by Troy weight at 60 cents per ounce. Did I gain or lose, and how much?

(236) By investing a certain sum of money in the 6 per cents. at $91\frac{1}{2}$ a man obtains an income of $320; what would

he obtain by investing an equal sum in the 5 per cents. at 80?

(237) A tradesman makes a deduction of 10 per cent. for ready money on a bill of \$28 due in 12 months, receiving \$25.20. Find the difference between this sum and the present worth of the debt, reckoning interest at 10 per cent.

(238) *M* invests one-third of his property in bank stock, one-sixth in Consols, and the remainder in railway shares. When he sells out he makes a profit of 5 per cent., 3 per cent., and 2 per cent. respectively on the investments, and realizes £6190. Required the amount of his property originally.

(239) Mr. A. sent \$3681 to his agent with instructions to deduct his com. at $2\frac{1}{4}$ % and invest the balance in flour at \$7.50 per bbl. If the cost of freightage and insurance amounts to \$119, at what must the flour be sold per bbl. so as to make a profit of 20 % ?

(240) How many bricks, 9 in. long, $4\frac{1}{2}$ broad, and 4 thick, will be required for a wall 60 ft. long, 20 ft. high, and 4 ft. thick, allowing $6\frac{1}{4}$ per cent. of the space for mortar?

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(241) What is the value of

$$\cdot 25 \text{ of } \tfrac{1}{14} \text{ of } \frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{2}-\frac{1}{3}} \text{ of 8 guineas?}$$

(242) A work can be accomplished by *A* and *B* in 4 days, by *A* and *C* in 6 days, by *B* and *C* in 8 days. Find in what time it would be accomplished by all working together.

(243) A man hired a laborer to do a certain amount of work, on the agreement that for every day he worked he should have \$1.50, but that for every day he absented himself he should lose 60 cents. He worked twice as many days as he absented himself, and received on the whole \$72. Find how long he was doing the work.

(244) A legacy of \$146000 is left to three sons in the proportion of $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$ respectively. How much will each receive?

(245) If \$10 is a proper discount off \$210 for 3 months, what should be a proper discount off the same sum for 1 year?

(246) The price of gold in this country is £3 17*s.* $10\frac{1}{2}$*d.* an ounce. Find the least number of ounces which can be coined into an exact number of sovereigns, and the number of sovereigns so coined.

(247) A merchant in Toronto instructed his agent in Montreal to sell a consignment of flour at $7.50 per barrel and invest the proceeds in Montreal bank stock at 174½, which pays half-yearly dividends of 7 %. If the merchant's first dividend is $445.50, and commissions of 1 % and ½ % be allowed on the transactions respectively, how many barrels of flour were sold ?

(248) State the connection between Troy and Avoirdupois weights. A ring weighs 1 dwt. 4 gr., and is worth £1 2*s.* If 1050 of such rings be packed in a box weighing 3½ lb., what would it cost to convey them 144 miles at the rate of 5*s.* per long ton per mile, insurance being demanded at the rate of ⅛ per cent. ?

(249) How long will it be before $2500, put out to compound interest at 10 per cent. per annum, will obtain $1727.58⅞ as interest ?

(250) The breadth of a room is two-thirds of its length and three-halves of its height, and the contents are 5832 cubic feet. Find the dimensions of the room.

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(251) Multiply 32856 by 121711, using 3 lines of multiplication only.

(252) Simplify $\dfrac{2\cdot8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot1\dot{3}\dot{6}} + \dfrac{4\cdot\dot{4} - 2\cdot8\dot{3}}{1\cdot\dot{6} + 2\cdot\dot{6}2\dot{9}} \text{ of } \dfrac{6\cdot8 \text{ of } 3}{2\cdot25}$.

(253) An agent received $21.70 for collecting a debt of $2480. What rate was his commission ?

(254) A man sells out of the U.S. 6's 5–20 of '85 at 92½ and realizes $25760. If he invests the proceeds in Erie R. R. stock at 45, which pays a yearly dividend of 3¼ %, what alteration in his income has ensued, brokerage on each of the two transactions being ½ % ?

(255) A farmer bought a horse for a bill of $292, due in 1 month, and sold him for a bill of $348, due in 4 months. What did he gain per cent., money being worth 4½ % ?

(256) A man and a boy are to work on alternate days at a piece of work which would have occupied the boy alone 13 days. If the boy take the first day the work will be finished half a day later than if the man commences. Find how long they would take to do it working together.

(257) Two men invest $300 and $100 in a machine ; it works 5 months for each of them. Determine what one must pay the other if they would have made 30 per cent. on the money by letting the machine.

(258) *A* owes *B* $2725, and offers to pay him at a certain rate of discount instantly, instead of at the end of two years, when the debt will be due. *B* can place out the money which he will receive at 5 per cent. interest, and by that means gain $25 on the transaction. At what rate is the discount calculated?

(259) If 36 men, working 8 hours a day for 16 days, can dig a trench 72 yards long, 18 wide, and 12 deep, in how many days will 32 men, working 12 hours a day, dig a trench 64 yards long, 27 wide, and 18 deep?

(260) A man discounts a bill of £180, drawn at 4 months, at 60 per cent. per annum, and insists on giving in part payment 5 dozen of wine, which he charges at 4 guineas a dozen, and a picture, which he charges at £19. How much ready money does he pay? If the cost to the man of wine and the picture be only one-fourth of the sum he charged for them, what is the real interest the man has be charged?

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(261) One-tenth of a rod is coloured red, one-twentiet orange, one-thirtieth yellow, one-fortieth green, one-fiftiet blue, one-sixtieth indigo, and the remainder, which is 30 inches long, violet. What is the length of the rod?

(262) The discount on a certain sum, due 9 months hence, is $20, and the interest on the same sum for the same time is $20.75. Find the sum and the rate of interest.

(263) Two persons, walking at the rate of 3 and 4 miles per hour respectively, set off from the same place in opposite directions to walk around a park, and meet in 10 minutes. Find the length of the walk round the park.

(264) In a hundred yards race *A* can give *B* four and *C* five yards' start. If *B* were to race *C*, giving him one yard in a hundred, which would win?

(265) A man buys an article and sells it again so as to gain 5 per cent. If he had bought it at 5 per cent. less, and sold it for $1 less, he would have gained 10 per cent. Find the cost price.

(266) If the difference between the simple and compound interest on a sum of money for two years at 5 per cent. be $3, find the sum.

(267) If 7 per cent. be gained by selling goods for $69.55, what will be gained or lost by selling them for $61.75?

(268) A draft on Dublin for £360 cost $1736.10. What

was the course of exchange, commission charged at the rate of $\frac{1}{4}$ per cent.?

(269) A banker, in discounting a bill due in 3 months at 8 per cent., charges $16 more than the true discount. Find the amount of the bill.

(270) A grocer mixes 18 pounds of coffee at 30 cents a pound with 12 pounds of chicory at 5 cents a pound. At what price must he sell the mixture to gain 25 per cent.?

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(271) The following rule has been given to divide by 3·14159: "Multiply by 7, divide by 11, then by 2, and add $\frac{1}{8}$th of $\frac{1}{1000}$th of the result." Find the error made in obtaining 1 ÷ 3·14159 by this process.

(272) Prove that $\frac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

(273) The estate of a bankrupt (value $21000) is to be divided among four creditors, whose claims are, *A*'s to *B*'s as 2 to 3, *B*'s to *C*'s as 4 to 5, *C*'s to *D*'s as 6 to 7. What must each receive?

(274) Which is the more profitable, to buy flour at $6.50 on 6 months, or $6.30 cash, money being worth 8 per cent.?

(275) If $10.50 be a person's income tax at $1\frac{1}{2}$ cents on the dollar, how much in the dollar is it when his income tax is $12.25?

(276) If 9 tons $7\frac{1}{2}$ cwt. of iron be sold for $1260, and the gain on it be 20 per cent., what was the cost per cwt.?

(277) I send to my agent in Montreal $3060 to invest in tea at 75 cents per lb.; he deducts his commission of 2 per cent., and invests the balance. At what must I sell per lb. so as to make a clear profit of 25 per cent., after paying freightage $30, and insurance at the rate of $\frac{1}{3}$ per cent.?

(278) A banker borrows money at $3\frac{1}{2}$ per cent., and pays the interest at the end of the year; he lends it out at 5 per cent., but receives the interest half-yearly, and by this means gains $200 per year. How much does he borrow?

(279) A dealer sends out 250 lbs. of tea at 80 cents per lb., and allows $2\frac{1}{2}$ per cent. on the price for the expense of carriage. Supposing the whole amount of carriage to come to $9.30, how much will the customer have to pay?

(280) A plate of gold, 3 inches square and one-eighth of an inch thick, is extended by hammering so as to cover a surface of 7 yards square. Find its proper thickness.

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(281) A man having a flock of sheep sold 8 per cent. of them to *A*, 90 to *B*, 3½ per cent. of the remainder to *C*, and 29 to *D*. He then had 550 left. How many had he at first?

(282) The product of the 1st and 2nd of three numbers is 176382; of the 1st and 3rd is 279152; of the 2nd and 3rd is 215496: what are the numbers?

(283) Find the rate of 2 trains 150 ft. and 180 ft. long respectively which pass each other going the same way in 15 secs., and going in opposite directions in 3 secs.

(284) By selling tea at 72 cents a pound a grocer clears ⅕ of his outlay. He then raised the price to 90 cents. What does he clear per cent. by the latter price?

(285) A grocer buys 1¾ cwt. of tea at 60 cents per lb. and 2¼ cwt. of tea at 50 cents per lb., and mixes them; he sells 2½ cwt. at 55 cents per lb.: at what rate must he sell the remainder to gain 20 per cent. on his outlay?

(286) In England gunpowder is made of 75 parts nitre, 10 sulphur, and 15 charcoal; in France of 77 parts nitre, 9 sulphur, and 14 charcoal: if half a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

(287) A ship 40 miles from the shore springs a leak, which admits 3¾ tons of water in 12 minutes. 60 tons would suffice to sink her; but the ships pumps can throw out 12 tons of water in an hour. Find the average rate of sailing that she may reach the shore just as she begins to sink.

(288) The receipts of a railway company are apportioned in the following manner: 48 per cent. for the working expenses, 10 per cent. on one-fifth of the capital, and the remainder, $32000, for division among the holders of the rest of the stock, being a dividend at the rate of 4 per cent.; find the capital and the receipts.

(289) If the discount on a sum due at the end of 2½ years is $\frac{80}{87}$ of the simple interest, at what rate is that calculated?

(290) If a crew, which can row from A to B in 60 minutes, can row from B to A in 55 minutes, compare the rates of the stream and boat.

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(291) Simplify

(*a*) $$\frac{2+\cfrac{1}{3-\cfrac{1}{5+\cfrac{1}{6}}}}{1\frac{5}{8}\div\left(1\frac{3}{4}\times 14\frac{1}{2}\right)}$$

(*b*) $$\frac{6\frac{3}{4}+5\frac{1}{3}\times 3\frac{1}{7}-7\frac{1}{4}}{3\frac{1}{5}+2\frac{1}{2}-4\frac{1}{10}}$$

(292) If 3 men and 5 women do a piece of work in 8 days which 2 men and 7 children can do in 12, find how long 13 men, 14 children, and 15 women working together will take to do it.

(293) A person possessing £10000 3 per cent. consols, sells out when they are at $93\frac{5}{8}$, and invests the proceeds in 4 per cent. stock at $101\frac{7}{8}$. Find the change in his income, allowing $\frac{1}{8}$ per cent. commission on each transaction.

(294) Five men do $\cdot 600\dot{6}$ of a piece of work in $2\cdot 12$ hours. How long will 6 boys take to finish it, it being known that 3 men and 7 boys have done a similar piece of work in 3 hours?

(295) A watch set accurately at 12 o'clock indicates 10 minutes to 5 at 5 o'clock. What is the exact time when the watch indicates 5 o'clock?

(296) *A* does $\frac{5}{6}$ of a piece of work in 20 days, and then gets *B* to help him. They work together for 2 days, when *B* leaves and *A* finishes the work in half a day more. How long would *B* have taken to do the whole?

(297) The wages of 5 men, 3 women, and 1 child amount to $34, a man receiving twice as much as a woman, and a woman three times as much as a child. What will be the wages of 6 men, 2 women, and 5 children?

(298) If 6 per cent. be gained by selling a horse for $132.50, how much per cent. is lost by selling him for $115?

(299) A person invests $6825 in 6 per cent. stock at 91; he sells out $5000 stock when it has risen to $93\frac{1}{2}$, and the remainder when it has fallen to 85. How much does he gain or lose by the transaction? If he invest the produce in M. B. S., which pays a dividend of 12 per cent., at 175, what is the difference of his income?

(300) The flooring of a room 14 ft. 3 in. long by 13 ft. 4 in. broad, is composed of $\frac{1}{2}$ in. planks, each 8 in. wide and 10 ft. long. How many will be required, and what will be the weight of the whole, if 1 cubic inch of wood weighs half an ounce?

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(301) Find the square roots of $4957\cdot 5681$ and $\dfrac{129\cdot 4947}{60\cdot 75}$.

(302) At what rate will $157.50 amount to $189 in 5 years?

(303) Two bills for $273.75 and $456.87½ are due on the 2nd and 22nd July respectively. What is their value on the

12th July, interest being reckoned at the rate of 5 per cent. per annum?

(304) If a cask contain 3 parts wine and 1 part water, how much of the mixture must be drawn off and water substituted for the mixture in the cask to become half and half?

(305) Three tramps meet together for a meal; the first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 8 half-pence; how ought they to divide the money?

(306) If the discount on a bill due 8 months hence at $7\frac{1}{2}$ per cent. per annum be $48.75, what is the amount of the bill?

(307) *A* began business with a certain capital. The first year he gained 20%, which he added to his capital; the second year he gained $37\frac{1}{2}$%, which he also added to his capital; the third year he lost 40%, and now found himself $200 worse than when he began business. Find the capital with which he began.

(308) A man sells two horses for $100 each, and by so doing gains 25 per cent. on one horse and loses 25 per cent. on the other. What did the horses cost him? Does he gain or lose on the whole?

(309) The difference between the interest and the discount on a certain sum of money for 6 months, at 4 per cent., is $2: what is the sum?

(310) A cistern without a top is 27 ft. long, 22 ft. wide, and 6 ft. 6 in. deep: what will it cost to paint it inside and out, at $4\frac{1}{2}$ cents a square yard?

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(311) Simplify

(*a*) $3 - \cfrac{1}{2 - \cfrac{1}{6 + \frac{1}{5}}}$ divided by $1 + \cfrac{1}{4 + \cfrac{1}{3 - \cfrac{1}{3 + \frac{1}{2}}}}$

(*b*) $\dfrac{5\frac{1}{3} - 3\frac{1}{2} \div 1\frac{8}{13} + 2\frac{1}{2}}{4\frac{2}{7} - 3\frac{1}{2} + 2\frac{3}{14}}$

(312) Three-fourths the selling price of goods is 20% less than cost. Find the gain per cent. at which the goods are sold.

(313) A sum of money amounts in 10 yrs. at $7\frac{1}{2}$% simple interest to $$787\frac{1}{2}$. In how many years will it amount to $990.

(314) I spent 25 % more than my income in a certain year; for each of the next four years I saved $6\frac{2}{3}$ % of it, and then I found that I had lived within it and had $50 besides. What was my income?

(315) A school rate of 5 mills per dollar and a general purpose rate of 8 mills in the dollar produce a tax of $101.40. Find the assessed value of the property.

(316) A grocer has 225 lbs. of tea, of which he sells 45 lb. at 72 cents per lb., and only gains 8 per cent. at this price. He now raises the price so as to gain 10 per cent. on the whole outlay. What is the price when raised?

(317) If I owe $2000, to be paid in 4 months' time, and I pay $500 now, what extension of time ought to be allowed me for the payment of the remainder, reckoning money to be worth 8 per cent. per annum simple interest?

(318) *A* and *B* run a mile race; at first *A* runs 11 yards to *B*'s 10, but after *A* has run a half a mile he tires and runs 9 yards in the time in which he at first ran 11, *B* running at his original rate. Which wins, and by how much?

(319) A woman buys a certain number of eggs at 21 a shilling, and the same number at 19 a shilling; she mixes them together and sells them at 20 a shilling. How much does she gain or lose per cent. by the transaction?

(320) A room whose height is 11 feet, and length twice its breadth, takes 143 yards of paper 2 feet wide for its four walls. How many yards of gilt moulding will be required?

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(321) Simplify

$$\frac{4\frac{5}{6} + 1\frac{11}{45} - 5\frac{11}{18}}{6\frac{1}{7} \times 3\frac{1}{2} - \dfrac{2\frac{1}{3}}{\frac{2}{7}} \times 1\frac{2}{7} + 1\frac{3}{5}}, \quad \frac{5}{6\frac{7}{8}} \times (1\frac{4}{7} \times 5\frac{1}{4}) + \frac{1}{3} + \frac{1}{1\frac{10}{17}},$$

and find their sum.

(322) Simplify (·006 of £2 1*s*. 8*d*. + $3{\cdot}4\dot{5}\dot{4}$ of £3 6*s*.) × $5\frac{5}{11}$.

(323) Two boys, *A* and *B*, come into school punctually by their own watches, which are quite right at 9 o'clock on Monday morning. *A*'s watch gains two minutes, and *B*'s watch loses a minute and a half every day. Find how much later *B* will be than *A* at Friday afternoon school, 2 P.M.

(324) Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 acres respectively. The first gang works 7 hours in the day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second gang complete theirs?

(325) A grocer buys some tea at 48 cents per lb., and some at 66 cents. In what proportion must he mix them that when he sells at 72 cents per lb., he may be making a profit of 20 per cent.?

(326) *A* pays $3.60 more tax than *B*, their incomes being equal; living in different towns, they are rated at $1\frac{1}{2}$ cents and $1\frac{1}{4}$ cents in the dollar respectively. What is *A*'s income?

(327) A bankrupt can pay 40 cents in the $; if his assets were $500 more he could pay 45 cents. Find his debts and his assets.

(328) If a piece of work can be done in 50 days by 35 men working at it together, and if, after working at it for 12 days, 16 of the men were to leave the work, find the number of days in which the remaining men could finish the work.

(329) Alfred owed Robert two-thirds of the amount that Robert owed Charles, and to settle matters Robert gave 10*d.* to Alfred, who then paid Charles. What did Robert owe Charles?

(330) The length, breadth, and height of a wooden box are 4 ft., $2\frac{1}{2}$ ft., 3 ft. respectively. Find the cost of painting the outside at 1*s.* 3*d.* a square yard.

(331) Simplify

$$\frac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}, \qquad \frac{3\frac{2}{3}}{4\frac{1}{7}} \times (3\frac{5}{8} \times 5\frac{4}{7}) - 17\frac{3}{4},$$

and find their sum.

(332) A man walks a certain distance, and rides back in 3 hours 45 min.; he could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways?

(333) I have to be at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be five minutes too late, if at the rate of 5 miles per hour, I shall be 10 minutes too soon. How far have I to go?

(334) *A*, *B*, *C*, and *D* enter into partnership; *A* and *B* contribute $1390, *B* and *C* $1590, *C* and *D* $1810, *A* and *D* $1610, *A* and *C* $1500. They gain $1152. What is the share of each?

(335) On a stream *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back in 5 hr. 15 min., and from *A* to *C* in 7 hr. How long would it take to go from *C* to *A*?

(336) I have a certain sum of money wherewith to buy a

certain number of nuts, and I find that if I buy at the rate of 40 for 10 cents I shall spend 5 cents too much; if at the rate of 50 for 10 cents, 10 cents too little. How much money had I?

(337) If *A* has \$38940 to invest, and can buy Toronto city 6 % bonds at $98\frac{1}{2}$, or Montreal Corporation Consolidated 7 % stock at $117\frac{1}{2}$, how much will the one transaction be better than the other, brokerage being $\frac{1}{2}$ %?

(338) What must be the face of the note for 3 mos., made on 18th Aug., so that discounted at $7\frac{1}{2}$ % on the day of making at the bank, the proceeds may be \$14315?

(339) If, in a meadow of 20 acres, the grass grows at a uniform rate, and 133 oxen consume the whole of the grass on it in 13 days, or that 28 oxen 5 acres of it in 16 days, how many oxen can eat up 4 acres of it in 14 days?

(340) In a constituency, in which each elector may vote for two candidates, half of the constituency vote for *A*, but divide their votes among *B*, *C*, *D*, *E*, in the proportions of 4, 3, 2, 1; of the remainder, half vote for *B*, and divide their votes among *C*, *D*, *E*, in the proportions of 3, 1, 1; two-thirds of the remainder vote for *D* and *E*, and 540 do not vote at all. Find the order on the poll, and the whole number of electors.

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(341) Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{\cdot24 + \cdot5\dot{3}}{2\cdot2 - \cdot6\dot{4}}\right)$.

(342) Simplify $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}$.

(343) When the New York gold market is at $104\frac{3}{4}$, what would I get for \$2304.50 currency?

(344) A person invests \$9450 in $5\frac{1}{2}$ per cent. stock, so as to receive an income of \$787.50. What was the price of the stock?

(345) Two pipes, *A* and *B*, would fill a cistern in 25 minutes and 30 minutes respectively; both are opened together, but at the end of $8\frac{2}{5}$ minutes the second is turned off. In how many minutes will the cistern be filled?

(346) A man for 5 years spends £40 a year more than his income. If he, at the end of that time, reduce his expenditure 10 per cent., in 4 years he will have paid off his debts and saved £120. Find his income.

(347) The sum of £177 is to be divided among 15 men, 20

women, and 30 children, in such a manner that a man and a child may receive together as much as two women, and all the women may together receive £60: what will they each respectively receive?

(348) If 8000 metres be equal to 5 miles, and if a cubic fathom of water weighs six tons, and a cubic metre of water 1000 kilogrammes, find the ratio of a kilogramme to a pound avoirdupois. (Long ton.)

(349) A mixture of soda and potash, dissolved in 2540 grains of water, took up 980 grains of aqueous sulphuric acid, and the weight of the compound solution was 4285 grains. Find how much potash and how much soda the mixture contained, assuming that aqueous sulphuric acid unites with soda in the proportion of 49 grains to 32, and with potash in the proportion of 49 to 48.

(350) A room is 21 ft. long, 15 ft. 6 in. wide, 10 ft. high; it contains 3 windows, the recesses of which reach to the ceiling, and are 4 ft. 6 in. wide; there are in it 4 doors, each 6 ft. 6 in. high and 3 ft. 3 in. wide; the fire-place is 6 ft. wide and 4 ft. high; a skirting 1 ft. 8 in. deep runs round the walls. Find the expense of papering the room at 5 cents a square foot.

ANSWERS.

Ex. (i.), p. 5.

(1) Seven; thirteen; forty-five; fifty-nine; three hundred and twenty-six; four thousand five hundred and seventy-eight.

(2) Ninety; one hundred and ten; two hundred and seven; four thousand three hundred; four thousand and thirty-six; four thousand three hundred and six.

(3) Seven hundred and eighty; six hundred and nine; five thousand three hundred and sixty; two thousand and twenty; one thousand one hundred and one.

(4) Thirty-six thousand four hundred and ninety-seven; forty-nine thousand five hundred and thirty-two; six hundred and fifty-four thousand three hundred and twenty-one; seven hundred and forty-three thousand two hundred and sixty-nine.

(5) Forty-five thousand; thirty-two thousand six hundred; seventy-five thousand two hundred and thirty; five hundred thousand.

(6) Eight millions five hundred and seventy-two thousand nine hundred and fourteen; three millions four hundred and sixty-nine thousand two hundred and eighteen; four millions six hundred and twenty-nine thousand eight hundred and seventeen.

(7) Nine millions; twenty-nine millions; seven hundred and fifteen millions.

(8) Nine hundred and ten millions three hundred and seven thousand two hundred and forty; three hundred and seven millions four thousand two hundred and five; three hundred and eighty millions five hundred and three thousand and forty.

(9) Two hundred and forty-three billions seven hundred and fifty-nine millions two hundred and sixty-eight thousand three hundred and forty-two; three hundred and seven billions four hundred and five millions six thousand two hundred and seventy.

Ex. (ii), p. 6.

(1) 9; 12; 17; 19; 13; 16; 11.
(2) 23; 27; 35; 38; 44; 40; 26; 34.
(3) 67; 75; 62; 83; 74; 92; 68; 95.
(4) 76; 22; 50; 15; 28; 61; 49; 18; 90; 73.
(5) 107; 130; 246; 372; 608; 740; 990.
(6) 836; 747; 410; 913; 750; 384.
(7) 818; 808; 206; 430; 512; 787.
(8) 7845; 9637; 12000; 8400; 6003; 85040.
(9) 5470; 3650; 8780; 1247; 4808.
(10) 6004; 7022; 3500; 9047; 2017; 19402.
(11) 70007; 60060; 14014; 70017; 12303; 16005.
(12) 356728; 640842; 900000; 800040.
(13) 7000000; 4576865; 75806940.
(14) 315000000; 5040000; 8000700; 18000020; 700000002.
(15) 315674018003; 35600000520.
(16) 7000000000; 5800000600047; 8000000043007.
(17) 305005004006003; 53000053053.
(18) 9000000000009; 9000000000900; 1900000019000;
1000001001101.

Ex. (iii.), p. 9.

1.

(1) Twenty-seven. (2) Forty-nine. (3) Sixty-eight.
(4) Seventy-three. (5) Ninety-two.
(6) One hundred and forty-four.
(7) One hundred and sixty-three.
(8) One hundred and ninety-nine.
(9) Six hundred and sixty-four.
(10) One thousand eight hundred and seventy-two.

2.

(1) XXXVII. (2) LIX. (3) LXII.
(4) LXXXVII. (5) XCV. (6) CXXXIX.
(7) CXLV. (8) CLXXIX. (9) DCCCXLVI.
(10) MDCCLXIII.

Ex. (iv.), p. 10.

(1) 11; 16; 20; 26. (2) 98. (3) 67.
(4) 60. (5) 1409. (6) 949.
(7) 738. (8) 4971. (9) 23406.
(10) 74338. (11) 2008. (12) 3310.
(13) 1671. (14) 880. (15) 28493.
(16) 33633. (17) 28206. (18) 18526.
(19) 208. (20) 1163. (21) 9289.
(22) 12932. (23) 106384. (24) 59223.

(25) 284271. (26) 1450741. (27) 680077891.
(28) 1843088. (29) 1979628. (30) 1184946.
(31) 3782272. (32) 3476908. (33) 799819.
(34) 50333150. (35) 20826857. (36) 14621293.
(37) 112251. (38) 764368. (39) 1825947.
(40) 227356697.

Ex. (v.), p. 14.

(1) 7. (2) 8. (3) 19.
(4) 29. (5) 34. (6) 54.
(7) 35. (8) 29. (9) 45.
(10) 66. (11) 509. (12) 82.
(13) 3808. (14) 2112. (15) 4228.
(16) 6222. (17) 61471. (18) 108.
(19) 2779. (20) 28828. (21) 2761.
(22) 381. (23) 46. (24) 32.
(25) 2042. (26) 6457. (27) 5780.
(28) 51195. (29) 10999. (30) 1096.
(31) 18467. (32) 60023. (33) 13378402.
(34) 1. (35) 999000. (36) 99900000.
(37) 9999998999. (38) 8. (39) 26.
(40) 610. (41) 593. (42) 159.
(43) 619.

Ex. (vi.), p. 18.

(1) 45. (2) 304. (3) 490.
(4) 870. (5) 684. (6) 861.
(7) 9273. (8) 11364. (9) 50 ; 75 ; 175 ; 225.
(10) 635 ; 1016 ; 1270 ; 1397 ; 2540 ; 8890.
(11) 9868 ; 14802 ; 27137 ; 29604 ; 1233500 ; 17269000.
(12) 336861 ; 411719 ; 449148 ; 1871450000 ;
2994320000000.

Ex. (vii.), p. 19.

(1) 345. (2) 1073. (3) 1620.
(4) 1820. (5) 3000. (6) 13734.
(7) 8815. (8) 30086. (9) 93940.
(10) 1546992. (11) 7417784. (12) 579826952.

Ex. (viii.), p. 20.

(1) 173432. (2) 123904. (3) 409354.
(4) 372302. (5) 2274048. (6) 2667640.
(7) 39342154. (8) 51212122. (9) 319766614.
(10) 152847420. (11) 58376823669.
(12) 348087421500. (13) 38871923744.
(14) 3340400440. (15) 295990965442.

(16) 609435012763918 (17) 13426705851000.
(18) 703004503. (19) 3590386740.
(20) 3454309838. (21) 4930038124.
(22) 61110346167. (23) 1407009621.
(24) 24259354428. (25) 248155914760.
(26) 13575555747. (27) 249493596792.

Ex. (ix.), p. 2i.

(1) 6840. (2) 1909680. (3) 1121111043844

Ex. (x.), p. 21.

(1) 225. (2) 576. (3) 1600.
(4) 3249. (5) 4761. (6) 5184.
(7) 7569. (8) 10000. (9) 12996.
(10) 56169. (11) 390625. (12) 804609.
(13) 622521. (14) 1331. (15) 2197.
(16) 15625. (17) 103823. (18) 314432.
(19) 804357. (20) 1000000. (21) 16974593.
(22) 45118016. (23) 156590819. (24) 348913664.
(25) 961504803.

Ex. (xi.), p. 24.

(1) 3. (2) 3. (3) 12. (4) 11.
(5) 14. (6) 14. (7) 24. (8) 103.
(9) 108. (10) 13. (11) 528. (12) 1032.
(13) 56285. (14) 241248. (15) 458097. (16) 7589523.
(17) 2104. (18) 17553. (19) 24000729. (20) 2019.
(21) 56169. (22) 5678094. (23) 4348432. (24) 5072.
(25) 317649. (26) 391525. (27) 39876548. (28) 30207.
(29) 3469805. (30) 68274625. (31) 472304974.
(32) 5642300741. (33) 8462974231. (34) 90807.
(35) 300071. (36) 29970.

Ex. (xii.), p. 25.

(1) 3426. (2) 6487. (3) 64008924.

Ex. (xiii.), p. 26.

(1) 3826. (2) 241987. (3) 2162558.
(4) 1749864. (5) 1243904. (6) 500603.
(7) 79267440. (8) 396547. (9) 659372.
(10) 444513674545. (11) 6947544611. (12) 30074902004
(13) 2131962, 1421308, 1065981.
(14) 310218774, 206812516, 155109387.
(15) 13770459132, 9180306088, 6885229566.
9035784, 5816115, 5169880.

(17) 196350840, 122719275, 109083800.
(18) 46913400, 29320850, 26063000.
(19) 1138764, 724668, 664279.
(20) 4224924, 2688588, 2464539.
(21) 962341116, 612398892, 561365651.

Ex. (xv.), p. 28.

(1) 3, 9. (2) 2, 3, 4, 8, 9. (3) 3, 5, 9, 11.
(4) 2, 3, 5, 10. (5) 2, 3, 4, 8, 9. (6) 3, 5, 9.
(7) 2, 4, 8. (8) 5. (9) 2, 3, 4.
(10) 2, 3, 4, 5, 8, 9. (11) 2, 11.

Ex. (xvi.), p. 29.

(1) 2, 3, 3. (2) 2, 2, 2, 3. (3) 3, 3, 3.
(4) 2, 2, 2, 2, 2, (5) 2, 2, 3, 3. (6) 3, 13.
(7) 2, 3, 7. (8) 3, 17. (9) 2, 3, 3, 3.
(10) 3, 19. (11) 2, 2, 3, 3. (12) 5, 17.
(13) 7, 13. (14) 3, 3, 11. (15) 2, 2, 5, 5.
(16) 3, 5, 7. (17) 2, 2, 3, 3, 3. (18) 2, 2, 2, 2, 7.
(19) 2, 2, 3, 11. (20) 2, 2, 2, 2, 11.
(21) 2, 2, 2, 2, 2, 3, 3. (22) 2, 2, 2, 2, 3, 3, 3.
(23) 3, 5, 5, 7. (24) 5, 5, 5, 5.
(25) 3, 3, 3, 3, 3, 3. (26) 3, 3, 3, 37.
(27) 2, 2, 2, 2, 3, 3, 3, 3. (28) 2, 2, 2, 2, 2, 5, 11.
(29) 2, 2, 2, 2, 2, 2, 2, 3, 3, 5.

Ex. (xvii.), p. 29.

(1) 4858. (2) 9306. (3) 147474.
(4) 305892. (5) 420077. (6) 1594432.
(7) 3104199. (8) 11976096. (9) 32661000.
(10) 4342356. (11) 48482280. (12) 10138680000.

Ex. (xviii.), p. 30.

(1) 2472. (2) 452736. (3) 41798032.
(4) 42370218. (5) 8642934. (6) 42765328.
(7) 74232657 (8) 14287262. (9) 204074.
(10) 436876. (11) 3781076. (12) 784978.
(13) 37296.

Ex. (xix.), p. 31.

(1) 94, rem. 14. (2) 11860, rem. 36.
(3) 18573, rem. 17. (4) 878, rem. 22.
(5) 105531, rem. 35. (6) 844380, rem. 85.
(7) 849, rem. 20. (8) 2392, rem. 134.
(9) 11447, rem. 72. (10) 965316, rem. 718.
(11) 10005, rem. 3569. (12) 10000821, rem. 1812.

Ex. (xx.), p. 32.

(1) 276, rem. 13. (2) 36724, rem. 11.
(3) 2378, rem. 9. (4) 20174, rem. 18.
(5) 28998, rem. 22. (6) 21074998, rem. 25.
(7) 85629, rem. 23. (8) 246925, rem. 21.
(9) 7429, rem. 7. (10) 129295, rem. 33.
(11) 2987635, rem. 19. (12) 4236, rem. 57.
(13) 423, rem. 72. (14) 504, rem. 123.
(15) 5687, rem. 207.

Ex. (xxi.), p. 33.

(1) 1. (2) 472339. (3) 624. (4) 3012.

Examination Papers. (Page 40.)

(I.)

(1) Four millions two hundred and thirty-seven thousand four hundred and ninety-six; 653812.

(2) 196181. (3) 7829.
(4) 4253111; 15362894. (5) 935977; 7429.

(II.)

(1) 25257630; four hundred and two millions fifty thousand four hundred and seven.

(2) 16992009. (3) 26438313; 99914800.
(4) 338091, rem. 53. (5) 1175427; 130603.

(III.)

(1) Ten billions ten millions two hundred and one thousand four hundred and one; 1023091; 10011224402; 2046002.

(2) 1546478344; 1577918816. (3) 2237069, rem. 11.
(4) 31405999. (5) 5226, rem. 33.

(IV.)

(1) 1888. (2) 7482229, rem. 93.
(3) 2, 2, 2, 7; 2, 3, 13; 2, 3, 19. (4) 12000590.
(5) 999899.

(V.)

(1) 65299476, rem. 5346. (2) 88652792964.
(3) 2, 2, 2, 5; 2, 3, 3, 5; 2, 3, 3, 7.
(4) xxiv; xlvii; clxxviii. (5) 12000.

(VI.)

(1) 619161890. (2) 8670344882024.
(3) 7283. (4) $14541.
(5) 7684 and 978.

(VII.)

(1) $269. (2) 72 days.
(3) 252. (4) 296237.
(5) $13300, $11900, $10500.

(VIII.)

(1) 7000. (2) 450 lbs.
(3) 31239. (4) $232.
(5) $30.

(IX.)

(2) 9999000025. (3) $37569.
(4) 11796 steps. (5) $21000 ; $5400.

(X.)

(2) 170680900742874252. (3) 2796219.
(4) 786543. (5) Rem. 12 ; Divisor 72 ; Quot. 432.

Ex. (xxii.), p. 44.

(1) 2. (2) 6. (3) 20. (4) 18. (5) 48.
(6) 7. (7) 3. (8) 16. (9) 16. (10) 3.

Ex. (xxiii.), p. 45.

(1) 48. (2) 32. (3) 3. (4) 3. (5) 3453.
(6) 36. (7) 936. (8) 355. (9) 23. (10) 2345.

Ex. (xxiv.), p. 46.

(1) 4. (2) 2. (3) 73. (4) 29. (5) 41. (6) 37.

Ex. (xxv.), p. 47.

(1) 54. (2) 2376. (3) 2532. (4) 9555.
(5) 17000. (6) 85800. (7) 23400. (8) 16128.
(9) 31759.

Ex. (xxvi.), p. 49.

(1) 360. (2) 1320. (3) 288. (4) 5040.
(5) 36036. (6) 27324. (7) 3570. (8) 2340.
(9) 27720. (10) 228150.

Examination Papers. (Page 49).

(I.)

(1) 3327. (2) 35 times. (3) 44496 rails.
(4) 7. (5) 84, 36 and 132.

(II.)

(2) Bags of 1, 2, or 3 bu. each; bins of 300, 150, or 200 bu.

(3) $1650. (4) 60 min. (5) 982832.

(III.)

(1) 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 25, 30, 36, 40, 45, 50, 60, 72, 75, 90, 100, 120, 150, 180, 200, 225, 300, 360, 450, 600, 900, 1800.

(2) 29.

(3) 3391 and 2699 are prime; 14787 and 1477 are composite.

(4) 60 hours; A, 300 mi.; B, 240 mi.; C, 180 mi.

(5) 40 grains.

(IV.)

(2) 203. (3) $9\frac{3}{4}$ mi. (4) 70560.
(5) 24 firkins.

(V).

(2) 60. (3) 3 and 6. (4) 44 times; 9284 trees.
(5) 3366000.

Ex. (xxvii.), p. 54.

(1) $\frac{3}{10}$. (2) $\frac{9}{35}$. (3) $\frac{1}{5}$. (4) $\frac{1}{3}$. (5) $\frac{3}{4}$.
(6) $\frac{7}{8}$. (7) $\frac{13}{15}$. (8) $\frac{20}{517}$. (9) $\frac{211}{347}$. (10) $\frac{14}{23}$.

Ex. (xxviii.), p. 55.

(1) $\frac{21}{28}$, $\frac{20}{28}$ (2) $\frac{24}{54}$, $\frac{15}{54}$, $\frac{14}{54}$. (3) $\frac{231}{385}$, $\frac{220}{385}$, $\frac{210}{385}$.
(4) $\frac{100}{240}$, $\frac{156}{240}$, $\frac{51}{240}$, $\frac{38}{240}$. (5) $\frac{408}{714}$, $\frac{630}{714}$, $\frac{364}{714}$, $\frac{455}{714}$.
(6) $\frac{30}{90}$, $\frac{54}{90}$, $\frac{15}{90}$, $\frac{25}{90}$. (7) $\frac{324}{1080}$, $\frac{200}{1080}$, $\frac{84}{1080}$, $\frac{33}{1080}$.

Ex. (xxix.), p. 56.

The fractions are arranged in *descending* order.

(1) $\frac{4}{5}$, $\frac{9}{13}$, $\frac{3}{7}$. (2) $\frac{5}{6}$, $\frac{7}{9}$, $\frac{12}{17}$. (3) $\frac{13}{15}$, $\frac{9}{11}$, $\frac{17}{21}$.
(4) $\frac{7}{40}$, $\frac{11}{70}$, $\frac{3}{20}$. (5) $\frac{7}{33}$, $\frac{9}{43}$, $\frac{11}{53}$. (6) $\frac{5}{34}$, $\frac{7}{51}$, $\frac{2}{17}$.

Ex. (xxx.), p. 57.

(1) $\frac{19}{35}$. (2) $\frac{37}{42}$. (3) $\frac{13}{21}$. (4) $\frac{58}{63}$. (5) $\frac{125}{156}$.
(6) $\frac{31}{32}$. (7) $\frac{1018}{1485}$. (8) $\frac{3043}{3465}$. (9) $\frac{2231}{2880}$.

Ex. (xxxi.), p. 57.

(1) $\frac{1}{30}$. (2) $\frac{48}{133}$. (3) $\frac{1}{156}$. (4) $\frac{48}{119}$. (5) $\frac{1}{756}$.
(6) $\frac{13}{418}$. (7) $\frac{218}{1911}$. (8) $\frac{1}{450}$. (9) $\frac{1061}{7659}$.

Ex. (xxxii.), p. 59.

(1) $\frac{5}{21}$. (2) $\frac{135}{308}$. (3) $\frac{2}{5}$. (4) $\frac{98}{165}$. (5) $\frac{9}{76}$.
(6) $\frac{275}{1104}$. (7) $\frac{1287}{2320}$. (8) $\frac{32}{405}$. (9) $\frac{50}{243}$.

Ex. (xxxiii.), p. 60.

(1) $\frac{4}{7}$. (2) $\frac{5}{6}$. (3) $\frac{5}{12}$. (4) $\frac{12}{29}$. (5) $\frac{2}{5}$.
(6) $\frac{49}{54}$. (7) $\frac{29}{77}$. (8) $\frac{5}{6}$. (9) $\frac{20}{27}$.

Ex. (xxxiv.), p. 61.

(1) $\frac{67}{9}$. (2) $\frac{1096}{47}$. (3) $\frac{6277}{29}$. (4) $\frac{173019}{1000}$.
(5) $42\frac{7}{10}$. (6) $3\frac{477}{1000}$. (7) $31\frac{46}{137}$. (8) $928\frac{55}{71}$.

Ex. (xxxv.), p. 62.

(1) $1\frac{16}{119}$. (2) $1\frac{2}{5}$. (3) $1\frac{2965}{3132}$. (4) $65\frac{5}{8}$.
(5) $43\frac{1}{3}$. (6) $178\frac{5}{7}$. (7) $5\frac{7}{12}$. (8) $12\frac{953}{1155}$.
(9) $38\frac{2}{15}$. (10) $2\frac{8}{35}$. (11) $8\frac{41}{72}$. (12) $\frac{163}{182}$.

Ex. (xxxvi.), p. 63.

(1) $27\frac{3}{5}$. (2) 744. (3) $718\frac{3}{4}$. (4) $\frac{16}{35}$.
(5) $1\frac{1}{3}$. (6) 4. (7) $2\frac{3}{4}$. (8) 26.

Ex. (xxxvii.), p. 64.

(1) $\frac{1}{35}$. (2) $\frac{1}{756}$. (3) $\frac{1}{24}$. (4) $\frac{1}{5460}$.
(5) $3\frac{1}{3}$. (6) $142\frac{1}{2}$. (7) $8350\frac{1}{3}$. (8) 66.

Ex. (xxxviii.), p. 67.

(1) $1\frac{1}{23}$. (2) $\frac{77}{206}$. (3) $\frac{12}{65}$. (4) $\frac{165}{254}$.
(5) $1\frac{7}{16}$. (6) $\frac{23}{64}$. (7) $\frac{26}{73}$. (8) $1\frac{7}{23}$.
(9) $2\frac{28}{31}$. (10) $\frac{12}{19}$.

Ex. (xxxix.), p. 68.

(1) $\frac{21}{25}$. (2) $\frac{2}{11}$. (3) $1\frac{4}{15}$. (4) $\frac{3}{5}$.

(5) $5\frac{37}{41}$. (6) $\frac{1}{2}$. (7) $\frac{685}{756}$. (8) $20\frac{103}{120}$.

(9) $1\frac{11}{108}$. (10) $1\frac{11}{17}$. (11) $7\frac{7}{20}$. (12) $\frac{3}{4}$.

Ex. (xl.), p. 68.

(1) $1\frac{43}{88}$. (2) $1\frac{6}{35}$, $\frac{20}{63}$. (3) $\frac{1}{9}$, $6\frac{2}{3}$.

(4) $\frac{13}{37}$, $\frac{111}{13}$, 3. (5) $4\frac{1}{5}$. (6) $11\frac{11}{12}$, 20.

(7) $\frac{15}{284}$, $\frac{1}{4}$. (8) 3. (9) $7\frac{1}{9}$.

(10) 3. (11) $10\frac{146}{171}$. (12) $1\frac{1}{2}$, $1\frac{1}{4}$.

(13) $\frac{3}{4}$, $\frac{7}{51}$. (14) $\frac{304}{357}$, $1\frac{2}{7}$. (15) 1, $\frac{3}{4}$.

(16) 5, 18. (17) 66. (18) $14\frac{42}{193}$.

(19) $\frac{75}{77}$. (20) 1. (21) $\frac{1}{2}$.

(22) $\frac{226}{299}$. (23) 2.

Examination Papers. (Page 71.)

(I.)

(2) $\$49\frac{1}{8}$. (3) $5\frac{82}{157}$. (4) \$13860. (5) $3\frac{27}{70}$ and $\frac{57}{70}$.

(II.)

(2) $\frac{23}{44}$, $\frac{29}{56}$, $\frac{7}{15}$, $\frac{8}{35}$. (3) $\frac{728}{1000}$. (5) Ship, \$24000; cargo, \$36000.

(III.)

(2) $\$18\frac{2}{3}$. (4) $\frac{473}{18500}$. (5) A, 20; B, 48; C, 84.

(IV.)

(2) $\frac{27}{35}$. (3) $34\frac{5}{17}$. (4) Horse, \$120; carriage, \$105; harness, \$25. (5) A, \$4334; B, \$1474; C, \$3080.

(V.)

(2) $3\frac{22}{39}$. (3) \$2015; 465 sheep; 390 calves; 806 pigs.
(4) \$180. (5) 18 ft.

(VI.)

(2) $\frac{3}{15}$, $\frac{5}{15}$, $\frac{7}{15}$. (3) 1. (4) \$40. (5) $\frac{5}{36}$ ft.

(VII.)

(2) $\$1333\frac{1}{3}$, $\frac{1}{30}$. (3) 1000000.

(4) 30 min.; A, 6 times; B, 5 times; C, 4 times.

(5) A, $\frac{5}{36}$; B, $\frac{5}{28}$; C, $\frac{10}{231}$; D, $\frac{15}{77}$.

(VIII.)

(2) $\frac{1}{6}$; $\frac{8}{91}$. (3) 36. (4) 30 min.; 4500 rods; 3600 rods; 3000 rods. (5) 252.

Ex. (xli.), p. 76.

(1) $\frac{1}{2}$. (2) $\frac{1}{4}$. (3) $\frac{3}{4}$. (4) $\frac{3}{8}$.
(5) $\frac{243}{100000}$. (6) $\frac{29}{400000}$. (7) $\frac{74}{5}$. (8) $\frac{20847}{200}$.
(9) $\frac{125001}{2500}$. (10) $\frac{100001}{1000}$. (11) ·9. (12) ·37.
(13) ·4579. (14) ·003. (15) 172·95. (16) ·0000059.
(17) ·025679. (18) 3·25793. (19) ·0019.

Ex. (xlii.), p. 78.

(1) ·7. (2) ·2464. (3) ·0012.
(4) 758·279832. (5) 385·260863. (6) 8741·2062.
(7) 6964·72672. (8) 970·17047.

Ex. (xliii.), p. 79.

(1) 51·211. (2) 1·543. (3) 48 2293. (4) ·001.
(5) ·0607. (6) 579·1274. (7) ·0000014.
(8) ·004385. (9) 9 9998. (10) ·00101.

Ex. (xliv.), p. 81.

(1) 35·25. (2) 18·9326. (3) ·100345.
(4) ·00041588. (5) 12·08980432. (6) ·9.
(7) 14977·92625425. (8) ·0000465131.
(9) ·0577468988828045. (10) 203·175662750726562.
(11) ·00984126. (12) 1·01. (13) ·00031304.
(14) ·15205806. (15) ·1009981674. (16) 20·570824.
(17) 150·0625.

Ex. (xlv.), p. 85.

(1) 12. (2) 14400. (3) ·0013.
(4) 12700. (5) 43·078. (6) 10000.
(7) 430. (8) 147. (9) ·0000002004.
(10) 98·476. (11) ·0065839. (12) 876540000.
(13) ·0000771039. (14) 299846000. (15) ·20162.
(16) 2469300000. (17) 3596. (18) ·00000029.
(19) 1290. (20) 3·59. (21) 457·61.
(22) 76·371. (23) 905741000.

Ex. (xlvi.), p. 87.

(1) 23·28125. (2) 1·119296875. (3) 3·4608.
(4) 33035·448... (5) ·00192. (6) ·0001736.

Ex. (xlvii.), p. 87.

(1) 26·654875. (2) ·0010902175. (3) 14498·8.
(4) ·00001614. (5) 175·03099875. (6) ·0000926.
(7) 154468·75. (8) 25000000. (9) ·00001.
(10) ·0000005005005.

Ex. (xlviii.), p. 88.

(1) 18478·260. (2) ·249. (3) ·092.
(4) 8658146·964. (5) ·095. (6) 32714·285.

Ex. (xlix.), p. 91.

(1) ·35. (2) ·44. (3) ·$\dot{8}5714\dot{2}$. (4) ·$\dot{0}\dot{1}$.
(5) ·$\dot{0}0\dot{1}$. (6) ·$\dot{0}243\dot{9}$. (7) ·$\dot{5}2380\dot{9}$. (8) ·$21\dot{6}$.
(9) ·$012\dot{3}\dot{6}$. (10) 2·$3\dot{4}\dot{5}$.

Ex. (l.), p. 94.

(1) ·09484. (2) ·002521. (3) 165·6995.
(4) 235·104. (5) 26·38702. (6) 1·611.
(7) ·0374. (8) 426·104. (9) 170·3367.
(10) ·928.

Ex. (li.), p. 95.

(1) $\frac{2}{3}$. (2) $\frac{3}{11}$. (3) $\frac{5}{111}$. (4) $\frac{347}{1111}$.
(5) $\frac{8}{1111}$. (6) $\frac{447}{1111}$. (7) $\frac{6}{11111}$. (8) $\frac{1}{11111}$.

Ex. (lii.), p. 96.

(1) $\frac{421}{990}$. (2) $\frac{1178}{2475}$. (3) $4\frac{19}{75}$. (4) $\frac{211}{49500}$.
(5) $53\frac{9}{3700}$. (6) $7\frac{2009}{9990}$. (7) $2\frac{1751}{3300}$.

Ex. (liii.), p. 98.

(1) 15·$\dot{8}43\dot{0}$. (2) 20·51$\dot{6}62025\dot{}$. (3) 1·7$\dot{7}8005\dot{2}$.
(4) ·02$\dot{0}6724\dot{9}$. (5) $20\frac{676}{891}$. (6) $\frac{13}{3960}$.
(7) $\frac{310}{369}$. (8) $\frac{37}{297}$.

Examination Papers. (Page 98.)

(I.)

(3) 1st. (4) 263 times ; $\frac{1}{3}$. (5) ·0000006 and ·0000009.

(II.)

(1) $\frac{5}{32}$, $\frac{77}{1100}$, $\frac{1820}{2912}$, $\frac{91}{560}$. (2) $14.90. (3) 3·715.
(4) $\frac{11}{9}$. (5) ·7142.

(III.)

(2) \$816$\frac{2}{3}$. (3) 13 : \$3000. (4) \$232$\frac{2}{3}$. (5) $\frac{13200}{2009}$.

(IV.)

(2) \$21.60. (3) 425, (4) 4000004·00000010000090007; Seventy-four millions, three hundred and six, and sixty millions and seven trillionths. (5) 82$\frac{1}{4}$ yd.

(V.)

(2) $\frac{39371}{2534400}$. (3) 10·7608 miles. (4) 13$\frac{23}{48}$.
(5) *A*, \$192.23$\frac{1}{2}$; *B*, \$145.53$\frac{1}{2}$; *C*, \$110.94$\frac{1}{2}$.

(VI.)

(3) \$34$\frac{1}{3}$. (4) 3·141592. (5) ·00097061

Ex. (lv.), p. 105.

(1) 14. (2) 23. (3) 32. (4) 75.
(5) 297. (6) 345. (7) 327. (8) 867.
(9) 440. (10) 835. (11) 6031. (12) 4698.
(13) 23456. (14) 72500. (15) 2031. (16) 739000
(17) 5678. (18) 437962.

Ex. (lvi.), p. 106.

(1) 4·1. (2) 16·79. (3) ·95. (4) ·51.
(5) ·25. (6) ·027. (7) 131·31 (8) 1·001.
(9) 210·75. (10) 137·65.

Ex. (lvii.), p. 107.

(1) 4·4721. (2) 5·4772. (3) ·9486. (4) ·3478.
(5) ·4110. (6) ·1264. (7) ·0252. (8) ·0347.
(9) 4·0305. (10) ·9999. (11) ·5025. (12) 6·4833.

Ex. (lviii.), p. 108.

(1) $\frac{9}{7}$. (2) $\frac{8}{11}$. (3) $\frac{17}{25}$ (4) $\frac{37}{87}$.
(5) $\frac{123}{427}$. (6) 2$\frac{1}{4}$. (7) 2$\frac{2}{5}$. (8) 1$\frac{10}{13}$.
(9) 8$\frac{1}{11}$. (10) 6$\frac{1}{5}$. (11) 4$\frac{1}{5}$ (12) 3$\frac{3}{7}$.
(13) ·7905. (14) ·6454. (15) 2·5298. (16) 3·0822.
(17) 8·7649.

Ex. (lix.), p. 110.

(1) 16. (2) 32. (3) 42. (4) 79.
(5) 85. (6) 64. (7) 34. (8) 73.
(9) 85. (10) 99. (11) 39· (12) 63.

Ex. (lx.), p. 111.

(1) 245. (2) 531. (3) 307. (4) 670.
(5) 128. (6) 179. (7) 463. (8) 103.
(9) 256. (10) 579. (11) 438. (12) 507.
(13) 686. (14) 708. (15) 888. (16) 512.
(17) 4968. (18) 8765.

Ex. (lxi.), p. 112.

(1) ·73. (2) ·364. (3) 30·02. (4) $\frac{11}{12}$.
(5) $\frac{5}{7}$. (6) 1$\frac{5}{7}$. (7) 7$\frac{2}{5}$. (8) 1·709.
(9) 8·320. (10) ·495. (11) 2·516. (12) ·822.
(13) ·908. (14) ·693. (15) 1·966. (16) 1·473.

Ex. (lxii.), p. 113.

(1) 27. (2) 45. (3) 6·3. (4) 13.
(5) 54 (6) 8·1.

Ex. (lxiii.), p. 117.

(1)-13 ; 30 ; 36 ; 47. (2) 108 ; 270 ; 615 ; 845.
(3) 3456 ; 4800 ; 2762 ; 16535. (4) 72 ; 58 ; 94 ; 105 ; 163.
(5) 960 ; 1228 ; 4253 ; 14087. (6) 41880 ; 103870 ; 305973.

Ex. (lxiv.), p. 118.

(1) 14$\frac{1}{4}$*d.* (2) 43$\frac{1}{4}$*d.* (3) 49$\frac{1}{4}$*d.*
(4) 7*s.* 5$\frac{1}{4}$*d.* (5) 9*s.* 11$\frac{3}{4}$*d.* (6) 15*s.* 6$\frac{3}{4}$*d.*
(7) £4 8*s.* 3$\frac{1}{2}$*d.* (8) £391 19*s.* 4$\frac{1}{2}$*d.* (9) £564 19*s.* 7*d.*

Ex. (lxv.), p. 120.

(1) £21 15*s.* 1*d.* (2) £31 8*s.* 0*d.* (3) £32 15*s.* 2*d.*
(4) £31 12*s.* 9*d.* (5) £23 13*s.* 8$\frac{1}{4}$*d.* (6) £33 18*s.* 4$\frac{3}{4}$*d.*
(7) £32 9*s.* 3$\frac{3}{4}$*d.* (8) £32 6*s.* 5*d.* (9) £169 5*s.* 1*d.*
(10) £181 18*s.* 6*d.* (11) £240 19*s.* 7*d.* (12) £168 11*s.*
(13) £200 17*s.* 11$\frac{1}{4}$*d.* (14) £220 6*s.* 9$\frac{3}{4}$*d.*
(15) £3602 17*s.* 6*d.*

Ex. (lxvi.), p. 122.

(1) £36 3*s.* 5*d.* (2) £28 0*s.* 10*d.*
(3) £8 18*s.* 11*d.* (4) £238 17*s.* 10$\frac{1}{4}$*d.*
(5) 14*s.* 1$\frac{3}{4}$*d.* (6) £1 16*s.* 7$\frac{3}{4}$*d.*
(7) 1$\frac{3}{4}$*d.* (8) £1519 12*s.* 9$\frac{1}{2}$*d.*
(9) £36108 17*s.* 6$\frac{3}{4}$*d.* (10) £1219 19*s.* 10$\frac{1}{2}$*d.*

Ex. (lxvii.), p. 123.

(1) £1 9*s*. (2) 5*s*. 10*d*. (3) 3*s*. 9*d*.
(4) £3 6*s*. 6*d*. (5) 18*s*. 8*d*. (6) £1 2*s*. 1*d*.
(7) £22 14*s*. 8*d*. (8) £14 11*s*. (9) £12 5*s*.
(10) £5 18*s*. $1\frac{1}{2}$*d*. (11) £21 12*s*. (12) £15 15*s*.
(13) £111 6*s*. 8*d*. (14) £5 18*s*. $1\frac{1}{2}$*d*. (15) £122 9*s*. 4*d*.
(16) £104 12*s*. (17) £5 12*s*. 6*d*. (18) £8 0*s*. $10\frac{1}{2}$*d*.
(19) £3 14*s*. 8*d*. (20) £48. (21) £36 6*s*.
(22) £39 7*s*. 6*d*.

Ex. (lxviii), p. 125.

(1) £6 10*s*. 6*d*. (2) £24 9*s*. $1\frac{1}{2}$*d*.
(3) £3 12*s*. $5\frac{1}{2}$*d*. (4) £5 18*s*. 4*d*.
(5) £29 13*s*. 4*d*. (6) £34 1*s*. $7\frac{1}{2}$*d*.
(7) £167 19*s*. 2*d*. (8) £15212 12*s*. 6*d*.
(9) £6189 5*s*. $7\frac{1}{2}$*d*. (10) £6022 0*s*. $7\frac{1}{2}$*d*.
(11) £8615 3*s*. 9*d*.

Ex. (lxix.), p. 127.

I. (1) 7*s*. $10\frac{1}{2}$*d*. (2) £5 12*s*. 6*d*. (3) 18*s*. $7\frac{1}{2}$*d*.
(4) £3 19*s*. 4*d*. (5) 12*s*. $3\frac{1}{2}$*d*. (6) £1 17*s*. $7\frac{1}{2}$*d*.
II. (1) £1 16*s*. $6\frac{1}{6}$*d*. (2) 4*s*. 3*d*. (3) 5*s*. 6*d*.
(4) 2*s*. 4*d*. (5) 1*s*. $6\frac{1}{2}$*d*. (6) 19*s*. 10*d*.
III. (1) £1 3*s*. 2*d*. (2) 3*s*. $4\frac{1}{4}$*d*. (3) £1 4*s*. $10\frac{1}{4}$*d*.
(4) £4 4*s*. $3\frac{1}{4}$*d*. (5) 6*s*. $4\frac{3}{4}$*d*. (6) £1 3*s*. $9\frac{1}{3}\frac{9}{9}$*d*.

Ex. (lxx.), p. 127.

(1) 100. (2) 22. (3) 42. (4) 79
(5) 231. (6) 10.

Ex. (lxxi.), p. 128.

(1) 3*s*. $6\frac{3}{4}$*d*. (2) 4*s*. $5\frac{3}{4}$*d*. (3) 6*s*. $6\frac{3}{4}$*d*.
(4) 1*s* (5) 1*s*. $9\frac{3}{4}$*d*. (6) £24 16*s*. 8*d*.
(7) 10*s*. 6*d*. (8) 14*s* 8*d*. (9) £13 6*s*. 6*d*.
(10) £48 1*s*. $4\frac{5}{[illegible]}$. (11) £77 5*s*. (12) £1 15*s*. $0\frac{3}{4}$*d*.
(13) £8 3*s*. $3\frac{1}{4}$*d*. (14) £8 12*s*. 1*d*.

Ex. (lxxii.), p. 129.

(1) £1412 11*s*. 8*d*. (2) £3226 0*s*. 6*d*.
(3) £28299 1*s*. 10*d*. (4) £31282 8*s*. 5*d*.
(5) £18873 1*s*. 6*d*. (6) £27877 13*s*. 3*d*.

Ex. (lxxiii.), p. 130.

(1) 22645 sec.; 61243 sec.
(2) 107020800 sec.; 544324 min.
(3) 33 da. 17 hr. 27 min.; 6 hr. 32 min. 47 sec.
(4) 8 da. 14 hr. 13 min. 12 sec.;
0 wk. 2 da. 0 hr. 24 min. 56 sec.
(5) 118; 151; 286; 120; 151.
(6) 76 hr. 34 min. 36 sec. (7) 136 da. 1 hr. 42 min.
(8) 26 wk. 2 da. 2 hr. (9) 22 yr. 293 da. 1 hr.
(10) 77 hr. 3 min. 41 sec.
(11) 250 da. 23 hr. 1 min. 13 sec.
(12) 2 hr. 54 min. 48 sec. (13) 83 da. 17 hr. 47 min.
(14) 6 da. 22 hr. (15) 298 da. 21 hr.
(16) 1 yr. 331 da. 21 hr.
(17) 5 da. 9 hr. 36 min. 46 sec.
(18) 463 hr. 35 min. 5 sec.; 740 hr. 46 min. 57 sec.
(19) 2 da. 6 hr. 14 min.; 12 min. 17 sec.

Ex. (lxxiv.), p. 132.

(1) 132 in.; 23166 ft. (2) 446418 in.; 5499 in.
(3) 13513 po. $3\frac{1}{2}$ yd.; 306 fur. 0 po. 4 yd. 2 ft. 6 in.
(4) 137 mi. 36 po. 3 yd. 1 ft.;
1309 mi. 4 fur. 32 po. 4 yd. 2 ft. 8 in.
(5) 107 yd. 1 ft. 8 in. (6) 154 mi. 2 fur. 20 po.
(7) 23 fur. 21 po. $4\frac{1}{4}$ yd. (8) 75 yd. 8 in.
(9) 50 mi. 2 fur. 35 po. (10) 35 po. [illegible] yd.
(11) 87 yd. 3 in.; 932 mi. 1 fur. 31 po.
(12) 1858 po. 3 yd.; 1783 mi. 3 fur. 5 po. 1 yd.
(13) 6 yd. 1 ft. 2 in.; 5 fur. $6\frac{5}{27}$ po.
(14) 2 yd. 1 ft. $5\frac{1}{2}$ in.; 1 fur. $29\frac{12}{55}$ po.

Ex. (lxxv.), p. 134.

(1) 36751875 sq. in. (2) 44425044 sq. in.
(3) 1210000 sq. yd.; 94608 sq. in.
(4) 4 sq. yd. 55 sq. in.; 3 ac. 28 po. 9 sq. yd.
(5) 1148 po. 2 sq. yd.; 14 po. 10 sq. yd. 7 sq. ft. 110 sq. in.
(6) 284 ac. 2 ro. 25 po.
(7) 163 sq. yd. 7 sq. ft. 91 sq. in.
(8) 112 ac. 3 ro. 33 po. $15\frac{1}{4}$ sq. yd.
(9) 27 ac. 2 ro. 36 po.
(10) 5 sq. yd. 8 sq. ft. 129 sq. in.
(11) 1 ac. 2 ro. 16 po. (12) 3 ac. 1 ro. 30 po.
(13) 6 sq. yd. 7 sq. ft. 22 sq. in.
(14) 66 ac. 3 ro. 36 po.
(15) 88 ac. 2 ro.; 931 ac. 3 ro. 9 po.
(16) 1 ro. 18 po.; 1 ro. 27 po.

Ex. (lxxvi.), p. 135.

(1) 202 cub. ft.; 1175183 cub. in.; 654558 cub. in.
(2) 43 cub. ft. 21 cub. in.; 9 cub. yd. 11 cub. ft. 372 cub. in.
(3) 244944 cub in.; 149904 cub. in.
(4) 270 cub. yd. 26 cub. ft. 1143 cub. in.
(5) 195 cub. yd. 3 cub. ft. 298 cub. in.
(6) 3558 cub. yd. 10 cub. ft. 284 cub. in.
(7) 8 cub. yd. 20 cub. ft. 1545 cub. in.
(8) 8 cub. yd. 1634 cub. in.
(9) 27 cub. yd. 7 cub. ft. 1472 cub. in.
(10) 707 cub. yd. 1323 cub. in.;
25049 cub. yd. 17 cub. ft. 518 cub. in.
(11) 6 cub. yd. 14 cub. ft. 1029 cub. in.; 8 cub. yd. 24 cub. in.

Ex. (lxxvii.), p. 136.

(1) 59 pts.; 109792 pts.
(2) 8 qr. 2 bu. 1 gall. 2 pt.; 47 qr. 4 bus. 3 pk. 1 gall.
(3) 41 gall. 1 pt. (4) 20 bus. 1 pk. 1 gall.
(5) 197 qr. 3 bus. (6) 2 qt. 1 pt.
(7) 3 pk. 1 gall. (8) 6 qr. 7 bus. 3 pk.
(9) 342 qr. 4 bus. 2 pk.; 1115 qr. 4 bus. 1 pk.
(10) 3 qt. 1 pt. 8 qr. 3 pk.

Ex. (lxxviii.), p. 137.

(1) 12960 gr. (2) 1680 dwt.; 3420 dwt.; 6185 dwt
(3) 22253 gr.; 42663 gr.
(4) 6 oz. 11 dwt. 1 gr; 7 lb 4 oz. 18 dwt.
(5) 12 lb. 6 oz. 19 dwt. 13 gr.; 13 lb. 6 oz. 6 dwt.
(6) 74 lb. 7 oz. (7) 30 oz. 4 dwt. 9 gr.
(8) 87 lb. 7 oz. 12 dwt. 18 gr (9) 3 oz. 4 dwt. 21 gr.
(10) 7 lb. 9 oz. 13 dwt. (11) 9 oz. 12 dwt. 23 gr
(12) 89 lb. 5 oz. 8 dwt.; 141 lb. 7 oz. 19 dwt.
(13) 401 oz. 7 dwt. 11 gr; 148 lb. 9 oz. 5 dwt. 21 gr.
(14) 2 lb. 12 dwt.; 6 oz. 6 dwt. $11\frac{2}{3}$ gr.
(15) 5 dwt. 8 gr.; 2 oz. 19 dwt. 20 gr.

Ex. (lxxix.), p. 138.

(1) 17600 oz.; 4352 dr.; 10000.
(2) 203200 oz.; 30050 lbs.
(3) 78416 dr.; 7507 lbs.
(4) 2 cwt. 3 qrs. 22 lbs. 11 oz.; 1 ton 17 cwt. 1 qr. 24 lbs

(5) 4 cwt. 2 qrs. 14 lbs. 8 oz.; 9 cwt. 2 qrs. 15 lbs. 15 oz. 14 drs.
(6) 53 lb. 12 oz. 1 dr. (7) 45 qr. 19 lbs. 15 oz.
(8) 88 cwt. 2 qr. 14 lbs. (9) 2 lb. 1 oz. 9 dr.
(10) 2 qr. 22 lb. 8 oz. (11) 1 cwt. 1 qr. 11 lbs.
(12) 7t. 19 cwt. 3 q. (13) 3 lbs.
(14) 34t. 18 cwt. 1 qr. 13 lbs.
(15) 120 cwt. 67 lbs. 2 oz.; 187 cwt. 65 lbs.
(16) 156 cwt. 1 qr. 15 lbs.; 390 oz. 13 dr.
(17) 1 cwt. 21 lbs. 8 oz.; 16 cwt. 1 qr. 13 lbs. 2 oz.
(18) 1 qr. 14$\frac{34}{35}$ oz.; 2t. 3 cwt. 3 qr. 6$\frac{5}{17}$ lbs.

Ex. (lxxx.), p. 139.

(1) 13 cwt. 1 qr. 2$\frac{1}{3}$ lb. (2) 13 lb. 14 oz. 12 dr.
(3) 80 mi. 1 fur. 22 po. (4) 679 yd. 1 ft. 5 in.
(5) 166 ac. 3 ro. 32 po. (6) 757 ac. 2 ro. 12 po.
(7) 78 sq. yd. 7 sq. ft. 6 sq. in.

Ex. (lxxxi.), p. 139.

(1) 2 cwt. 4 lb. (2) 10 oz. 5 dr.
(3) 1 mi. 5 fur. 3 po. (4) 3 yd. 6 in.
(5) 5 ac. 3 ro. 4 po. (6) 1 ac. 3 ro. 8 po.
(7) 5 sq. yd. 7 sq. ft. 87 sq, in.

Ex. (lxxxii.), p. 140.

(1) 13*s*. 4*d*.; £1 11*s*. 3*d*.; £2 10*s*. 9*d*.
(2) 6 fur. 16 po.; 30 po.; 3 qr. 8$\frac{1}{3}$ lb.
(3) £152 11*s*. 0$\frac{4}{5}$*d*.; £1 13*s*. 9*d*.; 2 mi. 2 fur.
(4) £514 16*s*. 15*s*. 9*d*. (5) £1 2*s*. 10$\frac{1}{2}$*d*.
(6) 13*s*. 6*d*. (7) 9 ac. 2 ro. 13$\frac{1}{2}$ po.
(8) 16 da. 3 hr. 35 min. (9) 2 fur. 37 yd. 1$\frac{1}{5}$ in.
(10) 4 cwt. 2 qrs. 11 lbs. 10$\frac{6}{7}$ oz.

Ex. (lxxxiii.), p. 141

(1) $\frac{1}{46}$. (2) $\frac{224}{247}$. (3) $\frac{11}{42}$. (4) $\frac{158}{211}$.
(5) $\frac{613}{2016}$. (6) $\frac{5}{8}$. (7) $\frac{3}{5}$. (8) $\frac{6526}{15481}$.
(9) $\frac{3}{11}$. (10) $\frac{1}{3}$. (11) $\frac{2880}{17}$. (12) $\frac{187}{119}$.

Ex. (lxxxiv.), p. 144

(1) 12*s*. 6*d*. (2) £15 5*s*. 6*d*.
(3) 2·3436*d*. (4) 3 qr. 18 lb. 12 oz.
(5) 12 dr. (6) £16. 0*s*. 6*d*.
(7) 1*s*. 5*d*. (8) £2 16*s*. 9·375*d*.
(9) 4*s*. 2*d*. (10) £2 10*s*. 7·6*d*.

(11) 22 lb. $6\frac{2}{5}$ oz. (12) £7 16*s.* $2\frac{1}{4}$*d.*
(13) 4 tons 16 cwt. 17·4 lb. (14) £26 17*s.* $10\frac{3}{30}$*d.*
(15) £2 5*s.* 9·8*d.* (16) 16*s.* 7*d.*
(17) £1 14*s.* 8*d.*

Ex. (lxxxv.), p. 145.

(1) ·3285. (2) $\cdot00208\dot{3}$. (3) ·1875.
(4) $\cdot4\dot{3}$. (5) 14·49. (6) ·24.
(7) 2·64. (8) 1·382890625. (9) $\cdot002\dot{7}$.
(10) $1\cdot4\dot{3}1\dot{8}$. (11) ·3. (12) ·00091875.
(13) $2\cdot44591\dot{6}$. (14) ·5581. . . . . (15) ·1406. . . . .

Examination Papers. (Page 146).

(I.)

(1) $366\frac{6}{59}$. (2) 3413 da. 9 hr. (3) 3 da. 2 hr. 20 min.
(5) 191.

(II.)

(2) $13\frac{1307}{1321}$. (3) 49 min. past 1 P.M.; $149\frac{1}{7}$ mi
(4) 19 mi. $1404\frac{27}{62}$ yd. (5) 28160.

(III.)

(2) 3600 ; £7 10*s.*
(3) *A*, 3 ac. 1 ro. 20 po. 21 yd. $77\frac{1}{7}$ in.; *B*, 6 ac. 3 ro. 1 po. 1[illegible] yd. 7 ft. 11[illegible] in.; *C*, 7 ac. 2 ro. 16 po. 17 yd. 1 ft. $29\frac{4}{7}$ in.
(4) 29 yd. (5) 16 ton 4 cwt.; 10 cwt. 3 qr. 5 lb.

(IV.)

(1) 11.37\frac{1}{2}$. (2) 259 bus. 2 pk. 1 gal. $1\frac{193}{231}$ pt.
(3) 41 bus. 3 pks. $2\frac{2}{5}$ qts. (4) 47 bags. (5) $96.93. . . . .

(V.)

(1) Loses $2. (2) $13\frac{387}{576}$ cents. (3) 20 grs. is largest weight.
(4) 12 tons 7 cwt. 3 qrs. 16·67 lbs. (5) 250 lbs.

Ex. (lxxxvi.), p. 150.

(1) £8061 7*s.* $3\frac{3}{4}$*d.* (2) £6022 0*s.* $7\frac{1}{2}$*d.*
(3) £5158 2*s.* $8\frac{1}{4}$*d.* (4) £53761 15*s.* 10*d.*
(5) £83720 19*s.* $5\frac{1}{2}$*d.* (6) £61386 16*s.* 7*d.*
(7) £169567 17*s.* $11\frac{1}{2}$*d.* (8) £1164 9*s.* 9*d.*
(9) £835 10*s.* $7\frac{1}{2}$*d.* (10) £1457 0*s.* $0\frac{15}{32}$.

Ex. (lxxxvii.), p. 151.

(1) £231 0*s*. $8\frac{112}{121}$*d*.
(2) £50 14*s*. $4\frac{178}{200}$*d*.
(3) £32 2*s*. $7\frac{1}{2}$*d*.
(4) £496 2*s*. $6\frac{47}{130}$*d*.
(5) £805 9*s*. $1\frac{1}{5}$*d*.
(6) £3127 0*s*. $7\frac{1}{2}$*d*.
(7) £63 6*s*. $5\frac{3}{4}$*d*.
(8) £1066 1*s*. $6\frac{3}{4}$*d*.
(9) £311 16*s*. $1\frac{23}{25}$*d*.
(10) 839 15*s*. $3\frac{3}{10}$*d*.

Ex. (lxxxviii.), p. 153.

(1) $1690.
(2) $$10.93\frac{1}{2}$.
(3) $164.74.
(4) $$74.14\frac{1}{2}$.
(5) $103.70.

Ex. (lxxxix.), p. 156.

(1) 6.
(2) $1\frac{1}{2}$.
(3) $47775.
(4) 25.
(5) $1502\frac{8}{17}$.
(6) 203 min.
(7) $3240.
(8) $14\frac{7}{13}$.
(9) £31 10*s*.
(10) 54 days.
(11) 20 cents.
(12) $$408\frac{1}{3}$.
(13) $9.15.
(14) $14.40.
(15) £18 15*s*. $7\frac{46}{47}$*d*.

Ex. (xc.), p. 157.

(1) $$7833.33\frac{1}{3}$.
(2) $5040.
(3) $18\frac{7}{10}$.
(3) $\frac{14}{165}$.
(5) 27 min.
(6) $$1182.12\frac{1}{2}$.
(7) $286\frac{2}{13}$.
(8) $400.
(9) £1 17*s*. 4·128*d*.
(10) The first.
(11) $22\frac{1}{2}$ cwt.

Ex. (xci.), p. 257.

(1) 480.
(2) $1152.
(3) 11268.
(4) 4000.
(5) $10.52.
(6) $$54.61\frac{1}{4}$.
(7) $360.
(8) 16.
(9) 20.
(10) $78.60.
(11) 27 laborers.
(12) 75 burners.
(13) 6 weeks.
(14) $396.
(15) 7 weeks.
(16) 4 days.
(17) $155\frac{3}{4}$ qr.
(18) $5\frac{1}{2}$ weeks.
(19) 84.
(20) 10 days.
(21) 2 days.
(22) 660.
(23) £120.

Ex. (xcii.), p. 161.

(1) $3\frac{3}{5}$ hr.
(2) $13\frac{37}{191}$ hr.
(3) $2\frac{22}{73}$. days
(4) $2\frac{2}{3}$ min,
(5) 10 days.
(6) 4 hr.
(7) 18 days.
(8) $\frac{28}{33}$.

Ex. (xciii.), p. 163.

(1) $21\frac{9}{11}$ min. past 4. (2) $32\frac{8}{11}$ min. past 6.
(3) $49\frac{1}{11}$ min. past 9.
(4) $5\frac{5}{11}$ min. and $38\frac{2}{11}$ min. past 4.
(5) $21\frac{9}{11}$ min. and $54\frac{6}{11}$ min. past 7.
(6) $10\frac{10}{11}$ min. and $43\frac{7}{11}$ min. past 11.
(7) $38\frac{2}{11}$ min. past 1. (8) $54\frac{6}{11}$ min. past 4.
(9) $10\frac{10}{11}$ min. past 8.

Examination Papers. (Page 164.)

(I.)

(1) 1800 lbs. (2) $18360. (3) $26.40. (4) $1042.40.
(5) £615.

(II.)

(1) $·029$\frac{23}{136}$. (2) 269.33\frac{1}{3}$. (3) $2408.40. (4) 300.
(5) 324 days.

(III.)

(1) 18 days. (2) 1200 men. (3) 35 days. (4) $33\frac{1}{3}$ days; $\frac{1}{5}$.
(5) 60 min.

(IV.)

(1) 24 days. (2) 360 days.

(3) 60 min. from A's starting point; 5 hrs. and 15 hrs. from starting.

(4) At 10 hrs. 15 min. a.m. on Saturday the watch is 5 min. $36\frac{7}{48}$ sec. too slow.

(5) $10\frac{10}{29}$ min. past 5 and $9\frac{21}{31}$ min. to 5.

(V.)

(1) 52 days. (2) 10 hrs. (3) A in $9\frac{3}{5}$ hrs.; B in $6\frac{6}{7}$ hrs.
(4) 3 hr. $54\frac{2}{5}$ min. p.m. (5) $\frac{3}{85}$ more.

Ex. (xciv.), p. 169.

(1) $825. (2) $1160. (3) $1215. (4) $4589.
(5) $95.70. (6) $156. (7) 164.02\frac{1}{2}$. (8) £4 9s. $2\frac{7181}{7300}$

Ex. (xcv.), p. 171.

(1) $4\frac{1}{2}$ %. (2) $5\frac{1}{4}$ years. (3) $1250.
(4) $375. (5) 20 years. (6) $5\frac{1}{4}$ %.
(7) $12\frac{1}{2}$ years (8) 5 %. (9) 97 days.
(10) $4\frac{3}{4}$ %, nearly. (11) $9\frac{1}{8}$ %. (12) Gained $50
(13) 6 %. (14) $600; $13\frac{1}{3}$ yrs. (15) October 6.

Ex. (xcvi.), p. 183.

(1) \$869.75. (2) \$902.83. (3) \$82.56.

Ex. (xcvii.), p. 176.

(1) \$59.109. (2) 175.28. (3) \$248.77. (4) \$297.89.

Ex. (xcviii.), p. 177.

(1) \$125·509. (2) \$238.81. (3) \$153.22.
(4) \$5.508. (5) £4 3*s.* $2\frac{74}{125}$*d.* (6) \$420.25.
(7) 92 cents. (8) \$15400. (9) \$24000.

Ex. (xcxix). p. 181.

(1) \$4600. (2) \$70. (3) \$562.50
(4) \$1050. (5) £537 10*s.* (6) \$36.
(7) \$456.80. (8) \$137.50 (9) £152 1*s.* 3*d.*
(10) £7 19*s.* 3*d.* (11) \$6000. (12) \$16·186...
(13) 5 per cent. (14) \$1105; Bill is due May 4.
(15) \$$45\frac{5}{6}$, \$$13\frac{2}{21}$. (16) 80 to 83; \$32.
(17) \$$16\frac{112}{113}$. (18) \$520; 6%.

Ex. (c.)

(1) \$0·285. (2) \$716.76. (3) \$314. (4) \$511.
(5) \$18.86...

Examination Papers, Page 184.

(I.)

(1) \$3121.60. (2) \$781.25. (3) 4.08%.
(4) \$77.90. (5) \$8108.326...

(II.)

(2) \$19.047 : 95 cents. (3) £9 3*s.* 4*d.* (4) $3\frac{1}{8}$%.
(5) \$100.

(III.)

(1) \$2500. (2) \$71·88... (3) \$$888.88\frac{8}{9}$. (4) \$$41\frac{33}{487}$.

(IV.)

(1) \$$7·11\frac{1}{9}$. (2) \$50000. (3) $6\frac{1}{4}$%; £574 13*s.*
(4) A's by \$224.05. (5) \$12000.

Ex. (ci.), p. 187.

(1) $7\frac{1}{2}$ mo. (2) $4\frac{7}{12}$ mo. (3) 5 mo.
(4) $3\frac{1}{2}$ mo. (5) $7\frac{1}{2}$ mo. (6) \$$666\frac{1234}{2341}$.
(7) $8\frac{2}{3}$ mo. (8) \$364.01. The equated time is May 5, 1869. All the bills are equivalent to \$362.35, but this will draw interest at 6% till June 2. (9) 28 May.

Ex. (cii.), p. 190.

(1) Aug. 6, 1875. (2) Nov. 20, 1877.
(3) Nov. 25, 1877.

Ex. (ciii.), p. 191.

(1) 21·25. (2) $738\cdot\dot{5}7142\dot{8}$. (3) $56087\cdot\dot{6}$.
(4) 26·9625. (5) 10·154875.

Ex. (civ.), p. 192.

(1) $120; 278 horses. (2) 33⅓; 2; 40.
(3) 75844. (4) 3¾; 12½; 15.
(5) 300; 23437½; 4300000.

Ex. (cv.), p. 193.

(1) $106·40. (2) $700. (3) $5·91.
(4) $1.87½. (5) 4½%. (6) $77.
(7) $7488. (8) $9800. (9) $38400.
(10) $20000; $50.

Ex. (cvi.), p. 194.

(1) $3·125. (2) 747·25. (3) £4488 15s.
(4) $415·25. (5) $473. (6) $9.80.
(7) $10000. (8) $4800.

Ex. (cvii.), p. 195.

(1) $56.70. (2) $0·017. (3) $1312500.
(4) 1¼ cents in the dollar.

Ex. (cviii.), p. 196.

(1) $458.10. (2) $88. (3) $450.
(4) $199.50. (5) $9500.

Examination Papers. (Page 197.)

(I.)

(1) 4·065. (2) $225. (3) $3640.
(4) 41825. (5) 10.

(II.)

(1) $1760. (2) $7119.80. (3) 21·75.
(4) $10935000. (5) *A*, $40; *B*, $45.

(III.)

(1) $8400. (2) 418 bales; $323.58
(3) As 40 and 31. (4) $8. (5) 2·97*d*.

(IV.)

(1) 305.78\frac{1}{8}$. (2) 16222·11 lbs. (3) 165 : 255 ; 330.
(4) Grain, $1020 ; groceries, $950.
(5) *A* gets $842.30 ; *B*, $918.87 ; *C*, $1598.83.

(V.)

(1) 2535\frac{15}{71}$. (2) $2040. (3) 100 bales.
(4) $255. (5) $3000.

Ex. (cix.), p. 203.

(1) 25. (2) $2000. (3) $4.35.
(4) £1 1*s.* 1*d.* per gall. (5) 3$\frac{13}{17}$ gain. (6) 33$\frac{1}{3}$.
(7) 8 per cent. (8) 10 per cent. (9) 5.
(10) 23. (11) 13·9...per cent. (12) 10.
(13) $300. (14) 3*s.* 7$\frac{1}{1}$*d.* per lb. (15) 4 lb.
(16) $3.45. (17) $3.60. (18) 33$\frac{1}{3}$ per cent.

Ex. (cx.), p. 212.

(1) $7262·75. (2) $7840. (3) $9065.25.
(4) £3542. (5) £523 16*s.* 9*d.* (6) $11200.
(7) $3000. (8) $850. (9) £2400.
(10) £6000. (11) $55.50. (12) $228.80.
(13) $385. (14) $150. (15) £276.
(16) $600. (17) $680. (18) $950.
(19) $3200. (20) 6$\frac{2}{3}$. (21) 5$\frac{5}{16}$.
(22) 5$\frac{10}{13}$. (23) 4$\frac{2}{3}$. (24) 111$\frac{1}{9}$.
(25) 108$\frac{2}{5}$. (26) 119$\frac{1}{21}$. (27) 90$\frac{10}{11}$.
(28) 80. (29) $5000. (30) $9600.
(31) $67500. (32) $41540. (33) 101$\frac{1}{2}$.
(34) 85. (35) 92$\frac{1}{8}$. (36) 83$\frac{1}{3}$.
(37) 6 per cents ; $\frac{4}{5}$%. (38) £4725. (39) 1$\frac{4}{5}$.
(40) £24960. (41) 90. (42) Nothing.
(43) Increased $56.55. (44) $10692 ; $21384
(45) Gain $125. (46) 5$\frac{3}{5}$ years. (47) Loss $45.22.
(48) 6000. (49) 6 per cents. (50) $30300.
(51) $44092. (52) 89$\frac{3}{4}$. (53) $95.
(54) 4\frac{1}{249}$: 4$\frac{9}{34}$. (55 $3200000. (56) 5040$\frac{1521}{1587}$.

Examination Papers. (Page 215).

(I.)

(1) $2568. (2) Loss 8$\frac{4}{7}$ per cent.
(3) 1.86\frac{12}{43}$ per lb. (4) 21$\frac{1}{2}$ per cent.
(5) Loss of $10·163...

(II.)

(1) $2.33\frac{1}{3}$. (2) $11\frac{1}{9}$; $8\frac{7}{8}$. (3) $9\frac{11}{96}$ cts. per oz.
(4) 90. (5) $229\frac{1}{3}$.

(III.)

(1) 4000 lbs.; $1.08\frac{1}{3}$. (2) 80. (3) *A*; $2.67\frac{6}{7}$;
$2.68\frac{3}{4}$; 40% and $39\frac{23}{43}$%. (4) $10. (5) 102·723...

(IV.)

(1) 12·99 gain. (2) Lost $71\frac{19}{91}$ (3) $6000;
$8160; $6528. (4) $510.59. (5) $2530.

(V.)

(1) $1000. (2) 40. (3) $9142\frac{6}{7}$.
(4) $43\frac{1}{3}$. (5) Loses $60; gains $180.

Ex. (cxi.), p. 220.

(1) $33; $27. (2) $250· $375; $875; $1000.
(3) $3300; $2200; $1650; $1320.
(4) 9 cwt. of saltpetre; $1\frac{1}{5}$ cwt. of sulphur; $1\frac{4}{5}$ cwt. of charcoal.
(5) 120 yd.; 160 yd.; 200 yd. (6) $240 to *A*; $80 to *B*; $320 to *C*. (7) 28; 32; 40.
(8) *A*, £102 3*s*. 9*d*.; *B*, £132 16*s*. $10\frac{1}{2}$*d*.; *C*, £183 18*s*. 9*d*
(9) 113; 339; 678; 791. (10) 30.
(11) $57\frac{41}{87}$; $40\frac{30}{87}$; $91\frac{83}{87}$. $10\frac{30}{87}$.
(12) *A*, 9*s*.. *B*, 12*s*.; *C*, 24*s*.
(13) Men, $5; women, $3; boys, $2.40.
(14) Men, $182.70; women, $182.70; children, $152.25.
(15) $6000. (16) *A*, $700; *B*, $2500; *C*, $1800.
(17) *A*, $1050; *B*, $1200; *C*, $1250; *D*, $1500.
(18) $\frac{414}{1573}$; $\frac{30}{143}$; $\frac{60}{286}$.
(19) $175.50; $218.40; $252.72; $117.00; $149.76.
(20) 1200 boys.

Ex. (cxii.), p. 222.

(1) First, $44.25; Second, $88.50.
(2) *A*, $4.50; *B*, $6.75; *C*, $11.25.
(3) *A*, $2062.40; *B*, $2320.20; *C*, $773.40.
(4) *A*, $656\frac{820}{1649}$; *B*, $286\frac{820}{1649}$. (5) *D*, $20; *E*, $56
(6) *A*, $87.50; *B*, $120; *C*, $202.50

(7) \$30; \$48; \$28. (8) \$15.30; \$14.25.
(9) *A*, \$245; *B*, \$225.
(10) Johnston, \$585; Wilson, \$487.50; Miller, \$175.50.
(11) *A*, \$34.30; *B*, \$53.90.
(12) *A*, \$735; *B*, \$490; *C*, \$367.50; *D*, \$294.
(13) 16$\frac{5}{7}$ gall. and 25$\frac{1}{14}$ gall.
(14) *A*, \$118.30; *B*, \$55.90; *C*, \$13.

Ex. (cxiii.), p. 225.

(1) Net gain, \$1974; *A*'s, \$2312; *B*'s, \$2172.
(2) Net loss, \$3165; *A*'s, \$2836; *B*'s, \$1154.
(3) Net loss, \$3550; *A*'s, \$1010; *B*'s, net insolvency, \$2730.

Ex. (cxiv.), p. 230.

(1) 5 lbs. of first, 7 lbs. of second.
(2) 30 bu. oats; 20 bu. rye; 20 bu. barley.
(3) 50 lbs. at 55 cts.; 30 lbs. at 75 cts. (4) 15 gall. water.
(5) 12 gall. kerosene. (6) 14 bu. rye; 14 bu. barley.
(7) 18 lb. at 14 cts.; 18 lb. at 18 cts.; 48 lb. at 30 cts.
(8) 36 lb. at 33 cts.; 36 lb. at 37 cts.; 48 lbs. at 45 cts.

Ex. (cxv.), p. 237.

(1) 109$\frac{1}{2}$. (2) \$44693.20. (3) 2 fr. 13 cent.
(4) 1760 copeks. (5) 9 fl. 20 kr. (6) \$3345.44.
(7) £576 12*s.* 6*d.* (8) £1 = \$4·8665. (9) London gives
25 fr. 45c. for £1. (10) £1 = 13$\frac{2}{9}$ marcs banco.
(11) \$4.86; £1. (12) 2602$\frac{1}{8}$ fl. (13) 53$\frac{1}{2}$*d.* per milree (nearly). (14) ·0102045 oz.; 25·17 francs.

Examination Papers.

(I.)

(1) 1·2372. (2) *A*, \$6075; *B*, \$5400; *C*, \$6000.
(3) \$5774.43. (4) Direct, \$14224.91; cir., \$14476.72; gain, \$251.81. (5) 2·341 % discount.

(II.)

(1) 7.8$\frac{3}{4}$ cents and 66$\frac{3}{4}$ cents. (2) *A*, \$4912; *B*, \$6168.
(3) £1 = 25·35$\frac{15}{23}$ fr. (4) *A*, \$2324; *B*, \$1708.
(5) 10 and 4.

(III.)

(1) 33$\frac{1}{3}$ lbs. of 8, 10, and 12 cents and 100 lbs. of 20 cents.
(2) \$1212. (3) \$1257$\frac{1}{7}$. (4) $\frac{81}{256}$. (5) £2 3*s.* 2$\frac{1}{2}$*d.* (nearly).

(IV.)

(1) \$2211$\frac{9}{11}$. (2) \$43.63. (3) \$5.
(4) Paris, \$14285.71$\frac{3}{7}$; London, \$14600; Amsterdam, \$14640. (5) 1 lb. at 8; 8$\frac{1}{2}$ lb. at 13; 8 lb. at 14.

(V.)

(1) 59, 17, and 106. (2) 118$\frac{4}{7}$ per cent.
(3) 8, 10 and 12 months. (4) 9176$\frac{8}{17}$..
(5) ·42; 23$\frac{2}{17}$ per cent.

Ex. (cxvi.), p. 242.

(1) $\frac{4}{9}$ is greater. (2) $\frac{10}{41}$ is greater.
(3) $\frac{13}{24}$ is greatest: $\frac{4}{7}$ is least. (4) 45 : 364.
(5) 112 : 405. (6) \$31.25. (7) $\frac{13}{40}$.
(8) $\frac{1}{2}$: 3$\frac{1}{3}$. (9) 128 · 1 (10) 9 : 13.

Ex. (cxvii.). p. 245.

(1) 4 : 3 : : 12 : 9. (2) 12$\frac{3}{4}$. (3) ·0076
(4) *A* : *C* : : 25 : 39. (5) 21. (6) $\frac{10}{21}$.
(7) ·048.. (8) 28. (9) $\frac{5}{27}$.
(10) *A* \$552; *B* \$460; *C* \$345; *D* \$230.

Ex. (cxviiii.) p. 247.

(1) £1285. (2) 10 h. 40 m. 36 $\frac{7}{48}$ sec. (3) $\frac{9}{13}$ mi.
(4) 3 h. 25 min. P.M. (5) 10 d.; 12$\frac{5}{7}$ d. (6) \$47.13.
(7) 78$\frac{3}{4}$. (8) 8 P.M. Thursday.
(9) 7722 stones. (10) 12800

Ex. (cxix., p. 249.

(1) 54 men. (2, 1050 men. (3) 18.
(4) 50 men. (5) Navvies did 6 times as much as soldiers.
(6) 12$\frac{54}{25}$ dronas. (7) 576. (8) 16$\frac{2}{3}$.
(9) 155. (10) 12 days.

Ex. (cxx)., p. 252.

(4) 5000 mm. (5) 1067.25 dcm.
(6) 15 milligrams. (7) 43·7 mm.; 4.37 cm.
(8) 155000 sq. cm. (9) 1086·42 sq. dcm.
(10) 1725 grams. (11) 100 milligrams; 10000 decigrams.
(12) 256·7 centigrams. (13) 5000 milligrams.
(14) 1·60931 kilometres (15) 567·875 cu. cm.
(16) 3720 litres.

Ex. (cxxi.), p. 255.

(1) 35 sq. ft. (2) 135 sq. ft. (3) $300\frac{3}{8}$ sq. ft.
(4) 12 sq. ft. (5) $452\frac{5}{6}$ sq. ft. (6) 224 sq. ft.
(7) 508 sq. ft. (8) $150\frac{3}{16}$ sq. ft. (9) $402\frac{1}{2}$ sq. ft.
(10) $30\frac{1}{4}$ sq. yd. (11) $1406\frac{1}{4}$ sq. yd. (12) $315\frac{1}{16}$ sq. ft.
(13) $870\frac{1}{4}$ sq. ft. (14) 91 sq. ft. 121 sq. in.
(15) $11\frac{1}{9}$ sq. ft. (16) 502 sq. ft. 72 sq. in.
(17) 2232 sq. ft. 81 sq. in. (18) 16 ft.
(19) 7 ft. 5 in. (20) 8 ft. 9 in. (21) 11 yd.
(22) 88 yd. (23) 99 yd. (24) 9 ft.
(25) 16 ft. (26) 103 ft. (27) 405 yd.
(28) 255 yd. (29) 360·5 yd. (30) 163·25 yd. nearly.
(31) $5\sqrt{2}$ in. (32) $625\sqrt{2}$ ft.

Ex. (cxxii.) p. 256.

(1) $28\frac{8}{9}$. (2) $46\frac{8}{27}$. (3) 67.
(4) 58. (5) $142\frac{52}{81}$. (6) $33.60.
(7) 90.93\frac{1}{3}$. (8) 83.89\frac{1}{2}$. (9) £11 9*s.* 8*d.*

Ex. (cxxiii.), p. 258.

(1) 630. (2) 855. (3) $875\frac{7}{8}$. (4) 798.
(5) $25.60. (6) $13.62. (7) £6 13*s.* $2\frac{1}{3}$*d.*
(8) £6 6*s.* $9\frac{9}{10}$*d.*

Ex. (cxxiv.), p. 258.

(1) 4.93\frac{19}{27}$. (2) 520.96\frac{1}{4}$. (3) 210 ft.
(4) 135 ft. (5) £13 10*s.* (6) $5.95.
(7) $1.20. (8) 12 ft. (9) 62 yd. 1 ft.
(10) $22\frac{1}{2}$ ac. (11) $17\frac{1}{2}$ ft. (12) 1 ft. $9\frac{1}{3}$ in.
(13) $12. (14) 5952 stones. (15) $9.00.
(16) $13.50. (17) 429 yds.; 715 yds.
(18) $10511\frac{1}{9}$ sq. yd.; $2955\frac{5}{9}$ sq. yd. (19) 26.
(20) $69.20. (21) 29.55\frac{5}{9}$. (22) $112.64.
(23) 2 ft. (24) 300.

Ex. (cxxv.), p. 262.

(1) 336 cub. ft. (2) $548\frac{5}{8}$ cub. ft. (3) $83\frac{73}{96}$ cub. ft.
(4) $850\frac{185}{216}$ cub. ft. (5) $1058\frac{17}{432}$ cub. ft. (6) 9600.
(7) 16335 tons. (8) 500 men. (9) 1 ft. 7 in.
(10) $2031\frac{1}{4}$ lb. (11) $\frac{1}{8}$ cwt. (12) $5\frac{1}{2}$ ft.
(13) 160. (14) $3\frac{3}{4}$ ft. (15) £38 19*s.* 2*d.*

Examination Papers.

(1) 113. (2) \$75. (3) 31; $\frac{11}{13}$; $\frac{24}{29}$; $\frac{7}{9}$.
(4) 30 inches. (5) 6 of each. (6) ·02; 2000; ·000002:
2000·020002; $\frac{1000010001}{500000}$.
(7) $\cdot 4\dot{3}\dot{2}$. (8) 7899 mi. 1 fur. 25 po. 3 ft. 6 in.
(9) 45 miles. (10) \$210. (11) 9405 steps.
(12) \$5670, \$7560. (13) $\frac{55}{28998}$; ·0189.
(14) $\frac{3}{4}$; ·75. (15) 14. (16) 17695260 in.; $\frac{331}{352}$.
(17) 12 days. (18) 108. (19) \$12000.
(20) *A*, \$2.49; *B*, \$15.81. (21) 1; $\frac{38}{49}$.
(22) $\frac{9601}{400}$; $\frac{13}{160000}$; ·21; 2100. (23) $\frac{197}{1687}$: twice.
(24) £142 12*s*. 6*d*.; £42 15*s*. 9*d*.; £14 5*s*. 3*d*.
(25) 12·96; $\frac{23}{49}$. (26) 5 h. 48 min.
(27) 21 yd. 2 ft. $2\frac{5}{6}$ in. (28) \$32·66$\frac{2}{3}$.
(29) \$166·66$\frac{2}{3}$. (30) \$49·50 and \$49.50.
(31) $3\frac{1}{2}$. (32) $\frac{6667}{3630}$.
(33) 4 per cents.; \$128700. (34) $7\frac{9}{13}$ knots.
(35) \$35.00 and \$52.50. (36) 35 cents less.
(37) 3 ft. (38) $5\frac{5}{11}$%; \$$17\frac{13}{58}$.
(39) 55·3 ft. (40) $75\frac{1}{6}$ yds. (41) 1.
(42) \$4200. (43) $2691\frac{13}{17}$ (44) 256.
(45) 16*s*. 6*d*. (46) 200, 189, 101.
(47) \$$2\frac{2}{3}$; 10 %. (48) \$1785. (49) 270 ft.
(50) £5 15*s*. $0\frac{1}{4}$*d*. (51) $\frac{144}{625}$. (52) 5*s*. 1*d*.
(53) \$$3733\frac{1}{3}$. (54) 36 days (55) 26 sec. loss.
(56) \$823.68. (57) \$9.37$\frac{1}{2}$. (58) \$$16\frac{7}{8}$.
(59) \$1.50. (60) \$$15\frac{5}{9}$. (61) $2\frac{2}{5}$.
(62) 33·6 in. (63) 1 hr. (64) \$3346.87$\frac{1}{2}$.
(65) 3 hr. (66) 8 hr. (67) 7 %.
(68) \$1680. (69) $16\frac{1}{2}$ ft. (70) 11 sq. ft.
(71) $1\frac{183}{670}$. (72) 7·976 d. (73) $89\frac{81}{196}$; ·000365.
(74) \$4906·25. (75) 16 day. (76) $107\frac{85}{193}$ days.
(77) *A* gets \$1925; *B* \$770; *C* \$154. (78) \$3600; $5\frac{5}{8}$ %.
(79) Loss of 40 %. (80) £4 1*s*. $6\frac{23}{27}$*d*.
(81) $\frac{2}{3}$ 6.33403. (82) $\frac{6}{37}$. (83) 8 days.
(84) 4 hr. 32 min. (85) 22 lb. of nitre. $4\frac{2}{3}$ lb. of
charcoal, $3\frac{1}{3}$ lb. of sulphur. (86) $95\frac{1}{16}$ cents.
(87) \$16. (88) \$12705. (89) 1·2535 lb.
(90) \$60.75; \$20.42$\frac{1}{2}$. (91) 1.
(92) 31116. (93) £1; $\cdot\dot{7}4\dot{0}$.
(94) $1\frac{3}{32}$ min. to 12. (95) $7\frac{1}{2}$ %.
(96) 4% & 5%. (97) \$805. (100) \$67.50.
(99) 900.

(101) 42238274625. (102) ·3937 in.

(103) *A* in $6\frac{5}{27}$ da.; *B* in $9\frac{13}{19}$ da.; *C* in $14\frac{2}{13}$ da.

(104) 90 miles. (105) $410 ; $800.

(106) $9.50. (107) Loses 4 %. (108) $214.

(109) 9.38\frac{8}{15}$. (110) 1*s.* $1\frac{8}{9}$*d.*

(111) 14 min. (112) $\frac{1}{9}$. (113) $8250.

(114) 30 mi.; 25 mi. per hr. (115) $7\frac{1}{2}$; £50 less.

(116) 4 days. (117) 6.17\frac{1}{2}$.

(119) On Tuesday P.M. when one clock marks 9 hr. 11 min. and the other 8 hr. 54 min. 30 sec.

(120) $1238.70. (121) 4497 times.

(122) $450. (123) $10, $18, $15.

(124) $1.12. (125) 30. (126) 4 times.

(127) *B* walks a mile in $13\frac{1}{4}$ min.; he loses by $11\frac{1}{4}$ min. and by $\frac{45}{53}$ mi. (128) $7\frac{1}{2}$ months.

(129) 90 ; $465. (130) 1200 gal.

(131) $4\frac{1}{21}$. (132) 46. (133) 4000 ft.

(134) *A* gets 88 cents ; *B*, $49\frac{1}{2}$ cents.

(135) $13\frac{1}{13}$ min. and $16\frac{4}{11}$ min. past 3.

(136) $13\frac{1}{3}$ years. (137) £136 9*s.* 2*d.* · $6\frac{1}{4}$ %.

(138) 10. (139) £60000. (140) 38.

(141) $1.76. (142) $3\frac{1}{2}$ days. (143) $3\frac{1}{3}$ hrs.

(144) $28\frac{4}{7}$ days. (145) 400 miles.

(146) $87\frac{1}{2}$. (147) *A* gets $1155 ; *B*, $572 ; *C*, $259.50. (148) $6\frac{1}{4}$.

(149) $111835\frac{3}{7}$ metres. (150) £44 13*s.* 3*d.* ; $\frac{1}{9}$ ft.

(151) 2. (152) 937 ; ·02268 of an inch.

(153) $7\frac{1}{3}$ miles. (154) $8\frac{1}{3}$. (155) £600. (156) $760.

(157) $16\frac{2}{3}$ miles. (159) 24·9 fr. = £1 ; 25·15 fr. = £1.

(160) 3 ft. $11\frac{1}{15}$ in. (161) 1. (162) $7442\frac{2}{35}$.

(163) 48 min. (164) $4\frac{1}{2}$ months.

(165) 9 of spirit to 31 of water. (166) $56\frac{1}{4}$ %.

(167) $2035. (168) 10. (169) 2·198 in. in a century.

(170) 6.30 P.M. (171) 103·67 ; 574. (172) 15 hr.

(173) 56 yd. (174) $6. (175) $\frac{1}{11}$ min.

(176) 23.17\frac{31}{32}$. (177) $13\frac{29}{31}$ min. (178) 1520 tons.

(179) 10861578, nearly. (180) 1120 yd. ; $262\frac{50}{121}$ ac.

(181) 15 masters, 345 boys.

(182) *A* gets $17.50 ; *B*, $52.50 ; *C*, $106 ; *D*, $175.

(183) $4500. (184) 18 cents. (185) 40 % of loss.

(186) $24360. (187) 6 cents. (188) $11835.75.

(189) $3.40. (190) $81.12 ; 7 yds. (191) 2.

(192) $1\frac{9}{17}$ hr. (193) $7500.

(194) $3750. (195) $8400.

(196) Each child gets $1920.60 ; each brother, $960.30.

(197) $215. (198) He gains $14\frac{6}{11}$ %.
(199) 50000000 quarters.
(200) 3 miles an hour ; $1\frac{1}{2}$ miles an hour. (202) $4\frac{4}{5}$.
(203) $37\frac{5}{51}$. (204) 1609·306 metres.
(205) $2.00. (206) 8·243 %.
(207) $5940 of increase. (208) 2 : 7.
(209) 33 lb. 7 oz. Avoir. (210) 2 ft. 3 in.
(211) £9 3*s*. 6*d*. (212) 5 % and 10 %. (213) 3 pints.
(214) 4 %. (215) $2450.
(216) $92 ; $115. (217) 30 days.
(218) $4000. (219) $115. (220) 474.
(221) ·05. (222) 124·001 and $\frac{8}{9}$.
(223) 9000 men. (224) Second greater by $50.
(225) $9125. (226) $7\frac{1}{5}$ hr.; 18 hr. · $5\frac{1}{7}$ hr.
(228) 10 hours. (229) $6120.
(230) Length, 32 ft.; breadth, 16 ft.; height, 8 ft.
(231) 5*s*. (232) 42000.
(233) $33.75. (235) Lost $1.
(236) $305. (237) $$\frac{14}{55}$. (238) £6000.
(239) $9.50. (240) 48000.
(241) 3*s*. (242) $3\frac{9}{13}$ days. (243) 90 days.
(244) $56000, $48000, $42000. (245) $35.
(246) 160 ; 623. (247) 1500 bbls.
(248) £1 11*s*. $4\frac{1}{5}$*d*. (249) $5\frac{1}{2}$ years.
(250) Length, 27 ft.; breadth, 18 ft.; height, 12 ft.
(251) 3998936616. (252) 9. (253) $\frac{7}{8}$ %.
(254) Increase $160. (255) $17\frac{6}{7}$ %.
(256) $4\frac{1}{3}$ days. (257) $30. (258) $4\frac{1}{2}$ per cent.
(259) 24 days. (260) £110 ; 150 per cent.
(261) 400 in. (262) $$553\frac{1}{3}$; 5 per cent.
(263) $1\frac{1}{6}$ mi. (264) B. (265) $200.
(266) $1200. (267) Loses 5 %. (268) $4.81, nearly.
(269) $40800. (270) 25 cents.
(271) Between ·0001 and ·0002. (273) *A*, $3200; *B*, $4800;
C, $6000; *D*, $7000. (274) $6.50.
(275) $1\frac{3}{4}$ cents. (276) $5.60. (277) $96\frac{7}{8}$ cents.
(278) $12800. (279) $4.30. (280) $\frac{1}{56443}$ in.
(281) 750. (282) 478, 369, and 584.
(283) 45 mi. and 30 mi. per hour.
(284) 50 per cent. (285) $82\frac{1}{3}$ cents.
(286) $15\frac{1}{5}$ cwt. of nitre ; $1\frac{9}{10}$ cwt. of sulphur ; $2\frac{9}{10}$ cwt. of charcoal. (287) $4\frac{1}{2}$ miles per hour.
(288) Capital, $1000000 ; receipts, $100000. (289) $3\frac{1}{2}$ %.
(290) 1 ; 23. (291) $36\frac{31}{39}$; $10\frac{55}{112}$. (292) $1\frac{11}{13}$.
(293) £66 13*s*. 4*d*. (294) $2\cdot 8\dot{5}2380\dot{9}$ hr.

(295) 5 hr. $10\frac{10}{28}$ min. (296) 32 days.
(297) $39.95. (298) 8 per cent.
(299) $$16\frac{2}{7}$. (300) $28\frac{1}{2}$; $427\frac{1}{2}$ lb.
(301) 70·41 ; 1 46. (302) 4 per cent.
(303) $$274.12\frac{1}{2}$; $456.25. (304) $\frac{1}{3}$.
(305) $3\frac{1}{2}$ and $\frac{1}{2}$. (306) $1023.75.
(307) $20000. (308) $80 ; $$133\frac{1}{3}$; Loss $$13\frac{1}{3}$.
(309) $5100. (310) $12 31. (311) $\frac{12903}{2907}$; $1\frac{8}{9}$.
(312) $6\frac{2}{3}$ per cent. (313) 16 years.
(314) $3000. (315) $7800.
(316) $73\frac{2}{3}$ cents. (317) $1\frac{1}{3}$ months.
(318) *B*, by 16 yds. (319) $\frac{1}{4}$ per cent. loss.
(320) 26 yards. (321) $\frac{1}{27}$; $6\frac{26}{27}$; 7.
(322) £62 5*s*. (323) 14 min. $43\frac{3}{4}$ sec.
(324) 9 days. (325) 1 ; 2.
(326) $1440. (327) $100000 ; $4000.
(328) 70. (329) 2*s*. 6*d*. (330) 8*s*. $2\frac{1}{3}$*d*.
(331) $\frac{7}{8}$; $\frac{1}{8}$; 1. (332) 5 hr. (333) 5 miles.
(334) $234 ; $266.40 ; $306 , $345.60.
(335) $3\frac{1}{2}$ hours. (336) 70 cents.
(337) First is $50. (338) $14600. (339) 25 oxen.
(340) *A*, 3240 ; *B*, 2916 ; *D*, 2052 ; *C*, 1944 ; *E*, 1728 ; in all, 6480.
(341) $12\frac{122}{175}$. (342) $4\frac{23}{64}$. (343) $2200.
(344) 66. (345) 18 min. (346) £1160.
(347) Man gets £4 4*s*. ; woman gets £3 ; child gets £1 16*s*.
(348) 27951 ; 12500.
(349) 375 grains of potash ; 390 grains of soda.
(350) $20.95.

APPENDIX I.

INTEREST, ANNUITIES, &c.

1. *To find the amount of a given sum, in any given time, at Simple Interest.*

If P be the principal in *dollars*, n the length of time in years, r the interest of \$1 for 1 year; then the interest of \$P for 1 year will be Pr, and for n years will be Prn; wherefore, if I be the interest, then

$$I = Prn.$$

If M be the amount, we have

$$\begin{aligned} M &= P + Prn \\ &= P(1 + rn). \end{aligned}$$

2. *To find the amount of a given sum, in any given time, at Compound Interest.*

Let P = the principal in *dollars;*
" r = the interest of \$1 for one year;
" n = the number of years;
" R = the amount of \$1 for 1 year $= 1 + r.$
then PR will be the amount of \$P for 1 year,
and this becomes the *Principal* for the 2nd year;

$$\therefore PR \cdot R = PR^2$$

will be the amount of \$P for 2 years,
and this becomes the *Principal* for 3rd year;

$$\therefore PR^2R = PR^3$$

will be the amount of \$P for 3 years, etc.;

$$\begin{aligned} \text{hence } M &= PR^n \\ &= P(1 + r)^n \end{aligned}$$

will be the amount of \$P for n years.

$$\begin{aligned} \text{Interest} &= PR^n - P \\ &= P(R^n - 1). \end{aligned}$$

3. *To show that the formula $M = PR^n$ is true when n is fractional.*

If n is fractional we can always find a whole number such that na is a whole number $= q$, suppose. Divide the year into α equal intervals, and let m be the amount of \$1 in one of these intervals, then the amount of \$1 in α intervals is m^{α}, and is equal to R; also the amount of \$1 in n years, that is $n\alpha$ intervals, is equal to $m^{n\alpha}$, and therefore equal to R^n; hence the amount of $\$P = PR^n$, therefore the formula is true for fractional values of n.

Thus, if r' is the nominal yearly rate of interest of \$1 payable q times a year, meaning that $\frac{r'}{q}$ is the interest payable at the end of each qth part of a year, then the amount of \$1 in a year is $\$\left(1+\frac{r'}{q}\right)^q$, and the *true* yearly rate of interest is $\$\left(1+\frac{r'}{q}\right)^q-1$

Ex. (1). Find the amount of \$100 in 2½ years at 8 per cent. Compound Interest.

$$M = 100\left(1+\frac{8}{100}\right)^{\frac{5}{2}}$$

$$= 100\left\{1+\frac{5}{2}\cdot\frac{8}{100}+\frac{5\cdot3}{1\cdot2\cdot2^2}\cdot\left(\frac{8}{100}\right)^2+\frac{5\cdot3\cdot1}{1\cdot2\cdot3\cdot2^3}\left(\frac{8}{100}\right)^3+\ldots\right.$$

$$= 100\,(1+\cdot02+\cdot012+\cdot000155+\ldots)$$

$$= \$121\cdot215\ldots$$

Ex. (2). Find the advantage when Compound Interest is reckoned, of having the interest paid half-yearly, quarterly, &c., instead of yearly.

The advantage per \$1 for a year, when the interest is paid half-yearly, and the half-yearly payment is half the yearly one.

$$= \left(1+\frac{r}{2}\right)^2-(1+r)$$

$$= 1+r+\frac{r^2}{4}+\ldots-(1+r)$$

$= \frac{r^2}{4}$ nearly, since r is a small fraction.

Similarly, when the interest is paid quarterly, the advantage $= \frac{3r^2}{8}$ nearly.

And generally, when the interest is paid p times a year the advantage

$$= \left(1 + \frac{r}{p}\right)^{p} - (1 + r)$$

$$= \left(\frac{p-1}{2p}\right) r^2 \text{ nearly.}$$

Ex. (3). Find the amount of a given sum at compound interest, the interest being supposed due every instant.

If the interest were paid q times per annum, then

$$M = P\left(1 + \frac{r}{q}\right)^{nq}$$

$$= P\left\{1 + nq \cdot \frac{r}{q} + \frac{nq(nq-1)}{1\cdot 2} \cdot \left(\frac{r}{q}\right)^2 + \dots\right\}$$

$$= P\left(1 + nr + \frac{n\left(n - \frac{1}{q}\right) r^2}{1\cdot 2} + \dots\right\}$$

Now, if q be indefinitely great, that is, the intervals between the payments indefinitely small, then, neglecting $\frac{1}{q}$ and its powers, we heve

$$M = P\left(1 + nr + \frac{n^2r^2}{1\cdot2} + \frac{n^3r^3}{1\cdot2\cdot3} + \dots\right\}$$

$$= Pe^{nr}, \text{ where } e = 2\cdot7182818.$$

Todhunter's Algebra, Art. 54 .

Ex. (4). If P represents the population of any place at a certain time, and every year the number of deaths is $\frac{1}{p}$th, and the number of births $\frac{1}{q}$th, of the whole population at the beginning of that year; required the amount of population at the end of n years from that time.

At the end of one year from the time the population was P,

$$\therefore \text{ the increase} = \frac{P}{q} - \frac{P}{p} = P\frac{p-q}{pq}$$

∴ population at end of 1st year

$$= P + P\frac{p-q}{pq} = P\left\{1 + \frac{p-q}{pq}\right\} = P_1, \text{ say}$$

Similarly population at end of second year

$$= P_1\left\{1 + \frac{p-q}{pq}\right\} = P\left\{1 + \frac{p-q}{pq}\right\}^2;$$

and so on as in Compound Interest.

Hence, population at end of nth year $= P\left\{1 + \dfrac{p-q}{pq}\right\}^n$.

4. *To deduce the formula for Simple Interest from the formula for Compound Interest.*

$$\begin{aligned} M &= PR \\ &= P(1 \\ &= P\left\{1 + nr + \frac{n(n-1)}{1\cdot 2}r^2 + \&c.\right. \end{aligned}$$

Now Compound Interest may be regarded as consisting of two parts :

(1) Interest on principal, and
(2) Interest on interest.

If from the value of M, given above, we take away the part that represents *interest on interest*, there remains the interest on the principal, or the *Simple Interest.* Now the third term contains r^2 or $r \times r$, that is interest on interest. Similarly for succeeding terms.

Therefore for Simple Interest we have

$$M = P(1 + nr), \text{ as before.}$$

Hence, any formula for Simple Interest may be deduced from the corresponding one for Compound, by neglecting r^2 and all higher powers.

Therefore, in general, we shall find the formula for Compound Interest, and deduce the corresponding formula for Simple Interest. Indeed this is the only rational method of treating the subject. There is but one kind of interest, viz., Compound Interest. Simple Interest is incorrect in principle, and of course may lead to very incorrect results. When any sum of money is due, it matters not whether it is called principal or interest, it is of value to the owner, and should bear interest. The results obtained by the principle of Simple Interest are merely approximations to the correct results obtained by the principle of Compound Interest.

Exercise I.

(1) A sum of \$P is put out at Simple Interest for n years. Find an expression for its amount at the end of that time.

(2) If R be the amount of \$1 in one year at any rate of interest, the amount of P dollars in n years will be PR^n, whether n be integral or fractional.

(3) If \$P at Compound Interest amount to \$M in t years, what sum must be paid down to receive \$P at the end of t years?

(4) If \$P at Compound Interest, rate r, double itself in n years, and at rate $2r$, in m years, show that $\frac{m}{n} > \frac{1}{2}$.

(5) In what time will a sum of money treble itself, at 5 per cent., Compound Interest?

$$\log. 3 = \cdot4771212,\ \log. 1\cdot05 = \cdot0211893.$$

(6) A sum of money, \$P, is left among A, B, C, in such a manner that at the end of a, b, c years, when they respectively come of age, they are to possess equal sums. Find the share of each at compound interest.

(7) Two men invest sums of \$4410 and \$4400 respectively, at the same rate of interest, the former at simple, the latter at compound interest; at the end of two years their properties amount to equal sums. Find the rate of interest.

(8) In a certain county the births in a year amount to an mth of the whole population, and the deaths to an nth. In how many years will the population be doubled?

(9) A person spends in the first year m times the interest of his property; in the second year, $2m$ times that of the remainder; in the third year, $3m$ times that at the end of the second, and so on; and at the end of $2p$ years he has nothing left. Show that in the pth year he spends as much as he has left at the end of that year.

(10) If interest be payable at every instant, in how many years would \$1 amount to \$6, reckoning interest at 5 per cent.?

(11) A person starts with a certain capital, which produces him 4 per cent. per annum compound interest. He spends every year a sum equal to twice the original interest on his capital. Find in how many years he will be ruined, having given $\log. 2 = \cdot3010300$, $\log. 13 = 1\cdot1139434$.

(12) The population of a county is 35743. There is no emigration or immigration. The annual deaths are 27 in

the 1000, and the births 62 in 1000. What will be the increase of the population in five years?

(13) If the population of a country be P, and every year the number of deaths is $\frac{1}{60}$th and the number of births $\frac{1}{45}$th, of the whole population at the beginning of the year; find in what time the population will be doubled.

$$\log 181 = 2{\cdot}25768,\ \log 3 = {\cdot}4771213.$$
$$\log 2 = {\cdot}30103.$$

(14) On a sum of money borrowed, interest is paid at the rate of 5 per cent. After a time $600 of the loan is paid off, and the interest on the remainder reduced to 4 per cent., and the yearly interest is now lessened one-third. What was the sum borrowed?

(15) If a debt a at compound interest is discharged in n years by annual payments of $\frac{a}{m}$, show that

$$(1 + r)^n\ (1 - mr) = 1.$$

DISCOUNT.

5. *To find the Present Worth and Discount on any sum for a given time. (1) Compound Interest. (2) at Simple Interest.*

The principal difference between Amount and Present Worth is that the former is reckoned *forwards* from a given date while the latter is reckoned *backwards* from the same date. Hence it is evident that if V represents the Present Worth, then,

$$V = P(1 + r)^{-n}$$
$$= \frac{P}{(1 + r)^n} \text{ for Compound Interest;}$$

expanding and neglecting r^2 and higher powers we have

$$\frac{P}{1 + nr} \text{ for Simple Interest.}$$

If D be the Discount, then

$$\begin{aligned} D &= P - V \\ &= P - \frac{P}{(1+r)^n}, \text{ Compound Interest.} \\ &= P - \frac{P}{1+nr}, \text{ approximately,} \\ &= \frac{Pnr}{1+nr}. \text{ Simple Interest.} \end{aligned}$$

6. If we expand P $(1+r)^{-n}$, and neglect r^2 and higher powers, we get

$$P\,(1 - nr)$$

which may be called the *common* present worth.

The *true* present worth is

$$\frac{P}{1+nr}; \text{ by division}$$

$$= P\,(1 - nr + n^2r^2 - n^3r^3 + \&c.$$

Subtracting the *common* from the *true* present worth, we get

$$Pn^2r^2\,(1 - nr + n^2r^2 - \&c.)$$

$$Pn^2r^2\,\frac{1}{1+nr};$$

and, therefore, when n is small the error committed in taking *common* for *true* discount is nearly proportional to the square of the time.

In the expression

$$P\,(1 - rn),$$

if $n = \frac{1}{r}$ the *common* present worth is nothing; while if $n > \frac{1}{r}$ it is a negative quantity. That is, the *common* present worth of a bill for \$100 due 20 years hence at 5 per cent. is nothing, and for any period beyond 20 years the holder of the bill would require to pay a certain sum to get quit of it, which is absurd. The *true* present worth of \$100 due in 20 years, as given by the formula $\frac{P}{1+nr}$, is \$50.

7. *The interest is greater than the discount.*

Since
$$D = \frac{Pnr}{1+nr},$$
$$\frac{1}{D} = \frac{1+nr}{Pnr}$$
$$= \frac{1}{Pnr} + \frac{1}{P}$$
$$= \frac{1}{I} + \frac{1}{P};$$
$$\therefore \frac{1}{D} > \frac{1}{I};$$
$$\therefore I > D.$$

8. Since
$$V = \frac{P}{1+nr},$$
and
$$D = \frac{Pnr}{1+nr}$$
$$= \frac{I}{1+nr},$$
we see that *the Discount is the Present Worth of the Interest.*

9. *The Discount is half the harmonic mean between the Principal and the Interest.*

$$D = \frac{I}{1+nr}$$
$$= \frac{PI}{P+Pnr}$$
$$= \frac{PI}{P+I}$$
$$= \tfrac{1}{2}\,\frac{2\,PI}{P+I}$$
= half harmonic mean between principal and interest.

Ex. (1). The Simple Interest on a certain sum of money for a certain time is \$28, and the discount for the same time at the same rate of simple interest is \$24.

What is the sum of money? If the time be $3\frac{1}{2}$ years, what is the rate per cent.?

From the above formula we have

$$24 = \frac{28\ \mathrm{P}}{\mathrm{P} + 28}$$

$$24\ \mathrm{P} + 24 \times 28 = 28\ \mathrm{P}$$

$$4\ \mathrm{P} = 24 \times 28$$

$$\mathrm{P} = \$168\,;$$

$\therefore$ the sum required is \$168.

$$\text{Again,} \qquad \mathrm{D} = \frac{\mathrm{I}}{1 + nr},$$

$$\text{or} \qquad 24 = \frac{28}{1 + 3\frac{1}{2}r}$$

$$3 = \frac{7}{2 + 7r}\,;$$

$$r = \frac{1}{21}\,;$$

$$\therefore \text{rate per cent.} = 100r = \frac{100}{21} = 4\tfrac{16}{21}.$$

Ex. (2). If the Simple Interest on a sum of money for a given time and rate is $\frac{1}{n}$th of that sum itself, the True Discount will be $\frac{1}{n+1}$ of the sum.

$$\mathrm{D} = \frac{\mathrm{PI}}{\mathrm{P} + \mathrm{I}}\,;$$

$$\text{but, in this case,} \qquad \mathrm{I} = \frac{1}{n}\,\mathrm{P}\,;$$

$$\therefore \mathrm{D} = \frac{\mathrm{P}\,\frac{1}{n}\mathrm{P}}{\mathrm{P} + \frac{1}{n}\mathrm{P}}$$

$$= \frac{\mathrm{P}}{n + 1}.$$

Similarly, if the interest be $\frac{a}{b}$ of the principal, the discount is $\frac{a}{a+b}$ of the principal.

Ex. (3). Bank Discount at 5 per cent. being \$130.90, find the True Discount on the same amount:

$$\frac{\mathrm{D}}{\mathrm{I}} = \frac{n}{n+1}, \text{ where } n = \tfrac{5}{100} = \tfrac{1}{20}$$
$$= \frac{20}{21};$$
$$\therefore \mathrm{D} = \frac{20}{21} \times \$130.90$$
$$= \$124.66\tfrac{2}{3}.$$

10. *Bank Discount exceeds True Discount by the Simple Interest on the True Discount.*

$$\text{Bank Discount} - \text{True Discount} = I - D$$
$$= Pnr - \frac{Pnr}{1+nr}$$
$$= \left(P - \frac{P}{1+nr}\right)nr$$
$$= \frac{Pnr}{1+nr}\,nr$$
$$= Dnr$$
$$= \text{Simple Interest on the True Discount.}$$

Ex. (4). The True Discount on a bill due in 1 year, and discounted at 8 per cent., being \$500, what would have been the Bank Discount thereon?

$$\text{Bank Discount} = \text{True Discount} + Dnr$$
$$= \$500 + \$500 \times \tfrac{8}{100}$$
$$= \$540.$$

Exercise II.

(1) Bank discount being 5 per cent., a person receives \$37.10 less than the nominal value of his bill. What should he receive for his bill if true discount only were deducted?

(2) A person possesses a sum of money, the simple interest of which at 4 per cent. is \$536.25. With this sum he purchases an estate, for which he pays by a note payable in 4 months' time, and which, being discounted at 4 per cent., is worth at present exactly the money he possesses. For how much is the bill drawn?

(3) True discount, at 4 per cent., on a sum of money being \$15, find the simple interest on the same sum at 5 per cent.

(4) The interest on a certain sum of money is \$180, and the discount on the same sum for the same time and the same rate of interest is \$150. Find the sum.

(5) If the interest on \$A for a year be equal to the discount on \$B for the same time, find the rate of interest.

(6) If the three per cents. are at 90 one month before the payment of the half-yearly dividend, what is the rate of interest?

(7) *A* gives *B* a bill for \$$a$, due at the end of m years, in discharge of a bill for \$$b$, due at the end of n years. For what sum should *B* give *A* a bill due at the end of p years to balance the account at Compound Interest?

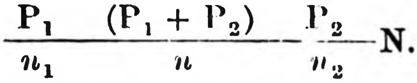
(8) Given *A* my income, a the premium for assuring \$100, r the rate of interest per cent. per annum, find what sum I must lay out in assuring my life, so that my executors may receive a sum whose interest will equal my reduced income.

(9) *A* sells goods to *B* and allows him 10 per cent. discount, if he pays in six months. What discount ought he to allow if payment be made in two months at 5 per cent. per annum, simple interest?

(10) The discount on a promissory note of \$100 amounted to \$7.50, and the interest made by the banker was 5·405 per cent. Find the interval at the end of which the note was payable.

EQUATION OF PAYMENTS.

11. *To find the equated time of payment of two sums due at different times at a given rate of interest.*



Let P_1, P_2, be the sums due at the end of the time n_1, n_2; r the rate of interest; take time N greater than n_2. Then it is manifest that the amounts of P_1, P_2, at the time N, should in equity be together equal to the amount of their sum $(P_1 + P_2)$, in the same time.

Whence,

$$P_1\Big(1 + r\Big)^{N - n_1} + P_2\Big(1 + r\Big)^{N - n_2}$$
$$= \Big(P_1 + P_2\Big)\Big(1 + r\Big)^{N - n};$$

or, dividing both sides by $\Big(1 + r\Big)^{N}$, we have

$$P_1\Big(1+r\Big)^{-n_1} + P_2\Big(1+r\Big)^{-n_2} = \Big(P_1 + P_2\Big)\Big(1+r\Big)^{-n} \ldots (1),$$

that is, the presents value of these sums, due at their respective times, are equal to the present value of their sum, due at the equated time.

If we expand, neglecting r^2 and higher powers, we have

$$P_1(1 - n_1 r) + P_2(1 - n_2 r) = (P_1 + P_2)(1 - nr)$$
$$\text{or, } P_1 n_1 + P_2 n_2 = (P_1 + P_2)n;$$
$$\therefore n = \frac{P_1 n_1 + P_2 n_2}{P_1 + P_2}$$

which is the rule given in Art. 184.

12. We have seen (Art. 6) that the expansion of $\Big(1 + r\Big)^{-n}$, neglecting r^2 and higher powers, gives *common* present worth instead of *true* present worth. The above process is, therefore, incorrect. It may easily be seen that we have taken the interest instead of the discount of sum paid before it is due, and thus, since interest is greater than discount (Art. 7), a small advantage has been given to the payer.

13. If we write equation (1) in the form,

$$\frac{P_1}{(1 + r)^{n_1}} + \frac{P_2}{(1 + r)^{n_2}} = \frac{P_1 + P_2}{(1 + r)^{n}},$$

and expand, neglecting r_2 and higher powers, we have

$$\frac{P_1}{1 + n_1 r} + \frac{P_2}{1 + n_2 r} = \frac{P_1 + P_2}{1 + nr},$$

which is the form of the equation for Simple Interest.

Solving for n we get

$$n = \frac{P_1 n_1 + P_2 n_2 + r(P_1 + P_2)n_1 n_2}{P_1 + P_2 + r(P_1 n_1 + P_2 n_2)};$$

which is the correct value of the equated time.

If r be a very small quantity, as in practice it usually is, and P_1, P_2, not very large, we shall have

$$n = \frac{P_1 n_1 + P_2 n_2}{P_1 + P_2}, \text{ as before.}$$

ANNUITIES.

14. The term Annuity is understood to signify any interest of money, rent, or pension, payable from time to time, at particular periods; and these payments may take place *yearly*, *half-yearly*, *quarterly*, *&c.*

15. *To find the Amount of an annuity to be paid for a given number of years, at Compound Interest.*

Let A be the annuity, n the number of years, R the amount on one dollar in one year, M the required amount.

We have

Amount due at the end of

$$\begin{aligned} 1 \text{ year} &= A \\ 2 \quad \text{``} &= A + AR \\ 3 \quad \text{``} &= A + AR + AR^2 \\ \&c. \quad \text{``} &= \&c. \\ \text{``} &= A + AR + AR^2 + \dots + AR^{n-1} \\ &= A\frac{R^n - 1}{R - 1} \end{aligned}$$

$$\text{Hence } M = A\frac{R^n - 1}{R - 1}$$

16. For Simple Interest, expanding and neglecting r^2 and higher powers, we get

$$M = \frac{A}{r}\left\{1 + nr + \frac{n(n-1)}{1\cdot2}r^2 + \ldots - 1\right\}$$

$$= A\left\{n + \frac{n}{2}(n-1)r\right\}.$$

17. *To find the Present Value of an annuity, to be paid for a given number of years, at Compound Interest.*

I. The amount of the annuity at the end of the first year is A, while the present value is AR^{-1}; similarly, the amount at the end of the nth year is AR^{n-1}, and the present value is AR^{-n}. Hence, in order to obtain the present value from the amount, we must first multiply the formula for the amount by R, and then change the sign of the index of R.

$$M = A\cdot\frac{R^n - 1}{R-1}.$$

Multiplying by R we get

$$A\cdot\frac{R^{n+1} - R}{R - 1}$$

Changing sign of index we have

$$P = A\cdot\frac{R^{-(n+1)} - R^{-1}}{R^{-1} - 1}$$

$$= A\cdot\frac{R^{-n} - 1}{1 - R}$$

$$= \frac{A}{r}(1 - R^{-n}).$$

II. We may obtain the same result by proceeding on the principle that if the present value P be put out to compound interest for n years, it ought to amount to the same as the annuity for that time.

$$\text{Hence} \qquad PR^n = A\cdot\frac{R^n - 1}{R - 1},$$

$$\therefore P = A\cdot\frac{1 - R^{-n}}{R - 1}$$

$$= \frac{A}{r}\left(1 - R^{-n}\right)$$

III. We will now proceed on the principle that the present value P is the sum of the present values of the respective annual payments.

Present value of A due 1 year hence $= AR^{-1}$,
" " 2 " $= AR^{-2}$,
&c. $=$ &c.,
" " n " " $= AR^{-n}$;

$$\therefore P = AR^{-1} + AR^{-2} + AR^{-3} \ldots . AR^{-n}$$

$$= AR^{-1} \cdot \frac{R^{-n} - 1}{R^{-1} - 1}$$

$$= \frac{A}{r}\left(1 - R^{-n}\right).$$

18. *For Simple Interest, expanding and neglecting* r^2 *and higher powers, we get*

$$P = \frac{A}{r}\left\{1 - \frac{1}{R^n}\right\}$$

$$= \frac{A}{r} \cdot \frac{(1 + r)^n - 1}{(1 + r)^n}$$

$$= \frac{A}{r} \cdot \frac{1 + nr + \frac{n(n-1)}{1 \cdot 2} r^2 + \ldots . - 1}{1 + nr}$$

$$= A \cdot \frac{n + \frac{n}{2}(n - 1)r}{1 + nr}$$

$$= \frac{nA}{2} \cdot \frac{2 + (n - 1)r}{1 + nr}.$$

19. *To find the present value of a perpetual annuity.*

I. Reckoning Compound Interest.

$$P = AR^{-1} + AR^{-1} + \ldots . \textit{ ad infinitum.}$$

$$= \frac{AR^{-1}}{1 - R^{-1}}$$

$$= \frac{A}{R - 1}$$

$$= \frac{A}{r}$$

II. Reckoning Simple Interest.

$$P = \frac{nA}{2} \cdot \frac{2 + (n - 1)r}{1 + nr}$$

$$= \frac{nA}{2} \cdot \frac{\frac{2}{n} + (1 - \frac{1}{n})r}{\frac{1}{n} + r}$$

Now when $n = \infty$, the limit of

$$\frac{\frac{2}{n} + (1 - \frac{1}{n})r}{\frac{1}{n} + r} = \frac{0 + (1 - 0)r}{0 + r} = 1.$$

Hence, the limit of P, when $n = \infty = \frac{\infty A}{2} = \infty.$

This result shows that an *infinite sum* of money is required to be left, in order to insure an equal annual payment for ever, which is absurd. It indicates, therefore, that the only correct method of computing annuities is on the compound interest principle.

20. *To find the Present Value of an annuity, to commence at the end of* p *years, and then to continue* q *years.*

The present values of the first, second, &c., qth payments, due at the end of $p + 1$, &c., $p + q$ years, respectively, will evidently be

$$AR^{-(p+1)} \qquad AR^{-(p+2)} \qquad \&c. \qquad AR^{-(p+q)};$$

whence the present value

$$P = AR^{-(p+1)}. \{1 + R^{-1} + R^{-2} + \ldots + R^{-(q-1)}$$

$$= AR^{-(p+1)}. \left\{\frac{1 - R^{-q}}{1 - R^{-1}}\right\}$$

$$= \frac{A}{R^{p+q}} \cdot \left\{\frac{R^q - 1}{R - 1}\right\}.$$

If the annuity is payable *for ever* after p years have expired, by summing the above series *ad infinitum*, we have

$$P = \frac{A}{R^p (R - 1)}.$$

These formula enable us to compute the values of *Reversions*, or *Annuities in Reversion*; and the latter determines the value of the *Fee Simple* of the freehold estate, which is to fall in at the expiration of p years.

Ex. (1). A sum of $\$a$ is borrowed for a period of m years, to be repaid by equal annual instalments, the first payment to be made after one year. Find the amount of the annual instalment.

Let A be the annual instalment.

Then the amount of this annual payment in m years

$$= \frac{A}{r}\left\{R^m - 1\right\}.$$

Again, if the sum a be allowed to accumulate for m years at compound interest, its amount

$$= a\,R^m$$

Now these two amounts ought to be equal.

Hence we have

$$\frac{A}{r}\left\{R^m - 1\right\} = a\,R^m;$$

$$\therefore A = \frac{a}{r}\cdot\frac{R^m}{R^m - 1}$$

$$= \frac{a}{r}\cdot\frac{1}{1 - R^{-m}}$$

Ex. (2). The present value of an annuity of $1, to continue q years, is $10; and the present value of an annuity of $1, to continue $2q$ years, is $16. Find the rate of interest.

Here, $$10 = \frac{1}{r}(1 - R^{-q}), \text{ Art. 17,}$$

and $$16 = \frac{1}{r}(1 - R^{-2q});$$

$$\therefore \frac{16}{10} = \frac{1 - R^{-2q}}{1 - R^{-q}}$$

$$= \frac{(1 - R^{-q})(1 + R^{-q})}{1 - R^{-q}}$$

$$= 1 + R^{-q};$$

or $$2 - \frac{16}{10} = 1 - R^{-q};$$

$$\therefore 1 - R^{-q} = \frac{2}{5}.$$

Subtituting in the first equation, we get

$$\frac{2}{5} \cdot \frac{1}{r} = 10;$$

$$\therefore r = \frac{1}{25},$$

$$\text{or } 100\,r = 4.$$

The rate is, therefore, 4 per cent.

Ex. 3. A mortgage of $5,000, interest at 6 per cent. per annum, has 7 years and 10 months to run; find its present value, interest at 10 per cent. per annum, payable half-yearly.

The first payment of interest is $300, and will be due in 10 months; its amount for seven years, at 10 per cent., payable half-yearly, will be $300(1{\cdot}05)^{14}$. Similarly, the amount of the second payment of interest at the end of the 7 years, will be $300(1{\cdot}05)^{12}$; and so on. The amount of the last payment will be $300.

Hence, the whole amount of the mortgage and interest will be

$$5000 + 300\,(1{\cdot}05)^{14} + 300(1{\cdot}05)^{12} + \ldots\ldots\ldots + 300.$$

$$= 5000 + 300\,\{(1{\cdot}04)^{14} + (1.05)^{12} + \ldots\ldots\ldots\ldots + 1\}$$

$$= 5000 + 300\left\{\frac{(1{\cdot}05)^{16} - 1}{(1{\cdot}05)^{2} - 1}\right\}$$

$$= 8462{\cdot}06.$$

Now, if the present value, P, be put out to compound interest at 10 per cent. per annum for 7 years and 10 mos., it ought to amount to the same as the mortgage for that time.

$$\therefore P\,(1{\cdot}05)^{15\frac{2}{3}} = 8462{\cdot}06$$

$$\therefore P = 3940.13.$$

The Present Value of the Mortgage is, therefore, $3940.13.

The value of $(1{\cdot}05)^{15\frac{2}{3}}$ may be found

(1) By means of a table of logarithms.

(2) By raising 1·05 to the 16th power, dividing this by 1·05 we obtain the 15th power, taking $\frac{2}{3}$ of the difference and adding to the 15th power, we get approximately the $15\frac{2}{3}$ power of 1·05.

(3) By the Binomial Theorem, as follows:

$(1\cdot05)^{\frac{1}{3}} = (1 + \frac{1}{20})^{\frac{1}{3}} = 1 + \frac{1}{60} - \frac{1}{3600} = 1\cdot0163$; then $(1\cdot05)^{16} \div 1\cdot0163$ will give a close approximation to $(1\cdot05)^{15\frac{2}{3}}$.

For additional information on this subject consult Loan Tables by Professors Cherriman and London.

Exercise III.

(1) A person's dividend from his Bank stock is \$530 a year. What is the present value of this income for five years to come, computing by simple, and also by compound interest, at 7 per cent.

(2) What annuity, to continue 20 years, can be purchased for \$10000, allowing compound interest at 5 per cent.

(3) For what sum might the Government of a country undertake to pay an annuity of \$1000 a year, for ever, on the supposition that money may always be invested at 6 per cent.

(4) For what sum might an annuity of \$400 a year, for ten years, to commence in 5 years, be purchased, allowing compound interest at 6 per cent.?

(5) A person who enjoyed a perpetuity of \$1000 per annum, provided in his will that, after his decease, it should descend to his only son for 10 years, to his only daughter for the next 20 years, and to a benevolent institution for ever afterwards. What was the value of each bequest at the time of his decease, allowing compound interest at 6 per cent.?

(6) A person at the age of 22 put \$100 at interest, at 6 per cent., and \$100 each year afterwards, until he was 40 years old. He also collected the interest annually, and converted the some into *principal;* what amount was, by these means, accumulated?

(7) A corporation borrows £3769 at 4 per cent., to be paid in 30 years by equal annual instalments. What will be the annual payment?

(8) A property is let out on lease for a years at an annual rental of \$$b$, and after c years the lease is renewed on paying a fine of \$$d$. What is the additional rent equivalent to this fine?

(9) A farm is let for n years at a fixed rent and a fine of \$$p$. When p years of the lease remain, what fine must be paid to extend these p years to q, at compound interest?

(10) If two joint proprietors have an equal interest in a freehold estate worth \$$a$ per annum, but one of them purchased the whole to himself by allowing the other an equivalent annuity of \$$b$ for n years, find the relation between a and b.

(11) Find the present value of an annuity of \$1, paid n times per annum, and continuing for m years, allowing compound interest at the rate of q per cent. per annum; and prove that, as n is indefinitely increased, this present value continually approaches the limit $\dfrac{1 - e^{-mq}}{q}$.

(12) A monthly instalment of \$10 has 2 years 1 month to run, what sums must be paid at once to reduce the period six months, money being worth one-half per cent. per month?

(13) A mortgage of \$4000, interest at 5 per cent. per annum, payable half-yearly, has 17 years and 8 months to run. Find its present value, interest 10 per cent. per annum, payable half-yearly.

(14) If two sums, s_1, s_2, due at times t_1, t_2, be paid together at an intermediate time t, t being determined from the equation

$$s_1 R^{-t_1} + s_2 R^{-t_2} = (s_1 + s_2) R^{-t}.$$

Show that whichever mode of payment be adopted

(1) At any antecedent period, the present values are the same;

(2) At any subsequent period, the amounts are the same;

(3) At the intermediate time of payment, the interest of the sum overdue is the discount of that not due.

* * *

APPENDIX II.

CUBE ROOT.

The extraction of the cube root, by the ordinary rule, is a troublesome process, seldom used and easily forgotten. The following process is much simpler and more easily remembered.

Let a be an approximate value of the cube root of N, so that $\sqrt[3]{N} = a + x$, x being very small ;
then $N = (a + x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$
$= a^3 + 3ax(a + x)$, nearly, since x is small.

First suppose $N = a^3 + 3a^2x$;

$$\therefore x = \frac{N - a^3}{3a^2};$$

$$\text{and} \quad \therefore a + x = \frac{N + 2a^3}{3a^2};$$

$$\text{therefore, more nearly, } N = a^3 + 3ax \cdot \frac{N + 2a^3}{3a^2},$$

$$\text{and} \qquad x = \frac{N - a^3}{N + 2a^3}a,$$

$$\text{and, therefore, } \sqrt[3]{N} = a + x$$

$$= \frac{2N + a^3}{N + 2a^3} \cdot a.$$

Suppose we wanted to find the cube root of any number N. In the first place we find some number a whose cube is somewhere near the given number. Then the fraction,

$$\frac{2N + a^3}{N + 2a^3} \cdot a$$

will be a nearer approximation to the cube root than a itself was. When we have found this value, we can take this as a and repeat the process.

Thus, to find the cube root of 241·804367, we observe that 216, the cube root of 6, is nearest to 241. Hence the first value of a is 6.

Therefore,
$$\begin{aligned}&\frac{2\mathrm{N} + a^3}{\mathrm{N} + 2a^3}\cdot a\\ &= \frac{699\cdot608734}{673\cdot804367} \times 6\\ &= \frac{4197\cdot636404}{673\cdot804367}\\ &= 6\cdot23, \text{ very nearly.}\end{aligned}$$

On trying 6·23, we find it is correct.

Ex. (1). Find the Cube Root of 47.

The nearest cube to 47 is that of 4.

Hence,
$$\begin{aligned}&\frac{2\mathrm{N} + a^3}{\mathrm{N} + 2a^3}\cdot a\\ &= \frac{94 + 64}{47 + 128} \times 4\\ &= \frac{632}{175}\\ &= \frac{2528}{700}\\ &= 3\cdot61, \text{ nearly.}\end{aligned}$$

Next, take 3·61 for a, and substitute in the formula, and we get 3·6088261, which is correct to seven places of decimals.

Ex. (2). Find the Cube Root of 10.

In this case,
$$\begin{aligned}&\frac{2\mathrm{N} + a^3}{\mathrm{N} + 2a^3}\cdot a\\ &= \frac{20 + 8}{10 + 18} \times 2\\ &= 2\cdot153.\end{aligned}$$

Next, substitute 2·15 instead of 2, and we get
$$\begin{aligned}&\frac{20 + 9\cdot938375}{10 + 19\cdot876750} \times 2\cdot15\\ &= 2\cdot1544346,\end{aligned}$$
which is correct as far as six places of decimals. This method has also the practical advantage that an error of work gets corrected at the next trial.

ANSWERS.

EXERCISE I.

(3) $\frac{P^2}{M}$ (5) 22·5 years, nearly.

(6) *A*'s share is $\frac{PR^{b+c}}{R^{a+b} + R^{a+c} + R^{b+c}}$.

(7) 5 per cent. (8) $\frac{\log.\ 2}{\log.\ (mn - m + n) - \log.\ mn}$.

(10) $n = 20\frac{\log.\ 6}{\log.\ e}$. (11) 17·67 years.

(12) 6708·471. (13) 125 years, nearly. (14) $3600.

EXERCISE II.

(1) \$706.66$\frac{1}{3}$. (2) $13585. (3) $19.50.

(4) $900. (5) $100\frac{B - A}{A}$ per cent.

(6) $3\frac{37}{71}$. (7) $aR^{p-m} - bR^{p-n}$.

(8) $\frac{Aa}{a + r}$. (9) 11·463 per cent. (10) $1\frac{1}{2}$ years.

EXERCISE III.

(1) At simple interest, $2237.77 ; at compound interest, $2173.10. (2) $802.42.

(3) \$16666.66$\frac{2}{3}$. (4) $2199.95.

(5) $7360.08 ; $6404.74 ; $2901.83. (6) $3000.56.

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